Effect of the Magnus Force in the PITCHf/x Tracking System

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A new method is proposed for calculating the pfx_x and pfx_z parameters in the PITCHf/x tracking system.

I. INTRODUCTION

The PITCHf/x system uses two cameras to track pitches between pitcher and batter, determining the coordinates of the ball at 1/60-sec intervals. The resulting trajectory x(t),y(t),z(t) (t is the time) is fit it to a nine-parameter (or "9P") fit corresponding to constant acceleration in each of the three dimensions. All quantities reported in the PITCHf/x data base, such as the pitch speed, the location of the pitch as it crosses the plate, the "break" of the pitch, etc., are derived from the fitted trajectory rather than from the original data. The nine parameters are the three initial positions x_0, y_0, and z_0; the three initial velocities v_{x0}, v_{y0}, v_{z0}; and the three accelerations a_x, a_y, and a_z. Here the coordinates refer to the usual PITCHf/x coordinate system, where the origin is at the point of home plate, $\hat{y}$ points towards the pitcher, $\hat{z}$ points vertically upward, and $\hat{x} = \hat{y} \times \hat{z}$ (i.e., the x axis points to the catcher's right). The 9P fit is an approximation to the actual equations of motion,

$$
\begin{aligned}
\ddot{x} &= -KC_D v v_x \,-\, KC_L v v_y \sin\phi \\
\ddot{y} &= -KC_D v v_y \,+\, KC_L v\,(v_x \sin\phi - v_z \cos\phi) \\
\ddot{z} &= -KC_D v v_z \,+\, KC_L v v_y \cos\phi \,-\, g\,.
\end{aligned} \qquad (1)
$$

Here g is the acceleration due to gravity (32.174 ft/s^2), C_D and C_L are the drag and lift coefficients, respectively, and $K = 5.44\times10^{-3}$ ft^{-1} is a numerical factor.[1] In these expressions, the spin axis is assumed to lie in the $x - z$ plane and makes an angle ϕ with the x axis, with a sign such that $\phi = 90^\circ$ corresponds to the spin pointing upward, along the z axis. Noting that v_y is negative, it is easy to see that the Magnus force makes an angle $\theta = \phi - 90^\circ$ with the x axis. Therefore $\phi = 0^\circ$ (topspin) results in a downward acceleration, and $\phi = 90^\circ$ (sidespin) results in an acceleration to the catcher's right, exactly as expected.

Two quantities calculated by PITCHf/x are pfx_x and pfx_z, the deviation of the pitch trajectory in the x and z directions from that expected in the absence of the Magnus force, as measured between y=40 ft and the front edge of home plate, y=1.417 ft. The exact way to calculate these quantities is to compare the actual trajectory with that computed by solving the equations of motion, Eqs. 1, with C_L=0. The PITCHf/x system uses an approximate method given by the prescription

$$
\begin{aligned}
pfx_x &= \frac{1}{2} a_{xM} t_{40}^2 \\
pfx_z &= \frac{1}{2} a_{zM} t_{40}^2\,,
\end{aligned} \qquad (2)
$$

where t_{40} is the time of flight between $y = 40$ and $y = 1.417$ ft and the accelerations due to the Magnus force, a_{xM} and a_{zM}, are given by

$$
\begin{aligned}
a_{xM} &= a_x \\
a_{zM} &= a_z + g\,.
\end{aligned} \qquad (3)
$$

This prescription assumes that a_x and $a_z + g$ are due entirely to the Magnus force. An inspection of Eq. 1 shows that this is a good approximation to the extent that the drag-related terms (i.e., those proportional

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to C_D) can be neglected in the expressions for the x and z accelerations. In this brief note, an alternate approximate prescription for pfx_x and pfx_z is proposed, then compared to the earlier prescription as well as to an exact calculation for a large selection of actual pitches.

II. ALTERNATE TECHNIQUE

The alternate technique uses an expression identical to Eq. 2 but with accelerations a_{xM} and a_{zM} modified to approximately remove the effects of drag as follows:

$$\begin{aligned} a_{xM} &= a_x - a_y \frac{< v_x >}{< v_y >} \\ a_{zM} &= a_z - a_y \frac{< v_z >}{< v_y >} + g \,, \end{aligned} \tag{4}$$

where the brackets indicate the time average over the full trajectory. The rationale behind this improved approximation is found in Eq. 1, where it is observed that the contribution of the drag to each component of the instantaneous acceleration is directly proportional to that component of the instantaneous velocity. The y component of the drag acceleration is approximated by a_y, an approximation that is expected to be quite good since $|v_x/v_y| \ll 1$ and $|v_z/v_y| \ll 1$. To get the time-averaged drag, the time-averaged velocities are needed and calculated using straightforward kinematics. First v_{yf}, the y component of velocity at $y_f = 1.417$ ft is calculated. Then the total flight time T is calculated. Then a_x and a_z are used to calculate v_{xf} and v_{zf}. Finally, the initial and final velocities are used to calculate the time-averaged values. The particular sequence of equations is as follows:

$$\begin{aligned} v_{yf} &= -\sqrt{v_{y0}^2 + 2a_y(y_f - y_0)} \\ T &= \frac{v_{yf} - v_{y0}}{a_y} \\ v_{xf} &= v_{x0} + a_x T \\ v_{zf} &= v_{z0} + a_z T \,, \end{aligned} \tag{5}$$

and

$$< v_x > = (v_{xf} + v_{x0})/2 \tag{6}$$

and similarly for $< v_y >$ and $< v_z >$. As an aside, it is noted that the angle ϕ can be found approximately from

$$\phi = \arctan\left(\frac{a_{zM}}{a_{xM}}\right) + 90^\circ \,. \tag{7}$$

III. COMPARING THE TWO TECHNIQUES

To facilite the notation, the quantities Δx and Δz are defined to be the difference between the new and old values of pfx_x and pfx_z, respectively. Also pfx is defined as $\sqrt{pfx_x^2 + pfx_z^2}$. The results of the analysis are shown in Fig. 1, where nearly 8000 pitches from games played in Toronto during the period April-June, 2007 are analyzed. The Δz plot shows a consistent systematic difference between the two techniques which is easily understood. Since the ball always follows a downward trajectory, the z component of drag is always upward (positive). Using the old system of calculating pfx_z, the upward drag produces a deflection which is systematically more positive than the exact value. That is, Δz (new value minus old value) is systematically negative. The effect on the x coordinate depends on the direction of the x velocity, and the two plots show a systematic shift of pfx_x in the positive direction (for a negative v_{x0}) or the negative direction (for a positive v_{x0}), with the reasoning being exactly the same as for the z deflection. The final plot shows the correlation between the total deflection pfx and the inferred value of the lift coefficient C_L for these trajectories, the latter calculated by doing a non-linear least-squares fit to the smoothed trajectories using the full equations of motion. Note that pfx should be linearly proportional to C_L and independent of the initial velocity. This latter point can be seen from Eq. 2, since the Magnus accleration is proportional to v^2 but the time is proportional to $1/v$. The figure shows that the values pfx calculated with Eq. 4 are perfectly correlated with C_L, indicating the method is a very good approximation to the exact solution. Indeed, the red curve is a nearly perfect straight line passing through the origin. On the other hand, the values calculated using Eq. 3 are less well correlated, as indicated by the scatter in the values.

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[1] The constant K scales with air density. The value given assumes normal temperature and pressure.

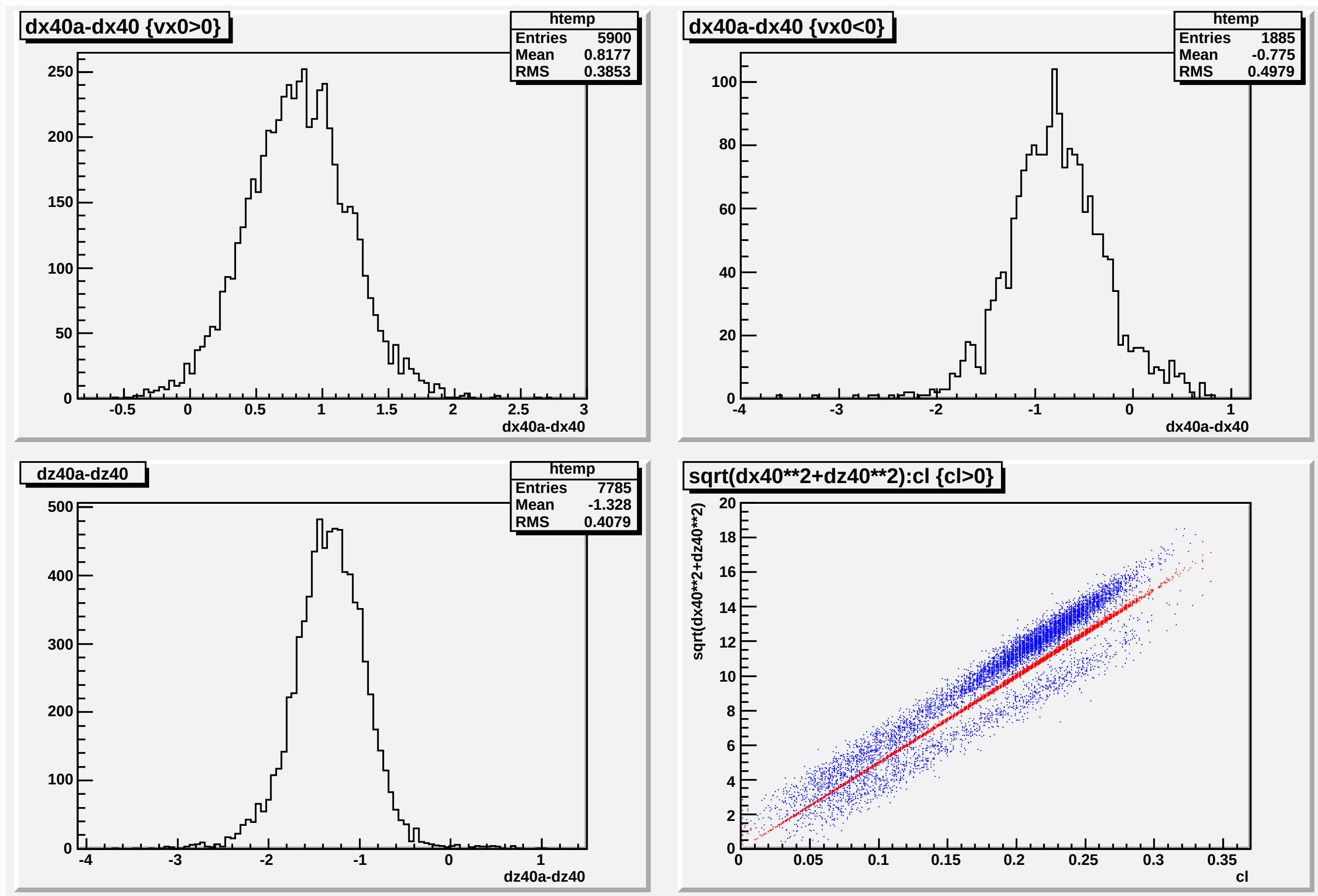

FIG. 1: (left) Plots of 7785 pitches from the Toronto PITCHf/x data. The upper row is a plot of Δx for $v_{x0} > 0$ (left) and $v_{x0} < 0$ (right). The lower left plot is Δz. The lower right plot shows the correlation between pfx and C_L, with the blue points and red points calculated using the prescriptions of Eq. 3 and Eq. 4, respectively.