What Is The Hawkeye Spin Data Teaching Us?
I: Determining Magnus and non-Magnus Movement

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I. INTRODUCTION

The introduction of Hawkeye as the video-based baseball tracking system for MLB provides a real opportunity to learn new things beyond what was learned from the previous Trackman-based tracking system. Both systems measure the full trajectory of pitched baseballs as well as the total spin rate ω. However, Hawkeye additionally measures the Cartesian components of spin, ω_x, ω_y, and ω_z, where $\omega = \sqrt{{\omega_x}^2 + {\omega_y}^2 + {\omega_z}^2}$. These new data allow for the first time a measurement of the spin efficiency as well as a quantitative study of movement beyond that due to the Magnus force.

The goal of this article is to introduce a formalism for separating the Magnus from the non-Magnus movement. By design it is mostly technical, since I want to collect all the necessary formulas in one place. At the end, I will present an example of how the formalism can be applied. Future articles will concentrate on that type of analysis.

I will be using the coordinate system established in the early days of PITCHf/x. Namely, the origin is at the corner of home plate, the x axis points to the catcher's right, the y axis points toward 2nd base, and the z axis points up.

II. FORMALISM FOR THE ANALYSIS

The publicly available pitch trajectory data have been parameterized by a constant acceleration model, resulting in nine parameters (9P) that characterize each trajectory. The resulting acceleration vector $\vec{a}$ can be decomposed into three parts:

- $\vec{g} = -g\hat{z}$, the acceleration due to gravity

- $\vec{a}_D$, the acceleration due to drag
- $\vec{a}_T$, the transverse acceleration

The drag acceleration is defined to be the component of $\vec{a} - \vec{g}$ in the -$\hat{v}$ direction, and it reduces the speed of the ball but does not change its direction. The transverse acceleration is the component of $\vec{a} - \vec{g}$ perpendicular to the $\hat{v}$ direction, and it changes the direction of the ball (a phenomenon called "movement") but does not affect the speed. Both the drag and the transverse acceleration are the result of aerodynamic forces. There are interesting things to be learned about both using the 3D spin axis, although this article will focus exclusively on the movement resulting from the transverse forces.

The separation of the acceleration into the three components has been described by Nathan[1] and results in a formula for the transverse part:

$$\vec{a}_T = \vec{a} - \vec{g} + [(\vec{a} - \vec{g}) \cdot \langle \hat{v} \rangle] \langle \hat{v} \rangle \,, \tag{1}$$

where $\langle \hat{v} \rangle$ is a unit vector in the direction of the *average* velocity, which is essentially the direction at the midpoint of the trajectory. Conventionally, the transverse force is identified with the Magnus force on a spinning baseball. However, recent work[2] has convincingly shown that there is movement other that due to the Magnus force. It has been speculated that this additional movement is due to a "Seam Shifted Wake" (SSW), related to the orientation of the seams relative to the air flow over the ball.[3] Accordingly, it is useful to decompose the transverse acceleration into components as follows:

$$\vec{a}_T = \vec{a}_M + \vec{a}_N \,, \tag{2}$$

where the M and N subscripts signify the accelerations due to the Magnus and non-Magnus forces, respectively.

The usual procedure is to use the x and z components of the transverse acceleration to calculate the x and z components of the total movement M:

$$M_{T,x} = \frac{1}{2} a_{T,x} t^2 \quad M_{T,z} = \frac{1}{2} a_{T,z} t^2 \quad \phi_T = \arctan\left(\frac{a_{T,z}}{a_{T,x}}\right) , \tag{3}$$

where t is the flight time from release to the front of home plate and ϕ_T is the direction of the movement in the xz plane As mentioned above, the movement is usually associated with the Magnus force. However, we now know that the movement may due to both Magnus and non-Magnus forces, and one goal is to separate that movement into these two contributions. Eq. 3 applies equally well for these contributions:

$$M_{M,x} = \frac{1}{2}a_{M,x}t^2 \quad M_{M,z} = \frac{1}{2}a_{M,z}t^2 \quad \phi_M = \arctan\left(\frac{a_{M,z}}{a_{M,x}}\right) \tag{4}$$

and

$$M_{N,x} = \frac{1}{2}a_{N,x}t^2 \quad M_{N,z} = \frac{1}{2}a_{N,z}t^2 \quad \phi_N = \arctan\left(\frac{a_{N,z}}{a_{N,x}}\right) \tag{5}$$

The total movement in the x and z directions is just the sum of the component parts:

$$M_{T,x} = M_{M,x} + M_{N,x} \quad M_{T,z} = M_{M,z} + M_{N,z}\,. \tag{6}$$

Since the total movement is known from Eqs. 1 and 3, the separation into Magnus and non-Magnus parts requires knowledge of the Magnus acceleration, which I will now review.

The Magnus acceleration is given by[1]

$$\vec{a}_M = KC_Lv^2\left[\frac{\hat{\omega}\times\hat{v}}{|\hat{\omega}\times\hat{v}|}\right]. \tag{7}$$

The factor K is given by

$$K = \frac{1}{2}\frac{\rho A}{m}, \tag{8}$$

where m and A are the mass and cross sectional area of the ball, respectively, and ρ is the density of the air.[4] The magnitude of the acceleration is directly proportional to the so-called lift coefficient C_L. The direction of the acceleration is given by the term in brackets, from which we find the expression for Magnus movement direction ϕ_M

$$\phi_M = \arctan\left(\frac{\omega_x v_y - \omega_y v_x}{\omega_y v_z - \omega_z v_y}\right). \tag{9}$$

Note that ϕ_M depends on the 3D spin components but is independent of C_L.

To find the Magnus movement requires knowing the x and z components of the Magnus acceleration

$$a_{M,x} = C_LK\langle v\rangle^2\left[\frac{\omega_y v_z - \omega_z v_y}{\omega v|\sin\theta|}\right] \quad a_{M,z} = C_LK\langle v\rangle^2\left[\frac{\omega_x v_y - \omega_y v_x}{\omega v|\sin\theta|}\right], \tag{10}$$

where

$$|\sin\theta| = \frac{1}{\omega v}\sqrt{(\omega_y v_z - \omega_z v_y)^2 + (\omega_z v_x - \omega_x v_z)^2 + (\omega_x v_y - \omega_y v_x)^2}\,. \tag{11}$$

Both components of the movement depend directly on C_L, which is not as well known as we would like and which directly affects our goal of separating the movement into Magnus and non-Magnus components. I will discuss this topic further in the next section.

The quantity θ in Eqs. 10 and 11 is the angle between the spin and velocity vectors, and $|\sin\theta|$ is commonly called the "spin efficiency". It is generally assumed that the spin vector, both magnitude and direction, is unchanged during the trajectory. However, since the velocity direction does change, the spin efficiency is not necessarily constant throughout the trajectory. Since all the available trajectory information is derived from the constant-acceleration parametrization (9P) of the actual trajectory, those parameters are sensitive to the *average* acceleration and hence the average Magnus acceleration. Therefore, we will choose the average velocity for defining both the spin efficiency in Eq. 11 and Magnus acceleration in Eq. 10 .[5] It is important to keep that in mind when analyzing the data.

The preceding equations simplify considerable in the approximation $|v_y| \gg |v_x|, |v_z|$, which is almost always satisfied for real pitches. This allows us to neglect terms involving v_x and v_z in Eqs. 9-11, resulting in

$$\begin{aligned} \phi_M &\approx -\arctan(\omega_x/\omega_z) \\ a_{M,x} \approx C_L K \langle v\rangle^2 \cos\phi_M \qquad a_{M,z} &\approx C_L K \langle v\rangle^2 \sin\phi_M \\ |\sin\theta| &\approx \sqrt{\omega_x^2+\omega_z^2}/\omega\,. \end{aligned} \tag{12}$$

Let's summarize where we now are. We know the total movement, both magnitude and direction. We know the direction of the Magnus movement from the 3D spin components but we need better knowledge of C_L to complete our goal. Nevertheless, it is still possible to make some progress, since the directions of both the total movement ϕ_T and the Magnus movement ϕ_M are known, independent of C_L. This allows a measurement of the "axis shift",

$$\text{Axis Shift:} \quad \Delta\phi = \phi_T - \phi_M \tag{13}$$

which is the difference between the directions of the total and Magnus movement. In a Magnus-only world, $\Delta\phi$ would be exactly zero. Therefore a non-zero value would indicate the presence of a non-Magnus force. Indeed, the approximation for ϕ_M in Eq. 12 was used by Smith, *et al.*[2] to demonstrate such forces from publicly available Statcast data. The "optical axis" (OA) in that article is the spin axis in the xz plane, which is the approximation for ϕ_M+90°, while the "inferred axis" (IA) is ϕ_T+90°.

III. DETERMINING THE LIFT COEFFICIENT

As discussed above, while the *direction* of the Magnus movement is known from Eq. 9, the magnitude is less well known. The magnitude is determined from the lift coefficient C_L, whose dependence on spin rate and spin efficiency is not as well known as we would like it to be. From laboratory experiments, we have some information on the magnitude of C_L and its dependence on spin rate, but only for unit spin efficiency. Even these data, shown in Fig. 1, have scatter of order ±20%. Given the limitations of the laboratory data, a method is sought using Statcast data to determine the dependence of C_L on spin rate for arbitrary spin efficiency. Specifically, there are two questions we seek to answer using Statcast data:

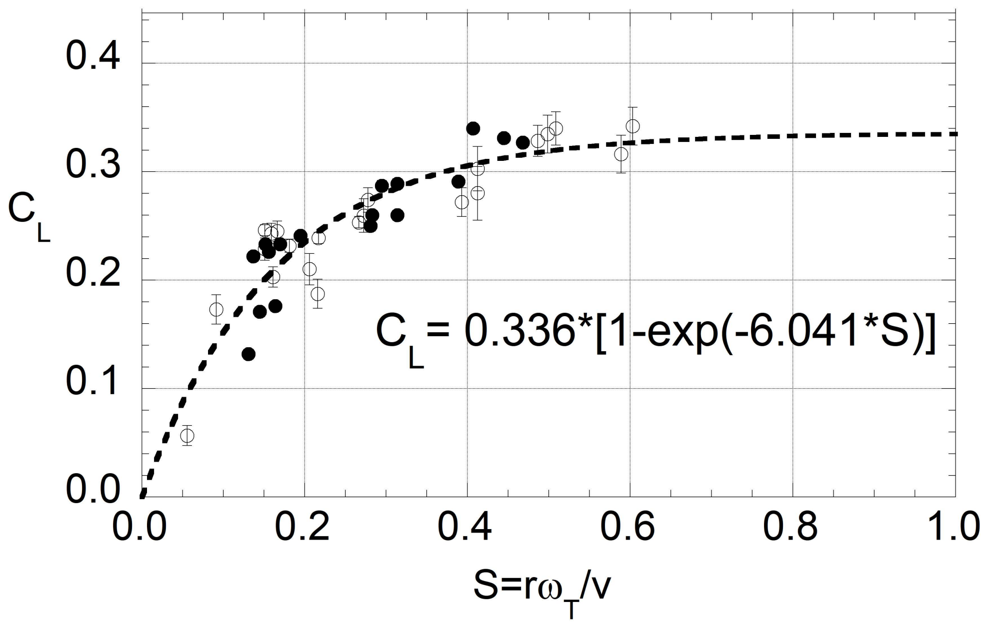

FIG. 1: Experimental values of C_L as a function of the spin factor $S = r\omega_T/v$, where ω_T refers to the transverse part of the spin.[1] The data were parametrized by the function $C_L = A(1 - e^{-BS})$, with A and B determined from a non-linear least-squares fit (black curve), with A=0.336 and B=6.041.

1. What is the relationship between C_L and spin for unit spin efficiency (i.e., for purely transverse spin)?

2. How is that relationship modified for non-unit spin efficiency (i.e., in the presence of gyrospin)?

The following is an outline of how one might proceed, assuming that the spin components are known.

- Define spin factor $S = R\omega/v$, where R is the radius the of ball
- For purely transverse spin (i.e., unit spin efficiency), $C_L{=}f(S)$, a smooth monotonically non-decreasing function that vanishes when $S{=}0$. An example of such a function is shown in Fig. 1.
- To determine $f(S)$, select pitches for which the spin efficiency is high (say, $\sin\theta \geq 0.95$) and the likelihood of non-Magnus transverse forces is small (say, $\mid \Delta\phi \mid \leq 2°$). This involves the implicit assumption that non-Magnus forces make no contribution when the axis shift is small. The validity of that assumption is under investigation.
- Having determined $f(S)$, determine which of the two prescriptions is most likely for arbitrary spin efficiency:[6]
  - prescription a: $C_L{=}f(S)\sin\theta$, or equivalently $C_L/\sin\theta = f(S)$
  - prescription b: $C_L{=}f(S\sin\theta)$
- Once the better prescription is known, then C_L will be known for arbitrary spin rate and spin efficiency, allowing a determination of the Magnus contribution to the movement and therefore to the non-Magnus contribution via Eq. 6.

IV. AN EXAMPLE USING PSEUDO-DATA

An example of how to apply this formalism to real analysis is given in Fig. 2. The example utilizes the publicly available data for Alex Cobb's sinker, the mean parameters of which are given in Table I. For the Magnus parameters, the approximations of Eq. 12 were used, with C_L assumed to lie on the curve of Fig. 1, thereby implicitly assuming unit spin efficiency. With those assumptions, the Magnus movement can be determined and, when combined with the actual movement, the non-Magnus movement can also be determined (Eq. 6). The pitch-by-pitch results are shown in the figure. Note that in this example, the net effect of the non-Magnus force is to decrease the total movement by $\sim 5\%$ and to shift the direction by

$\sim 18°$ toward more arm-side and less upward. The direction of the non-Magnus movement is approximately perpendicular to the Magnus direction.

All the calculations of the movement in Fig. 2 utilized very straightforward R code. However, I have also created a downloadable spreadsheet template[7] to do these calculations, using the 9P parameters and the 3D spin components as the input, in addition to information needed to calculate the constant K. The spreadsheet uses a particular parametrization of $f(S)$, which is needed to calculate the lift coefficient.

TABLE I: Mean parameters of Alex Cobb's sinker.

ω (rpm)	S	ϕ_T (deg)	ϕ_M (deg)	$\Delta\phi$ (deg)	$M_{T,x}$ (inch)	$M_{T,z}$ (inch)
2071	0.195	141.6	123.8	17.8	-16.2	+12.9

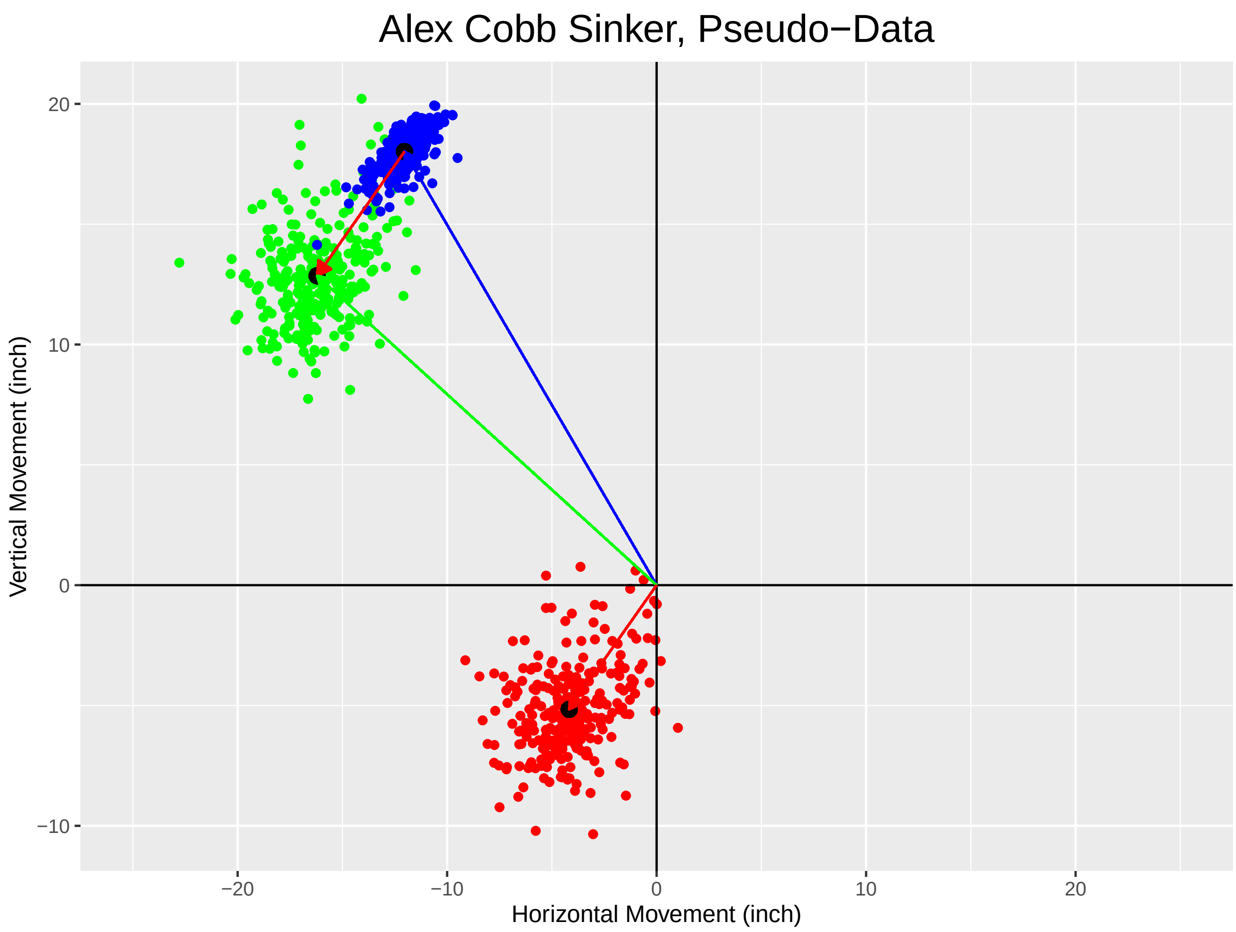

FIG. 2: Movement in the xz plane, shown from the catcher perspective, for Alex Cobb's sinker. The green, blue, and red clusters are the total, Magnus, and non-Magnus movements, respectively, while the black dot is the mean value of each cluster. The arrows are lines connecting the mean values and show the vector nature of their mutual relationship (i.e., the red plus the blue equals the green). The green points and arrow (actual movement) and the direction of the blue arrow (Magnus direction) are determined directly from the data and are independent of C_L. The axis shift (17.8°), which is the angle between the green and blue arrows, is also independent of C_L. The length of the blue arrow and red points and arrow (non-Magnus) depend on C_L, which was assumed to lie on the curve in Fig. 1.

Thanks to a number of people for some very useful feedback on this article. I suspect some would rather remain anonymous, so I won't name names. They know who they are.

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[1] Alan M. Nathan, Determining the 3D Spin Axis from Statcast Data, May 24-25, 2020 update.

[2] Barton Smith, Alan Nathan, and Harry Pavlidis, Not Just About Magnus Anymore, Baseball Prospectus, November 5, 2020.

[3] See Cliff Notes Seam Shifted Wake, Posted December 23, 2020.

[4] To determine the lift coefficient C_L requires knowing the atmospheric data that affect air density, and therefore K, such as temperature, pressure, and relative humidity (including stadium elevation). It is preferable to know this information on a short time scale, say every 15 minutes. For stadiums with a retractable roof, one must know the roof status during the game, since the atmospheric conditions inside a covered stadium are likely to be very different from those outside. Wind can also affect the inferred lift coefficient, but it is very hard to take that into account, since it often varies in the four dimensions (the three spatial dimensions and time).

[5] As of this writing, it is still not obvious which velocity vector is best used to calculate the spin efficiency for the purpose of finding the Magnus movement. I have chosen the average velocity for the studies here. Another possibility is the release velocity.

[6] Alan M. Nathan, The 3D Spin Axis: An Addendum, August 30, 2020.

[7] Template for Calculating Movement.