# The 3D Spin Axis: An Addendum

Alan M. Nathan*

*Department of Physics, University of Illinois, Urbana, IL 61801*

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## I. INTRODUCTION: WHAT PROBLEM ARE WE TRYING TO SOLVE?

A spinning baseball is deflected from its trajectory due to the Magnus force. The resulting acceleration is given by

$$\vec{a}_M = KC_L v^2 \frac{\hat{\omega} \times \hat{v}}{|\hat{\omega} \times \hat{v}|}, \tag{1}$$

where $C_L$ is called the lift coefficient, $\hat{\omega}$ is the direction of the spin axis, and $\hat{v}$ is the direction of the velocity. The factor $K$ is given by

$$K = \frac{1}{2}\frac{\rho A}{m}, \tag{2}$$

where $m$ and $A$ are the mass and cross sectional area of the ball, respectively, and $\rho$ is the density of the air. The direction of the Magnus force is given by the vector cross product $(\hat{\omega} \times \hat{v})$, which is perpendicular to both the velocity vector and the spin axis.

The Magnus force, and therefore the lift coefficient, is expected to depend on the so-called spin factor $S$, defined as

$$S = \frac{R\omega}{v}, \tag{3}$$

where $R$ is the radius of the ball and $\omega$ is the total spin rate. Moreover, it is expected to depend not on the total spin rate $\omega$ but rather on the "transverse" or "active" spin rate $\omega_T$, which is the component of $\omega$ perpendicular to the velocity:

$$\omega_T = \omega \sin\theta, \tag{4}$$

where $\theta$ is the angle between the spin direction $\hat{\omega}$ and the velocity direction $\hat{v}$:

$$\sin\theta = |\hat{\omega} \times \hat{v}|. \tag{5}$$

The quantity $\sin\theta$ is popularly referred to as the "spin efficiency" or "active spin ratio". When the spin axis is perpendicular to the velocity direction, then $\sin\theta = 1$, the spin is purely "transverse", and $C_L$ is maximized. When the spin axis is along or opposite to the velocity direction, then the spin is purely "gyrospin" and $C_L = 0$. Finally, for a given $\theta$, $C_L$ is expected to be a monotonically increasing function (actually, non-decreasing) function of $S$.

Given these constraints, one possible way to express the dependence of $C_L$ on spin is as follows:

$$C_{L,a} = f(S)\sin\theta \qquad \text{prescription}\, a\,, \tag{6}$$

where $f(S)$ is a smooth monotonic function of $S$ (Eq. 3) that vanishes when $S = 0$ (i.e., when the total spin is zero). An alternate way is as follows:

$$C_{L,b} = f(S\sin\theta) \qquad \text{prescription}\, b\,. \tag{7}$$

These two expressions, $C_{L,a}$ (prescription $a$) and $C_{L,b}$ (prescription $b$), are identical for the two extreme cases of purely gyrospin ($\sin\theta$=0), in which case $C_L = 0$, and purely transverse spin ($\sin\theta$=1), in which case $C_L$ is maximized. However, between these two extremes the expressions are not identical leading to the obvious question: Which one is correct?

That is the question I pose but do not answer in this brief article. I will start by examining what we know from experimental data about the function $f(S)$. I will then show how the ambiguity raised by the differences between prescriptions $a$ and $b$ are manifest in both Trackman and Rapsodo data. Finally, I will discuss the opportunity presented by Hawkeye data to resolve the ambiguity.

## II. WHAT DO WE KNOW ABOUT $f(S)$?

We actually know quite a bit about $f(S)$ from laboratory experiments. Before turning to these, I found it helpful to consider a special case whereby $f(S)$ is a linear function of $S$ with zero intercept: $f(S)$=$kS$. In this case, $C_{L,a}$ and $C_{L,b}$ are both equal to $kS\sin\theta$ and are indistinguishable from each other. However, life is never that simple and, as we will now see, $f(S)$ is highly nonlinear.

One way to determine $f(S)$ is from laboratory experiments using purely transverse spin, $\sin\theta$=1 to measure the relationship between $C_L$ and $S$. The data from two such experiments are shown in Fig. 1, along with the function $f(S)$ that was adjusted to fit the data:

$$f(S) \;=\; A\,[1-\exp(-BS)]\;, \tag{8}$$

with the numerical values $A$=0.336 and $B$=6.041. Note that $f(S)$ satisfies our critera that it vanishes when $S$=0 and is monotonically non-decreasing.

Having determined $f(S)$, we can now rewrite Eqs. 6-7 as follows:

$$C_{L,a} \;=\; A\,[1-\exp(-BS)]\sin\theta \qquad \text{prescription}\,a\,, \tag{9}$$

and

$$C_{L,b} \;=\; A\,[1-\exp(-BS\sin\theta)] \qquad \text{prescription}\,b\,. \tag{10}$$

Given the clearly nonlinear behavior of $f(S)$, we expect the two prescriptions to produce different results.

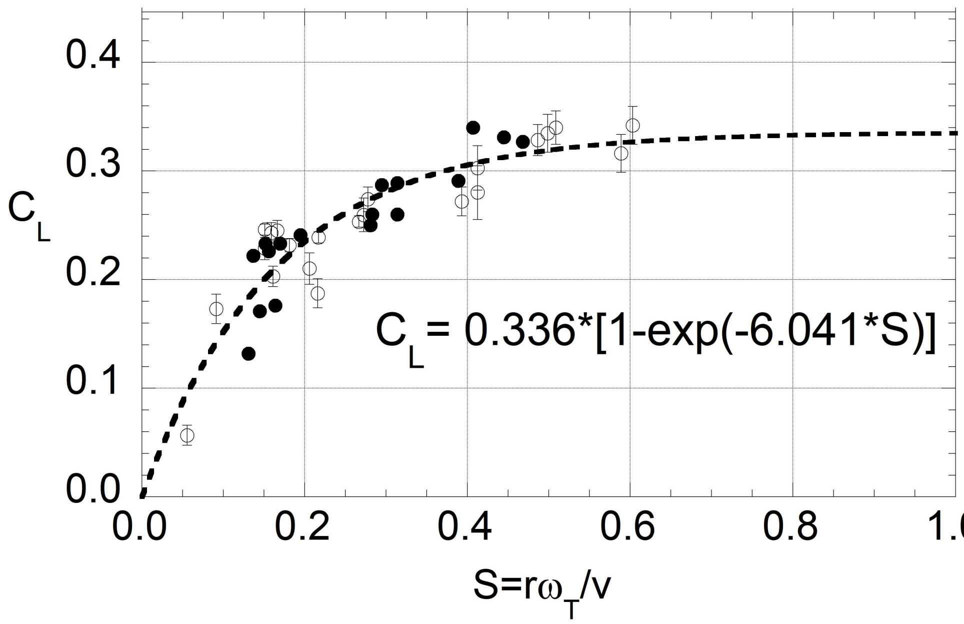


FIG. 1: Experimental values of $C_L$ as a function of $S$ for the special case purely transverse spin, $\sin\theta$=1. The data come from motion capture experiments taken at speeds in the range 80-100 mph, including data from Alaways[1,2] (closed points) and Nathan (open points).[3] The data were parametrized by the function $C_L = f(S) = A(1-e^{-BS})$, with $A$ and $B$ determined from a non-linear least-squares fit (black curve), with $A$=0.336 and $B$=6.041.

As an extreme example, consider the case of high $S$, on the asymptotic part of the $f(S)$ curve, so that a further increase in $S$ produces very little change in $f(S)$. As a specific example, suppose $S$=0.6 and the spin is purely transverse ($sin\theta$=1), in which case $C_L$=0.327. Now suppose the pitcher adds an equal amount of gyrospin, keeping the transverse spin the same, in which case $S$ will increase by a factor $\sqrt{2}$, $\sin\theta$ will decrease by a factor $\sqrt{2}$, keeping $S\sin\theta$ unchanged. Under prescription $a$, $C_L$ will decrease to 0.236 while under prescription $b$ it will increase slightly to 0.334. Admittedly this is not a particularly realistic scenario, since it is very difficult for a pitcher to achieve $S$=0.6. Nevertheless, it does vividly demonstrate the ambiguity, as the more realistic examples below will further demonstrate.

## III. WHAT CAN WE LEARN FROM PITCH-TRACKING DATA?

### A. Trackman

Trackman, the primary pitch-tracking tool used by MLB during the 2015-2019 seasons, measures the full trajectory of each pitch and the total spin $\omega$, from which both $C_L$ and $S$ can be determined.[4] The goal is to determine the spin efficiency $\sin\theta$. I proceed with a numerical example.

Suppose $C_L$=0.20 and $S$=0.24, the latter the value found for a total spin rate of 2400 rpm and speed of 90 mph. From Eq. 8, we find $f(S)$=0.257. We then solve Eqs. 9-10 to find $\sin\theta$. Skipping all the algebra, we obtain

- prescription $a$: $\sin\theta$=0.778
- prescription $b$: $\sin\theta$=0.625

Quite obviously, these two values for the spin efficiency are very different. Which is correct? Fig. 2 shows a simple geometric interpretation of the two prescriptions.

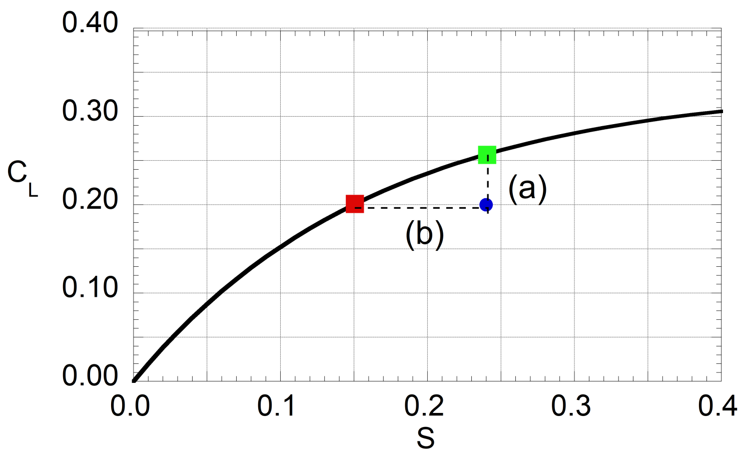


FIG. 2: Example showing the ambiguities between prescriptions $a$ and $b$. The blue dot shows the values of $C_L$=0.20 and $S$=0.24 while the curve is $f(S)$, representing the maximum value of $C_L$ for a given $S$. The ratio of $C_L$ values of the blue dot to the green square is the spin efficiency from prescription $a$. The ratio of $S$ values of the red square to the blue dot is the spin efficiency from prescription $b$. The two prescriptions are clearly different.

### B. Rapsodo

Rapsodo is a pitch-tracking system that uses a combination of radar to measure the relesase speed and high-speed video to measure the spin rate, the spin axis, and the speed and direction. The combination of these determines the spin efficiency $\sin\theta$ and $S$. The goal is to determine $C_L$. As a numerical example, suppose $S$=0.24, as in the preceding example, and $\sin\theta$=0.625. In this case, we again use Eqs. 9-10, using the measure values of $S$ and $\theta$ to solve for $C_L$:

- prescription $a$:: $C_{L,a}$=0.161
- prescription $b$:: $C_{L,b}$=0.200

Once again, these two values are very different. Which is correct?

### C. Hawkeye

To my knowledge, no experiment has yet been done that distinguishes between the two prescriptions. The laboratory experiments have mostly been done with purely transverse spin or at sufficiently low $S$ and limited precision so that $f(S)$ is approximately linear. As discussed above, under such conditions the two prescriptions are identical. Trackman measures $C_L$ and $S$ but not $\sin\theta$. Rapsodo measure $S$ and $\sin\theta$ but not $C_L$. So we have a dilemma. One way to resolve the dilemma is in an experiment in which Trackman and Rapsodo are used simultaneousely. Another way is with Hawkeye.

Hawkeye is the optically-based ball-tracking system used by MLB starting in the 2020 season. Hawkeye measures the pitch trajectory, from which $C_L$ can be determined. Using high-speed video, Hawkeye also measures both the spin rate and the spin axis, from which both $S$ and $\sin\theta$ can be determined. The combination of all these things can then be used to figure out whether prescription $a$ or $b$ is the correct relationship connecting $C_L$ to $S$ and $\sin\theta$. Hopefully all the necessary data will soon be publicly available.

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* Electronic address: a-nathan@illinois.edu

[1] L. W. Alaways, "Aerodynamics of the Curve Ball: An Investigation of the Effects of Angular Velocity on Baseball Trajectories," Ph.D. thesis, University of California, Davis (1998).

[2] L. W. Alaways and M. Hubbard, "Experimental determination of baseball spin and lift," J. Sports Sci. **19**, 349-358 (2001).

[3] See a summary in A. M. Nathan, Am. J. Phys. **76**, 119-124 (2008). A copy of the paper can be downloaded at `http://baseball.physics.illinois.edu/AJPFeb08.pdf`. The data shown in Fig. 5 of the article have been scaled up by 3% from the published values based on a reanalysis of the air density and ball diameter.

[4] Alan M. Nathan, "Determining the 3D Spin Axis from Statcast Data, May 2020 update", `http://baseball.physics.illinois.edu/trackman/SpinAxis.pdf`.