The Effect of Spin-Down on the Flight of a Baseball

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A heuristic model is presented for the spin-down rate of a baseball in flight. The model is compared to data for a golf ball. Implications for the trajectory of a baseball are explored.

I. THE MODEL OF ADAIR

The model is essentially that described by R. K. Adair in his book[1]. We start with the expression for the magnitude of the lift force on a spinning baseball[2],

$$F_L = \frac{1}{2}\rho A C_L v^2 \,, \tag{1}$$

where ρ is the density of air, A is the cross sectional area of the ball, v is the speed of the ball, and C_L is the so-called lift coefficient. We suppose that the line of action of F_L passes a perpendicular distance kR from the center of the ball, where R is the ball radius and $0 \leqslant k \leqslant 1$ is a dimensionless constant that we call the "torque parameter." Therefore, F_L gives rise to a torque of magnitude kRF_L which slows down the spin ω:

$$I\frac{d\omega}{dt} = -\frac{1}{2}kR\rho A C_L v^2 \,, \tag{2}$$

where $I = 0.4MR^2$ is the moment of inertia of the ball. Using $M = 0.145$ kg, $R = 0.0364$ m, and $\rho = 1.27$ kg/m^2, we arrive at the expression

$$\frac{d\omega}{dt} = -0.250kv^2 C_L \,, \tag{3}$$

with v in mph and ω in rad/s. Defining the spin parameter $S = R\omega/v$ and using the approximate expression $C_L \approx S$, we find

$$\text{baseball :} \qquad \frac{d\omega}{dt} = -0.020kv\omega \,, \tag{4}$$

so that the time constant for exponential decay is 50/kv s. As a numerical example, if the torque parameter is 0.1, then the spin-down time constant is 5 s at the fixed speed v=100 mph.

II. MODELS AND DATA FOR GOLF BALLS

A. The Model of Smits and Smith

Smits and Smith[3] have reported wind-tunnel measurements of the lift and drag coefficients and the spin-down rate on a golf ball (mass=0.04593 kg, radius=0.02134 m). Their data, shown in Fig. 1, demonstrate that the parameter $\dot{\omega}R^2/v^2$ is an approximately linear function of S, and independent of Reynold's number (for fixed S) in the range (1.0-2.5)x10^5. Numerically, the spin-down rate is given by

$$\text{golf ball (Smits)}: \qquad \frac{d\omega}{dt} = -4.0 \times 10^{-6} \frac{v^2}{R^2} S\,, \tag{5}$$

with v in mph[5]. For $v = 100$ mph, $\dot{\omega} = \omega/23.8$, implying a spin-down time constant of 23.8 s.

Note that $\dot{\omega}$ scales with v^2S/R^2 in the Smits model and with v^2RC_L/M in the model described in Sec. I. The two models would therefore appear to be different. However, it should be noted that M scales with R^3 and C_L scales with S, so the scaling of $\dot{\omega}$ with R, v, and S is essentially identical. In effect, the Smits model provides empirical evidence for the more physically based model of Sec. I. It is useful to apply the latter model to the golf ball, then use the Smits data to fix the torque parameter k. Putting in the mass and radius appropriate to a golf ball and assuming that $C_L \approx S$ for a golf ball, an expression identical to Eq. 4 can be derived with the numerical factor equal to 0.0215. This means that for comparable v and k, the time constant for spin decay for a baseball will about 8% larger than that of a golf ball . Using the golf data, we fix the value $k = 0.020$, a factor of 5 smaller than that hypothesized by Adair[1], corresponding to a factor of 4 larger spin decay time constant.

B. The model of Tavares

Tavares *et al.*[4] have proposed a model for spin decay in which the torque responsible for the spin decay is parametrized as

$$I\frac{d\omega}{dt} = -R\rho AC_M v^2\,, \tag{6}$$

where C_M is the so-called coefficient of moment. The spin decay measurements of Tavares, which utilizes a novel radar gun to measure the time-dependent spin, show that $C_M \approx 0.012S$. Using $I = 0.4MR^2$ and the values of M and R appropriate to a golf ball, Tavares' result can be expressed as

$$\text{golf ball (Tavares)}: \qquad \frac{d\omega}{dt} = -5.0 \times 10^{-6} \frac{v^2}{R^2} S \,, \tag{7}$$

with v in mph. This equation is identical in form to Eq. 5 with a numerical factor 25% larger. For example, the spin-down time constant for $v = 100$ mph will be 18.9 sec.

More generally, if $I{=}\alpha MR^2$ and if $C_M = \beta S$,[6] then one rearrange Eq. 6 to derive an expression for the spin decay time constant τ

$$\tau \equiv \frac{\omega}{\dot{\omega}} = \left[\frac{M}{R^2}\right] \frac{\alpha}{\pi\rho\beta v} \,. \tag{8}$$

Therefore for a given v and fixed values of α and β, the spin decay time constant scales with M/R^2, allowing a comparison among different spherical balls. For example, a golf ball and baseball have M/R^2= 101.2 kg/m^2 and 109.4 kg/m^2, respectively, so that the time constant for a baseball will be about 8% larger than for a golf ball, as we found earlier.

III. NUMERICAL CALCULATIONS OF THE TRAJECTORY

We investigate the trajectories of hit baseballs for two different initial conditions: one appropriate for a long fly ball and another appropriate for a popup. All calculations utilize the parameterizations of lift and drag coefficients given by Sawicki *et al.*[2] For the fly ball, we assume the ball leaves the bat at a height of 3 ft, a speed of 100 mph, a takeoff angle of 30°, and backspin of 2000 rpm. In Fig. 2, we show the calculated trajectory and spin for values of k equal to 0 (i.e., no spin-down), 0.02 (twice the value taken from the golf measurements), and 0.1. For the popup, we assume the ball leaves the bat at a height of 3 ft, a speed of 75 mph, a takeoff angle of 70°, and backspin of 5000 rpm. In Fig. 3, we show the calculated trajectory and spin for values of k equal to 0 (i.e., no spin-down), 0.02 (the value taken from the golf measurements), and 0.1 (the value estimated by Adair). For both the fly ball and popup, the calculations with k=0 and 0.02 are barely distinguishable and differ slightly from that with $k = 0.1$. Given that the latter value is almost surely unrealistically large, we conclude that the spin decay plays only a minor role in the trajectory of a hit baseball.

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1 R. K. Adair, *The Physics of Baseball* (HarperCollins, New York, 2002) 3rd ed., pp 25-26.

2 G. S. Sawicki, M. Hubbard, and W. Stronge, "How to hit home runs: Optimum baseball bat swing parameters for maximum range trajectories," Am. J. Phys. **71**, 1152-1162 (2003).

3 A. J. Smits and D. R. Smith, "A new aerodynamic model of a golf ball in flight," Science and Golf II, Proceedings of the 1994 World Scientific Congress on Golf, edited by A. J. Cochran and M. R. Farraly(E&FN Spon., London, 1994), pp. 340-347.

4 G. Tavares, K. Shannon, and T. Melvin, "Golf ball spin decay model based on radar measurements," Science and Golf III, Proceedings of the 1998 World Scientific Congress on Golf, edited by M. R. Farraly and A. J. Cochran(Human Kinetics, Champaign IL, 1999), pp. 464-472.

5 The numerical factor is 2.0×10^{-5} if v is in m/s

6 Note that β=0.0096 or 0.012 for the Smits and Tavares measurements, respectively

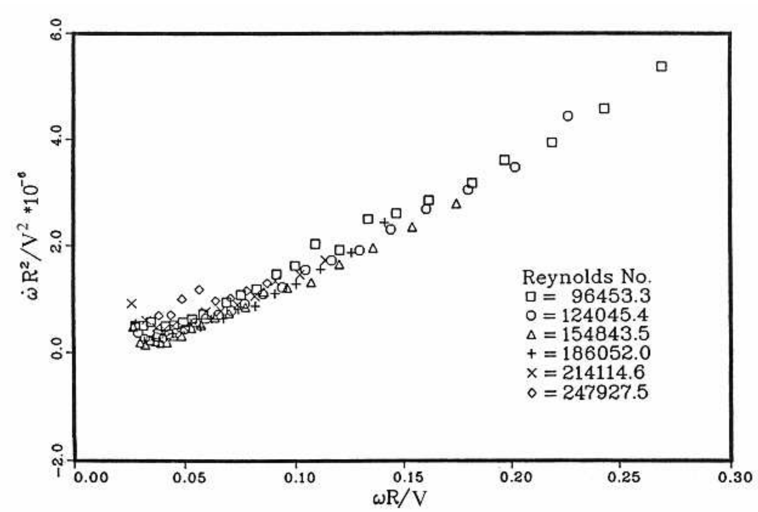

FIG. 1: Measurements reported by Smits and Smith[3] of the spin-down rate of a golf ball.

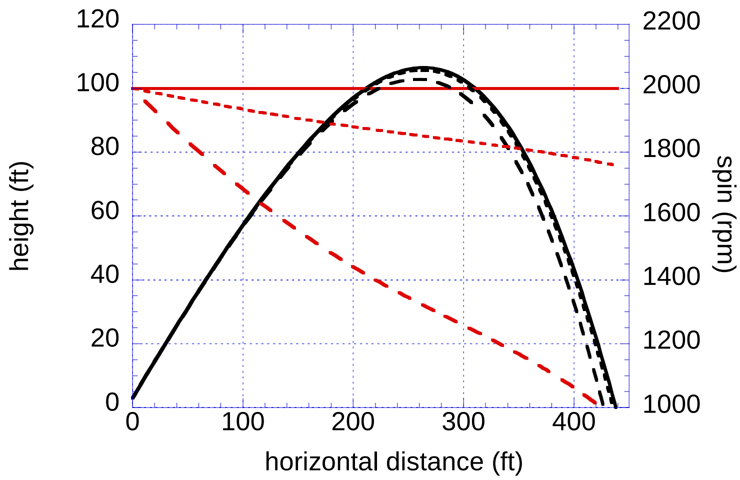

FIG. 2: Calculated trajectory (black) and spin (red) of a long fly ball, with initial parameters given in the text. The torque parameter k is 0 (solid), 0.02 (short dashed), and 0.1 (long dashed).

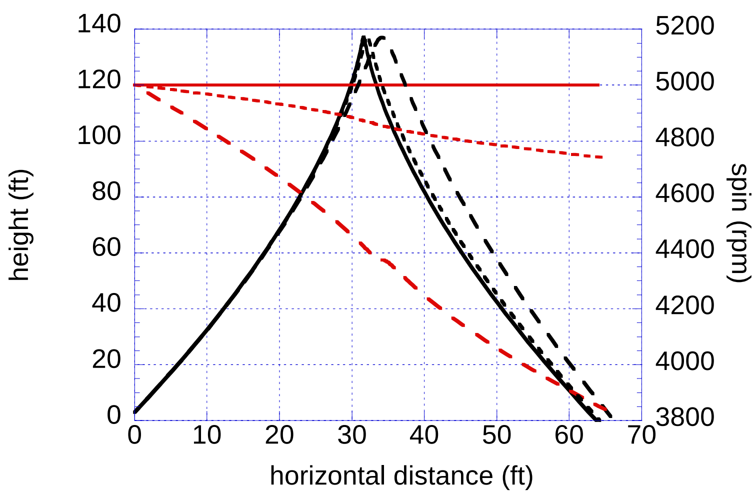

FIG. 3: Calculated trajectory (black) and spin (red) of a popup, with initial parameters given in the text. The torque parameter k is 0 (solid), 0.02 (short dashed), and 0.1 (long dashed).