[6]J. L. Synge and B. A. Griffith, *Principles of Mechanics* (McGraw-Hill, New York, 1949), pp. 154–157.
[7]S. Timoshenko and D. H. Young, *Advanced Dynamics* (McGraw-Hill, New York, 1948), pp. 94–103.
[8]E. J. Routh, *A Treatise on Dynamics of a Particle* (Dover, New York, 1960), pp. 95–99.
[9]W. D. MacMillan, *Statics and the Dynamics of a Particle* (Dover, New York, 1958), pp. 254–258.
[10]E. T. Whittaker, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies* (Cambridge U.P., London, 1961), pp. 229–230.
[11]H. Cabannes, *General Mechanics* (Blaisdell, Waltham, 1968), pp. 233–234.
[12]cf. R. A. Becker, *Introduction to Theoretical Mechanics* (McGraw-Hill, New York, 1951), p. 392.
[13]See, for example, P. A. Stark, *Introduction to Numerical Methods* (MacMillan, New York, 1970), pp. 198–202.

The lateral force on a spinning sphere: Aerodynamics of a curveball

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The lateral force on a spinning baseball in a wind tunnel has been measured. The magnitude of the force is nearly independent of the orientation of the seams of the ball. The drag coefficient appears to be at most weakly dependent on Reynolds number and to be principally a function of the ratio of the rotational speed of the equator of the ball to the wind tunnel speed. This is to be compared to the work of Briggs, which implies a strong effect of Reynolds number on the drag coefficient.

I. INTRODUCTION

The fact that tennis balls curve because of the spin imparted to them was noted as early as 1671 by Sir Isaac Newton.[1] Two hundred years later, Lord Rayleigh,[2] also in a paper describing the irregular flight of the tennis ball, credited the German engineer G. Magnus with the first explanation of the lateral deflection of a spinning ball. From this the phenomenon derives its name: the Magnus effect. It has been pointed out by Barkla and Auchterlonie[3] that Magnus' explanation was not published until 1853, more than a century after a similar explanation had been given by B. Robins in his book *New Principles of Gunnery*[4] published in 1742. Both Magnus and Robins were interested in the trajectories of cannon and musket balls. Robins experimented with the effect of spin on the curvature of the paths of musket balls by firing them from a gun with slightly curved barrel. With the barrel bent to the left, the ball was forced into contact with the right side of the bore, thus imparting spin on the ball in the clockwise direction when viewed from above. The paths of the musket balls fired in this way were curved to the right.

The explanations given by Robins and Magnus were, in fact, not entirely correct. Their arguments went as follows: A spinning ball induces in the air around it a kind of whirlpool of air in addition to the motion of the air past the ball as the ball flies through the air. This circulating air slows down the flow of air past the ball on one side and speeds it up on the other. In accordance with Bernoulli's theorem, when the kinetic energy of a fluid increases, its pressure decreases. Thus, the side of the ball on which the air speed is lower experiences a higher pressure than the other side. The resulting pressure (and force) imbalance causes the ball to move laterally toward the low-pressure (high-speed) side.

What actually happens is somewhat more complicated than this. As pointed out by Briggs,[5] boundary layer separation is apparently delayed on the side of a spinning ball that is moving in the same direction as the free stream flow of air, while separation occurs prematurely on the side moving against the free stream flow. The wake region of the ball therefore shifts toward the side moving against the free stream flow, deflecting the flow past the ball in that direction. The resulting change in momentum causes a force on the ball in the opposite direction.

The most systematic experimental determination of the forces acting on a spinning baseball that was reported in useful detail in the scientific literature was conducted by Lyman Briggs.[5] A spinning baseball was dropped across a horizontal wind tunnel in which the velocity of the air was known, and the deflection of the ball's path caused by the spin was measured. The ball was mounted at the lower end of a shaft at the top of the wind tunnel, held in place by a suction cup device. It was initially shielded from the wind by a small hollow cylinder. By rotation the shaft using a small electrical motor, the ball could be spun with the axis of rotation in the vertical direction. The rate of rotation was measured with a strobotac. With the ball spinning at a known rate, the suction acting through the suction cup device was turned off, so that the ball fell out of its protective shield and dropped a distance of six feet across the wind tunnel. The air in the tunnel pushed the ball downstream, but, presumably because of the spin on the ball, it was also deflected to one side or the other, depending on the direction of rotation.

Using the measured lateral deflections reported by Briggs, it is a simple matter to calculate the necessary lateral forces. The results are shown in Fig. 1. Briggs reported that the lateral force is proportional to the product of the square of the wind tunnel speed (V) and the rotation rate of the ball (ω).

According to Joseph F. Drury,[6] aerodynamicist Igor Si-

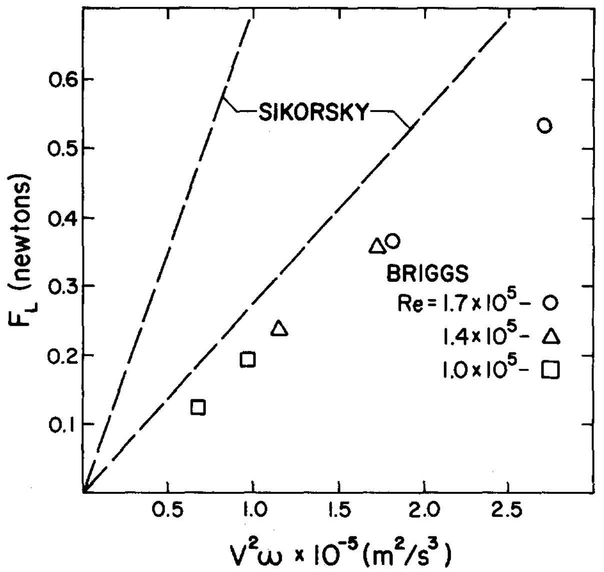

Fig. 1. The data of Briggs and Drury's description of the Sikorsky data.

korsky measured the force on a spinning baseball by placing spinning baseballs inside a wind tunnel at air speeds between 80 and 110 mph. By supporting the balls on a slender spike connected to the shaft of a small motor mounted on a balance, the ball could be rotated and forces on the ball measured. The details of what Sikorsky found are somewhat obscured by the fact that they were apparently never reported in the scientific literature. The article cited above, however, states that Sikorsky found that the deflection (d) of a spinning baseball on its path toward home plate is directly proportional to the rotation rate (ω) and to the square of the velocity (V) of the ball times the time (t) required to reach home plate, and inversely proportional to the mass (m) of the ball;

$$d = K\, V^2 t^2\, \omega / m. \tag{1}$$

Drury also reported that Sikorsky had found that a baseball thrown at a speed of 80 mph with a rotation rate of 600 rpm would be deflected by 7.5 to 19 in. *depending on the orientation of the seams relative to the axis of rotation of the baseball* (emphasis ours). From this it can easily be determined that $1.37\times 10^{-6} < K < 3.48\times 10^{-6}$ (s^3/m), depending on the orientation of the seams of the baseball.

If a constant (lateral) force F_L is applied in a direction perpendicular to the forward motion of the baseball it can be shown using elementary physics that the ball will be deflected in the direction of the force by a distance

$$d = F_L\, t^2 / 2\, m. \tag{2}$$

(This formula is derived under the assumption that the displacement d is much smaller than the total distance traveled by the ball. The actual trajectory of the ball when acted upon by a single force perpendicular to V is a circle.) By combining Eqs. (1) and (2) it is easily shown that the force on the ball in the Sikorsky experiment must have been:

$$F_L = 2\, K\, V^2\, \omega. \tag{3}$$

Thus, the lateral force acting on a rotating baseball was found to be directly proportional to the rotation rate and to the square of the translational velocity, in agreement with Briggs' conclusion.

The lateral force on a spinning baseball based on Drury's description of the Sikorsky results is shown in Fig. 1 along with the data of Briggs. Two lines are shown representing the Sikorsky results, one based on the maximum force, i.e., the largest value of K, and one based on the minimum force. Although both Briggs and Sikorsky (as reported by Drury) indicated that the lateral force is proportional to $V^2\omega$, the prediction based on the Sikorsky experiment overestimates the Briggs data by a large amount, even when the seams of the baseball are placed so that the force is at its minimum.

Apart from the obvious differences in the magnitudes of the forces implied by the Briggs data and the results of Sikorsky (as reported by Drury), a new question is raised because of the possible dependence of the force on the orientation of the seams of a baseball. Drury states that Sikorsky discovered a large effect, yet Briggs reported none at all.

There is another curious aspect of these results. According to the Kutta–Zhukovskii theorem,[7] whenever a two-dimensional object is moving through an inviscid fluid, and there is a net circulation of the fluid about the object, there results a force mutually perpendicular to the velocity vector and the vorticity vector associated with the circulation. This force has a magnitude that is proportional to the product of the velocity and the circulation. Experimental measurements of lift on rotating cylinders appear to bear this out.[8] It seems reasonable that the lift force on a rotating sphere would also depend on ωV rather than on ωV^2. Recent experiments on golf balls by Bearman and Harvey[9] indicate that this is the case.

In the following section, we describe a new set of experiments designed to determine the effect of the orientation of the seams on the lateral force on a rotating baseball, and also to determine whether the lift force depends on ωV^2 or on ωV.

II. APPARATUS AND EXPERIMENTS

The experimental arrangement that we used consisted of a subsonic wind tunnel, a device for measuring lift on a spinning ball, and devices for measuring the rotational and free stream velocities. Three baseballs were impaled on 6.3-mm-diam shafts with the seams in the positions shown in Fig. 2. The shaft of a particular ball was then mounted in a

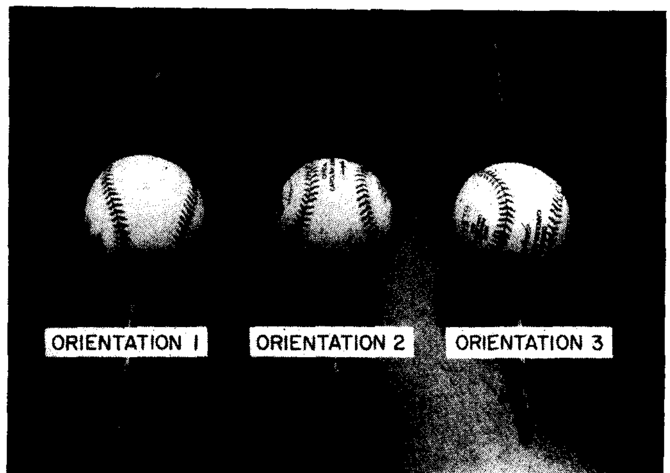

Fig. 2. The three seam orientations tested.

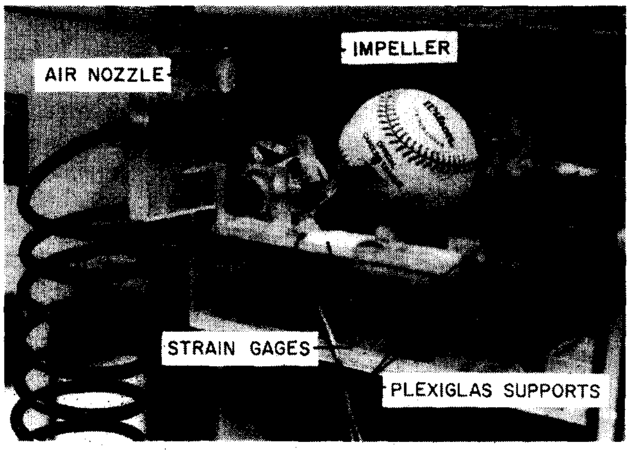

Fig. 3. The experimental device.

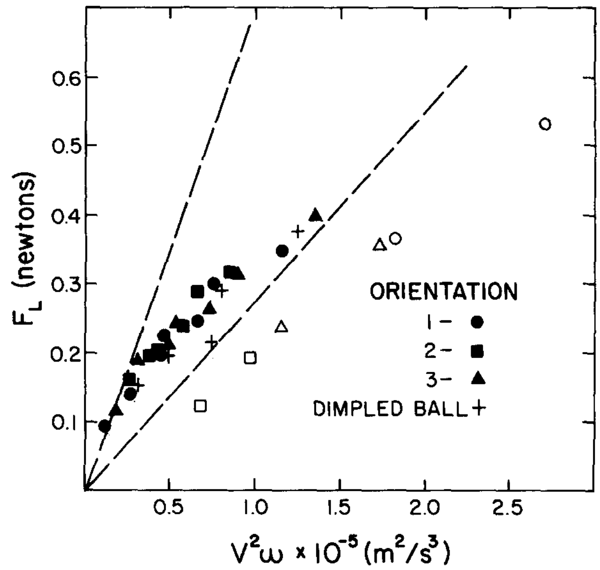

Fig. 4. Lift data according to these experiments.

frame as shown in Fig. 3. An impeller mounted on the end of the shaft was used for creating rotation by the use of high-speed air from a nozzle. The entire device was mounted in the test section of a subsonic wind tunnel. Air in the tunnel flows from right to left in Fig. 3. Drag and lift forces were simultaneously manifest through bending stresses in the Plexiglas supports. These stresses were measured by strain gages mounted on each side of the two Plexiglas supports. The device was calibrated by hanging known weights from the shaft and measuring the strain using a microstrain indicator.

With the wind speed in the tunnel at a constant level, the ball was set spinning. The rotation rate was regulated by varying the speed of air from the nozzle and measured using a phototachometer. Owing to the position of the shaft, the strain in each of the Plexiglas arms was the result of both drag and lift forces. Two experiments were performed at each wind (free stream) velocity/rotation rate combination, one with clockwise and the other with counterclockwise rotation. The strain indicator measurements could then be subtracted and the effect of the drag force was canceled. The difference between the two measurements divided by two was taken to be the effect of the lift force alone.

The wind tunnel used was one commonly used in the undergraduate laboratory at Tulane University.

III. THE LIFT FORCE

The data resulting from our measurements are shown in Fig. 4. We were at first surprised to find essentially no dependence of lift force on orientation. In retrospect, this does not seem so surprising. The ball apparently operates as a fully rough sphere regardless of where the seams are located. To check this hypothesis we tested a rough sphere the same diameter as a baseball but without seams. Balls of this type are used in baseball pitching machines. The balls have very nearly the same weight and diameter as baseballs but the surface is covered with dimples in the manner of golf balls. The data obtained is shown in Fig. 4 and labeled "dimpled ball." It agrees quite well with the standard baseball data.

The lift data of Briggs and the lines bounding the Sikorsky data are shown in Fig. 4 for comparison. At a given value of $V^2\omega$, our measured lift forces are significantly larger than those of Briggs, and fall between the Sikorsky limits. Assuming F_L approaches zero as $V^2\omega$ approaches zero, it appears clear that our data do not show a linear dependence of F_L on $V^2\omega$. Furthermore, the data of Bearman and Harvey on golf balls indicate that the lift force depends not on $V^2\omega$ but on $V\omega$. To lay a groundwork for the discussion of this data and its connection with our own data, we turn to dimensional analysis.

IV. PRESENTATION OF THE DATA AS DIMENSIONLESS GROUPS

Fluid mechanics data is most appropriately presented in terms of dimensionless groups. In the present case, the appropriate dimensionless groups are the lift coefficient $C_L = F_L/\frac{1}{2}\rho V^2 A$, the Reynolds number $\mathrm{Re} = VD/\nu$, the ratio of the speed of the surface of the ball relative to its center to the translational speed, $\pi D\omega/V$, and some measure of the roughness of the sphere, for example, the ratio of the mean roughness height to the sphere diameter ϵ/D. Here F_L is the lift force, ρ is the density of air, V is the speed of air in the wind tunnel, A is the cross-sectional area of the ball, D is the diameter of the ball, ν is the kinematic viscosity of air, ω is the rotation rate of the ball, and ϵ is the mean roughness height. Thus we suppose that

$$C_L = f\left(\frac{\pi D\omega}{V}, \mathrm{Re}, \frac{\epsilon}{D}\right). \tag{4}$$

Presented in this form, the result should be valid for any sphere, not only for baseballs. Data on spinning golf balls by Bearman and Harvey,[9] on golf balls and smooth spheres presented by Davies[10] can be compared to the present data, as can the pioneering smooth sphere data of Maccoll.[11]

Data of Bearman and Harvey, Davies, Briggs, and Maccoll are presented along with our data in Fig. 5. Unfortunately, Davies' data was obtained at a single translational speed, corresponding to a Reynolds number of 9×10^4. All of Briggs' data correspond to larger Reynolds numbers, while all of our data is for smaller Reynolds numbers. Speed limitations in the wind tunnel available to us prevented us from obtaining data at Reynolds numbers comparable to those of Briggs. These same speed limitations also prevented us from obtaining data at values of $\pi D\omega/V$ as small as those of Briggs.

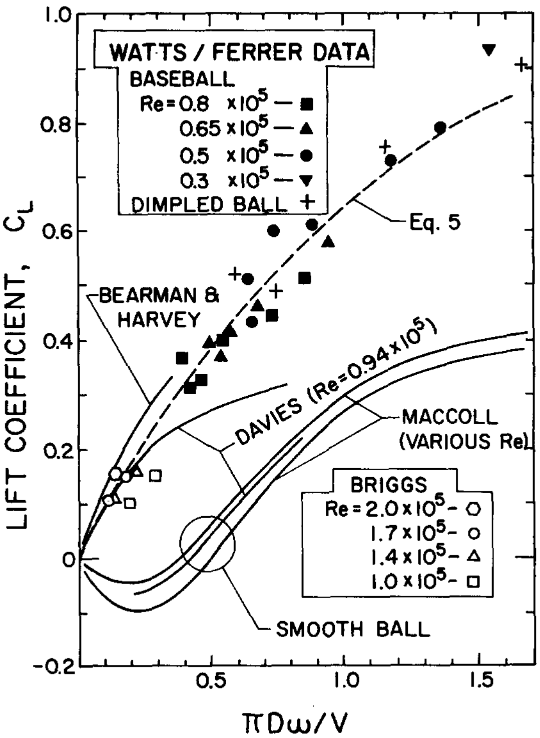

Fig. 5. Lift coefficient data.

According to the Kutta–Zhukovskii theorem, the lift force should be directly proportional to ωV. This means that the lift coefficient should be directly proportional to $\pi D\omega/V$. If the lift were a linear function of $V^2\omega$, as indicated by Briggs, then it must follow that C_L is proportional to the product of $\pi D\omega/V$ and the Reynolds number,

$$C_L = \alpha \frac{\pi D\omega}{V}\frac{VD}{\nu}. \tag{5}$$

This is most definitely in conflict with our data and with the Bearman and Harvey data. The smooth sphere data of Maccoll also show only a very weak dependence of C_L on Reynolds number. The smooth sphere data of both Maccoll and Davies indicate that, although the dependence is obviously not linear, C_L varies primarily with $\pi D\omega/V$ and is less strongly dependent on the Reynolds number. Briggs' data for baseballs, on the other hand, show a strong linear dependence of C_L on *Re*. The rough sphere (dimpled golf ball) data of Davies indicate a much larger lift coefficient than the Briggs data at a similar Reynolds number. Bearman and Harvey give larger C_L than Davies.

The data of Bearman and Harvey span a Reynolds number range of $0.4\times10^5 < \mathrm{Re} < 2.4\times10^5$. Bearman and Harvey comment that a few data points taken at low Reynolds numbers and low spin rates imply that under those conditions the lift coefficient may be a weak function of Reynolds number. However, whenever $\mathrm{Re} > 0.6\times10^5$, C_L is largely independent of Reynolds number and a function of $\pi D\omega/V$ only.

Our data, taken at higher values of $\pi D\omega/V$ than either the Briggs or Davies data because of the speed limitations of our wind tunnel, show very little dependence of the lift coefficient on Reynolds number. Our values of C_L appear to be consistent with the Davies data at low values of $\pi D\omega/V$, but our values continue to increase with $\pi D\omega/V$, at least up to $\pi D\omega/V = 1.5$, while the Davies data shows practically no increase after $\pi D\omega/V = 0.5$. The data of Bearman and Harvey appear to be consistent with our data in that C_L is a very weak function of Reynolds number.

V. SOME COMMENTS ON THE PATHS OF SPINNING BASEBALLS

The deflection of a spinning baseball on its path to home plate is given by

$$d = F_L t^2/2m. \tag{6}$$

The time required to reach home plate is

$$t = L/V, \tag{7}$$

where L is the distance between the pitcher's release and home plate, approximately 18 m. If, as Briggs reported, the deflecting force is $K_1V^2\omega$, then

$$d = K_1L^2\omega/2m, \tag{8}$$

independent of the speed of the pitch. On the other hand, if the deflecting force is $K_2V\omega$, as we claim, then

$$d = K_2L^2\omega/2mV \tag{9}$$

and is inversely proportional to the speed of the pitch.

According to Briggs' data, $K_1 = 2\times10^{-6}$ ns^3/m^2. Hence, a pitch rotating at 2000 rpm (227 rad/s) would be deflected by 0.5 m. (The mass of a standard baseball is 0.145 kg.) According to our data, $K_2 = 5.5\times10^{-4}$ kg, and with $\omega = 2000$ rpm and $V = 80$ mph (35.76 m/s), the ball would be deflected by 0.4 m.

The rotation rates and speeds used above are reasonably representative of those associated with curveballs thrown by professional baseball pitchers.[5,12] The calculated deflections are also reasonable according to Selin.[12] Therefore these calculations do not form a strong argument in favor of either the Briggs data or our own and that of Bearman and Harvey. It is perhaps of interest to note that baseball players apparently believe that slower pitches curve further than faster ones.[13] This, however, could also result because when throwing the ball more slowly the pitcher can put more effort into spinning the ball and therefore obtain a higher rotation rate with a slower pitch.

A comment concerning the apparent absence of any strong effect of seam orientation on the lateral force seems warranted, especially since baseball pitchers believe the effect to be important. We can only guess that the way in which a baseball is gripped allows the pitcher to apply more spin to the ball with some string orientations than with others.

VI. CONCLUSIONS

We have discovered only three sets of data on the lift force on rough spinning spheres, those of Bearman and Harvey, Briggs, and Davies. In addition, some rather vague general statements have been attributed to Igor Sikorsky by Joseph Drury. These data sets appear not to be consistent with each other. We have obtained some new data which are not consistent with either the Briggs data or the Davies data, but are consistent with the Bearman and Harvey data. According to our data, the lift coefficient for rough spheres is a function of the ratio $\pi D\omega/V$ and is at most a weak function of Reynolds number. This seems more consistent with the Kutta–Zhukovskii theorem than do the results of Briggs, although the Kutta–Zhukovskii theorem is strictly applicable only to two-dimensional inviscid flow.

It appears that the force on a curveball does not depend strongly on the orientation of the seams as suggested by Drury in his description of Sikorsky's experiments. In general, it appears that the lift coefficient is principally a function of $\pi D\omega/V$ and the roughness of the sphere and is only a weak function of the free stream Reynolds number.

[1]I. Newton, Philos. Trans. R. Soc. 7, 3078 (1771–2).
[2]Lord Rayleigh, Messenger of Mathematics 7, 14 (1877).
[3]H. Barkla and L. Auchterlonie, J. Fluid Mech. 47, 437 (1971).
[4]B. Robins *New Principles of Gunnery* (Hutton, London, 1742).
[5]L. Briggs, Am. J. Phys. 27, 589 (1959).
[6]J. Drury, "The Hell It Don't Curve," from *The Fireside Book of Baseball*, edited by Charles E. Einstein (Simon and Shuster, New York, 1956).
[7]R. Granger, *Fluid Mechanics* (Holt, Rinehart and Winston, New York, 1985).
[8]E. G. Reid, NACA Tech. Note No. 209 (1924).
[9]P. W. Bearman and J. K. Harvey, Aeronaut. Q. 27, 112 (1976).
[10]J. Davies, J. Appl. Phys. 20, 821 (1949).
[11]J. Maccoll, J. R. Aeronaut. Soc. 32, 777 (1928).
[12]C. Selin, The Research Quarterly 30, 232 (1959).
[13]Personal Communication from Ray Miller (former Baltimore Orioles pitching coach).

The dynamic shear modulus and internal friction of a fiber vibrating in the torsional mode

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A torsional pendulum consisting of an 8-mil boron–tungsten fiber and inertial member is described. The torsional oscillations are excited electrostatically and detected by a frequency modulation technique. Both the dynamic shear modulus and the internal friction can be measured. In addition, this system exhibits the resonance behavior characteristic of a high Q oscillator.

INTRODUCTION

Many novel experiments have been developed to measure the dynamic shear and Young's modulus for a variety of solids.[1–5] These systems usually involve longitudinal, flexural, or torsional vibrations of a uniform rod or beam driven at resonance. Some of these apparatuses also allow for the determination of the internal friction inherent in any oscillating body. In this paper we describe an electromechanical system that allows one to measure the dynamic shear modulus and the internal friction of an electrically conducting fiber or thin rod.

For a simple torsional pendulum with low damping, the natural frequency, f, of oscillation is

$$f = (1/2\pi)\sqrt{K/I}, \tag{1}$$

where I is the moment of inertia of the inertial member and for a uniform rod (or fiber)

$$K = G\pi a^4/2L. \tag{2}$$

Here, G is the shear modulus, a is the radius of the fiber, and L is the free length between the inertial member and the clamped end of the fiber. Thus, for a given fiber, the natural frequency of torsional vibration is related to its length according to

$$f \alpha 1/\sqrt{L} \tag{3}$$

and a measurement of f vs $1/\sqrt{L}$ can yield a precise determination of the shear modulus. In addition, if the system behaves as a discrete mass-spring oscillator with the inertial member corresponding to the mass and the restoring torque and internal damping resulting from the fiber, then the amplitude of the torsional vibration can be expressed as[6]

$$A = \frac{A_0\omega_0/\omega}{[(\omega_0/\omega - \omega/\omega_0)^2 + Q^{-2}]_{1/2}}. \tag{4}$$

Here ω is the angular driving frequency, ω_0 is the natural frequency of the undamped oscillator, and Q is the quality factor. By fitting Eq. (4) to an experimentally determined resonance curve, one can obtain the Q of the fiber.

EXPERIMENTAL PROCEDURE

The torsional pendulum is constructed by cementing one end of the fiber in the middle of a thin rectangular shaped aluminum foil ($0.0051 \times 0.48 \times 1.60$ cm^3) and firmly clamping the other end at various lengths L (see Fig. 1). Placing metal screws in an insulated holder at symmetric positions near opposite ends and sides of the aluminum vane forms two coupled parallel plate capacitors, hereafter referred to as the sample capacitor. Establishing a potential between the two screws and the vanes forms a couple. Excitation of the torsional mode of oscillation results on application of an ac voltage across this sample capacitor. Since

