# EVALUATION OF DOPPLER RADAR BALL TRACKING AND ITS EXPERIMENTAL USES 

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A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING WASHINGTON STATE UNIVERSITY
Department of Mechanical and Materials Engineering

December 2012

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## ACKNOWLEDGMENT

I would first like to thank my beautiful wife for listening to my engineering problems and pretending like she understood/cared. Without her I would have never been able to complete this chapter in my life. I would also like to thank my mother and brother for providing support when things got tough. Thank you Nana and Papa for teaching me life's little lessons, "Jason, if it's worth doing, it's worth doing well" rang in my head daily.

I would like to thank Janus from TrackMan A/S for answering the same questions over and over again. Without the financial support from ASA this project would not have been possible.

Lastly, and most importantly I would like to thank Dr. Smith and the employees in the Sports Science Lab. Without Dr. Smith's guidance and patience this project would have never been a success. There are too many individuals in the lab that helped, you know who you are, and I thank you from the bottom of my heart.

# EVALUATION OF DOPPLER RADAR BALL TRACKING <br> AND ITS EXPERIMENTAL USES 

Abstract<br>by JASON JOHN MARTIN, M.S.<br>Washington State University<br>December 2012

Chair: Lloyd V. Smith
Doppler radar is being implemented in baseball and softball venues around the world to track the trajectory of pitched and hit balls. The units are positioned behind home plate and can track the pitch and hit speeds, spin rates, angles, and ball positions. The data is used to measure the performance of players, as well as keep statistics on the game. The radar units are also used by sporting goods manufacturers to measure the performance of equipment.

The goal of this study was to determine the accuracy of data reported by the radar. When compared to high speed video, the radar measured the speeds of the hit and pitch near home plate to within $2.8 \%$, and $2.3 \%$ respectively. The associated angles were within $1.2^{\circ}$. Hit distances of the radar were within $5 \%$ of a laser range finder. The hit and pitch speeds and angles near home plate were within $0.2 \%$ and $0.6^{\circ}$, respectively when compared to infrared video tracking. Balls hit at increasing horizontal angles experienced increasing radar noise and an optimal vertical angle of $15^{\circ}$ minimized noise. An error of $4 \mathrm{~m}(13.1 \mathrm{ft})$ in the initial ball position resulted in an error of the velocity and angles of a pitch of $4 \mathrm{~m} / \mathrm{s}(8.9 \mathrm{mph})$ and $8^{\circ}$.

An analysis of game and field study data provided a measure of the vertical and horizontal release point of the pitch. When the starting point of the pitch was held constant, a standard deviation of $0.03 \mathrm{~m}(0.09 \mathrm{ft})$ and $0.04 \mathrm{~m}(0.13 \mathrm{ft})$ in the horizontal and vertical position
was observed. Game data was used to calculate the aerodynamic properties of a low seam softball and raised seam baseball. The lift and drag coefficients were, $0.25<C_{d}<0.4$ and $0.05<$ $C_{L}<0.4$ for flat seamed softballs and $0.3<C_{d}<0.5$ and $0.05<C_{L}<0.24$ for raised seam baseballs. The $C_{d}$ values were shown to have a minimal dependence on spin rate, while the $C_{L}$ values were shown to have a stronger dependence on spin rate.

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## Dedication

A poet once said, "this is dedicated to all the homies that have been down since day one!"

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## Chapter 1 - Introduction

Technology continues to change the way baseball is viewed, heard, and understood. Baseball began over a hundred years ago with a simple ball made of rubber and horse hide and a bat made of wood. Over the years the game has become more popular and people are taking interest in unraveling some of the mysteries of how the ball moves, or how a homerun is hit. This curiosity has opened the door for new products that use modern technology to quantify and help visualize the game. Little has been done, however, to examine these new technologies to understand and help show how they work. Doppler radar is one such technology.

Recently Doppler radar has been implemented in baseball and softball stadiums around the country. It is used to track both the pitches and hits through their trajectory. The data is used by statisticians, scouts, and players to evaluate the games of baseball and softball and quantify player performance. It is also used by equipment manufacturers to measure the performance of bats and balls. Most recently, the data reported by radar has been used to measure the forces of lift and drag on balls along the trajectory of a hit. Because this system is being used extensively by sporting professionals to measure player performance and by manufacturers to determine if their equipment is safe, an assessment of the technology is needed.

The assessment includes examining how the system records data for pitches and hits, how noise affects the data, how the data is filtered, and finally a measure of the accuracy of the data. To show how the system records data, a brief discussion of radar theory is presented. This includes describing the raw data, and how position and angles are found from it. This
showed where in the trajectory this data was most affected by noise. Because noise is the limiting factor to any radar system, some time was spent explaining the different types of noise and what impact they had on the data.

The radar data that was evaluated included the beginning and end of the trajectories of the pitch and hit. This was done using high speed video tracking positioned around home plate and in the outfield. A second system of infrared cameras was also used to track the pitch and hit at home plate allowing for comparison to a second system. Game data was also used to measure the repeatability of the radar and to determine the effectiveness of using radar to measure aerodynamic forces of hit balls.

## Chapter 2 - Literature Review

In determining the accuracy of the radar it is important to first look at how radar works. This involves introducing some key terms and equations that will provide an understanding of how data is calculated and where in the trajectory it is affected by noise. Because noise is the limiting factor of any radar, it will be discussed in detail. The wide range of radars and their uses makes it difficult to define radars in their entirety, therefore only information applicable to the radar used for this work will be discussed.

## Section 2.1 History of radar

The basic theory of radar was first observed by the experiments conducted by the German physicist Heinrich Hertz from 1885 to 1888 (1). His experiments used a similar principle to that of a pulse-wave system operating at frequencies of about 455 MHz . He observed that radio waves mimic characteristics of light waves and that they could be reflected from metallic objects. Although his work was ground breaking, he never developed it for practical applications.

It wasn't until a German named Christian Hulsmeyer filed a patent in London in 1904 for a device that detected ships at sea that radar took on its practical application (2). He marketed the technology to the German Navy and to shipping companies, but interest for a ship collisionavoidance system was not understood at the time. During the 1920's others made use of radar type technologies, but it wasn't until the appearance of heavy military bombers in the late 1920's and early 1930's that the need for radar was established. These bombers could carry heavy pay loads long distances making it essential for an early warning system. In the 1930's
radar was developed by several different countries almost simultaneously. Most of these radars operated at frequencies between 100 to 200 MHz and were characterized as bistatic continuous wave (CW) radars, distinguished by the receiver and emitter being widely separated (3) (4).

The first pulse radar reflections from aircraft were observed in 1934 by the Naval Research Laboratory (3). It wasn't until 1936 that the observations became useful because of the duplexer, which allowed the use of a common antenna for both emitting and receiving. The signals used in the 30's where on the low end of the spectrum, and it wasn't until the 1940's that frequencies above 400 MHz could be achieved. This was primarily due to the application of the magnetron allowing much higher transmitting powers. The higher frequency spectrum allowed smaller antennas to be used with narrow beam widths. The 1940's also paved the way for the monopulse principle and its applications in measuring accurately the position of a target (3).

The 1950's saw the use of the klystron amplifier in radars, allowing for much higher power outputs than offered by the magnetron (5). This also allowed for pulses of different waveforms to be explored. The biggest achievement of the decade came from the statistical theory of detection, published in 1952 (3). This paper showed the need to look at radar detection from a statistical point of view, and expressed the importance of determining the probability of detection and probability of false alarms. The next major breakthrough did not come until 1965 when phased array antennas were used by the United States Navy (5). This paved the way for modern radar systems.

During World War II, radar technology grew rapidly and much advancement was made to make the systems smaller and more accurate. Following the war, radar use and radar technology continued to grow. The current development of radar has been focused on digital control and processing. This allows the devices within the radar to be smaller and allows the amount of information processed to be much quicker and use less energy.

Today many different types of radar are used for different applications. While the applications have progressed the basic operation has remained unchanged over the last 70 years. The major progression in radar technology has been in the digital processing and as new technologies and computing processing power increase so will the way in which radar signals are evaluated.

## Section 2.2 - Introduction to Radar

Radar is an electromagnetic system used to detect or track objects in the environment (4). The word radar was first used by the US Navy in 1940 and stands for radio detection and range (6). It is robust, capable of operating in many different weather conditions, distances, and track many different objects. Its ability to accurately measure distance and speed in different conditions is one of its most important aspects. There are many different types of radars used and depending on what operational requirements are needed the units will vary significantly. Probably the most recognized would be the Doppler radar you see on the local weather channel. These units are normally ground based, using a pulse-wave signal to analyze the density, speed, distance and size of oncoming weather patterns (7). Regardless of the type of
radar, or the object being tracked, the basic components of any radar are the same and are illustrated in Figure 2.1.


Figure 2.1: Simplified radar system.

The radar begins by emitting a signal which is transmitted through its directive antenna. The radiated signal is intercepted by many different objects and reflected back in many different directions. The reflected signals are received by either the same antenna that emitted the originating signal or a separate directive antenna. If it is determined that the signal requires further processing it is sent to different processing equipment delineated as the amplifier in Figure 2.1. The signals are then processed to filter out the required signals from the unwanted reflections of other objects. A combination of electronic signal processing and data processing are done to determine the presence of an object, its speed if moving, and location (6).

This is a simplified description of a radar system, but it helps outline the basic operation. Even a simple radar system has many more components such as amplifiers, pulse modulators, duplexers, filters, and a means to display the signal. All of which are necessary in the operation of the system. For the purpose of this work, only descriptions of radar signals, antennas, calculations, and the associated error will be discussed.

## Section 2.3 The Radar Signal

The signal being transmitted by the radar is very important and determines the resolution of the unit being used. Radars operating at higher frequencies have greater potential to characterize the target's physical features (8). The frequency fof an electromagnetic wave is related to the wavelength $\lambda$ by,

$$
\begin{equation*}
c=f \lambda \tag{2.1}
\end{equation*}
$$

where $c$ is the speed of light and is approximately $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ (9). It will be shown later that the signal wavelength relates to the accuracy of the velocity.


Figure 2.2: Wave propagation of two waves showing destructive interference.

A snap shot of a radar signal showing the fundamentals of wave propagation are shown in Figure 2.2. In the figure two wave forms are shown, each with equal amplitude and frequency. The amplitude is a measure of the peak power transmitted by the wave (10 W ) and the frequency is described as $f=1 / \mathrm{T}$, where T is the period of the wave. The two waves are said
to be $180^{\circ}$ out of phase from one another, indicated by the shift of Wave 2 in time. The particular shift shown above indicates destructive interference or a superposition of two waves that create a resultant wave with smaller amplitude. The case shown in Figure 2.2 describes a wave that has a phase change of $\pi$ radians or $180^{\circ}$ resulting in a net amplitude of zero. Conversely two waves can also have constructive interference in which the waves add together providing a larger amplitude than either individual wave. In this case a maximum constructive interference occurs when the phase change is equal to $2 m \pi$ radians, where $m$ is any integer (10).

Radars operate at frequencies as low as a few megahertz or frequencies higher than 240 GHz (4). The highest frequencies of tracking systems are those of laser systems called lidars (light detection and ranging) and have an upper frequency limit of approximately $3 \times 10^{14} \mathrm{~Hz}$ (6). This has changed from radars of the World War II era that operated at frequencies ranging from 100 MHz to 36 GHz .

During World War II a coding system was established to designate the different frequency bands. The original purpose was one of secrecy, but because of the ease in describing a region of radar operation the coding was continued and is still used today. Table 2.1 lists the radar frequency letter band designation approved as an IEEE Standard (11). The ranges are allocated by the International Telecommunications Union (ITU). Therefore some of the frequencies within the allocated range are not allowed for operation. For example the Lband is designated for between $1-2 \mathrm{GHz}$, but actual operational frequencies are only allowed to operate within the range of $1.215-1.4 \mathrm{GHz}$ since that is the band assigned by the ITU (4).

The best way to determine the frequency needed for an application may be to discuss some problems with using different frequencies. Low frequencies, those lower than the HF band;

- require very large antennas
- the atmosphere produces large amounts of scatter, creating unwanted echoes
- the time required to process velocity information due to the Doppler shift take much longer due to the small changes in frequency when signals are scattered from moving targets
- logistical problems with obtaining transmitting licenses due to many frequencies being used for other purposes.

At the higher end of the spectrum, most problems encountered are due to atmospheric absorption and the limits of technology.

Table 2.1: IEEE standard radar-frequency letter-band nomenclature.

| Band Designation | Nominal Frequency Range | Specific Frequency Range for <br> Radar based on ITU <br> Assignments in Region 2 |
| :---: | :---: | :---: |
| HF | $3-30 \mathrm{MHz}$ |  |
| VHF | $30-300 \mathrm{MHz}$ | $\begin{aligned} & 138-144 \mathrm{MHz} \\ & 216-225 \mathrm{MHz} \end{aligned}$ |
| UHF | $300-1000 \mathrm{MHz}$ | $\begin{aligned} & \hline 420-450 \mathrm{MHz} \\ & 850-942 \mathrm{MHz} \\ & \hline \end{aligned}$ |
| L | $1-2 \mathrm{GHz}$ | $1215-1400 \mathrm{MHz}$ |
| S | $2-4 \mathrm{GHz}$ | $\begin{aligned} & 2300-2500 \mathrm{MHz} \\ & 2700-3700 \mathrm{MHz} \end{aligned}$ |
| c | 48 GHz | $5250-5925 \mathrm{MHz}$ |
| X | $8-12 \mathrm{GHz}$ | $8500-10680 \mathrm{MHz}$ |
| $\mathrm{K}_{\text {u }}$ | $12-18 \mathrm{GHz}$ | $\begin{aligned} & 13.4-14.0 \mathrm{GHz} \\ & 15.7-17.7 \mathrm{GHz} \end{aligned}$ |
| K | $18-27 \mathrm{GHz}$ | $24.05-24.25 \mathrm{GHz}$ |
| K, | $27-40 \mathrm{GHz}$ | $33.4-36 \mathrm{GHz}$ |
| V | 40.75 GHz | $59-64 \mathrm{GHz}$ |
| W | $75-110 \mathrm{GHz}$ | $\begin{array}{r} 76-81 \mathrm{GHz} \\ 92-100 \mathrm{GHz} \\ \hline \end{array}$ |
| mm | $110-300 \mathrm{GHz}$ | $\begin{aligned} & 126-142 \mathrm{GHz} \\ & 144-149 \mathrm{GHz} \\ & 231-235 \mathrm{GHz} \\ & 238-248 \mathrm{GHz} \end{aligned}$ |

The term electromagnetic wave is used to describe the transmission of energy both electric and magnetic (10). The energy has both wave and particle properties of which the frequency covers a wide range. Conventional radars operate in the microwave region (12), which is loosely defined as lying between the radio and infrared regions or $10^{6} \mathrm{~Hz}$ to $10^{12} \mathrm{~Hz}$. Figure 2.3 shows the different spectrums of electromagnetic waves and the corresponding frequencies.


Figure 2.3: Frequency spectrum for electromagnetic waves.

## Section 2.4 Radar antennas

The antenna is usually the part of the system that is visible and best understood by the general public. It is arguably the most important device of a radar system and it is the one component that is shared by all radar systems. For this reason the antenna is the only component of the radar that will be discussed in this work.

Antennas come in many different sizes and shapes and have different functionality, but all antennas have the same basic job of emitting and receiving the radar signal. They can be summarized by four main functions (6) (4).

1. To project power from the transmitter to space, concentrating it in the direction of the target.
2. Collect the reflected energy scattered back to the radar from a target.
3. To allow the measurement of the angular position of the object from the reflected signal.
4. Provide the desired volumetric coverage of the radar.

In describing antennas it is pertinent to define several important antenna parameters. The first parameter is the antennas directive gain $G_{D}$, or directivity. The directivity is descriptive of the nature of the radiation pattern (1), and for the transmitting antenna is defined by,

$$
\begin{equation*}
G_{D}=\frac{4 \pi P(\theta, \phi)_{\max }}{\iint P(\theta, \phi) d \theta d \phi} \tag{2.2}
\end{equation*}
$$

where $P(\theta, \phi)_{\max }$ is the maximum power radiated per unit angle and the denominator represents the total power radiated by the antenna. The maximum power radiated per unit
angle is obtained from inspection, while the total power radiated is found from an integration of the volume under the radiation pattern. The ratio of the two is sometimes referred to as the beam width $(B)$ and it is defined by,

$$
\begin{equation*}
B=\frac{\iint P(\theta, \phi) d \theta d \phi}{P(\theta, \phi)_{\max }} \tag{2.3}
\end{equation*}
$$

Therefore equation 2.3 can be simplified to,

$$
\begin{equation*}
G_{D}=\frac{4 \pi}{B} \tag{2.4}
\end{equation*}
$$

Directivity is very important and defines how much radiated power is directed in a particular direction. A good example of this would be to compare a cell phone antenna with a satellite dish antenna. A cell phone antenna would have a very low directivity because it needs to be able to send and receive radiation in many different directions, while a satellite dish is very directive, emitting and receiving in a specific direction.

A second parameter is called the power gain, $G$, commonly referred to as the gain. The gain is similar to the directive gain except that it takes into account dissipative losses (12). The mathematical description is similar to the $G_{D}$ except the denominator is the net power received by the antenna from the transmitter. The mathematical expression for the power gain is,

$$
\begin{equation*}
G=\frac{4 \pi P(\theta \phi)_{\max }}{P_{t}(\text { received })} \tag{2.5}
\end{equation*}
$$

Because the gain takes into account dissipative losses it is the typical descriptor of any radar antenna and is the parameter used by engineers when designing radars. The relationship between the two is defined by,

$$
\begin{equation*}
G=\rho_{r} G_{D} \tag{2.6}
\end{equation*}
$$

where $\rho_{r}$ is the radiation efficiency (4). While the directive gain is always larger than the power gain, the difference between the two is usually small for most types of antennas.

The relationship between the gain of the antenna and the power at a specific distance from the radar is called the power density $\left(P_{\rho}\right)(4)$ and is defined by,

$$
\begin{equation*}
P_{\rho}=\frac{P_{t} G}{4 \pi R^{2}} \tag{2.7}
\end{equation*}
$$

where $P_{t}$ is the transmitter power and the denominator represents the surface area defined by a sphere and range of the object from the receiver. The power density that is returned to the antenna is of great importance since the energy is reradiated in many different directions. This is equated using,

$$
\begin{equation*}
P_{\rho}(\text { returned })=\left(\frac{P_{t} G}{4 \pi R^{2}}\right)\left(\frac{\sigma}{4 \pi R^{2}}\right) \tag{2.8}
\end{equation*}
$$

where $\sigma$ is the radar cross section of the target and is used to describe the objects ability to reflect energy back in the direction of the radar. This should not be confused with the physical cross sectional area of the object. It can be calculated for complex targets by solving Maxwell's equations with the correct boundary conditions or by using computer modeling (4). The radar cross section for a sphere is shown in Figure 2.4, as a function of $2 \pi r / \lambda$, the circumference measured in wavelengths. The cross section is shown here to be normalized by the projected physical area of the sphere, $\pi r^{2}$. The definitions described above were given in terms of a transmitting antenna but apply to receiving antennas as well.


Figure 2.4: Normalized radar cross section of a sphere as a function of its circumference

It is common to discuss gain, both power and directive, as a function of the antenna's emitted angle or beam angle. Beam angles are expressed in two ways, the elevation and azimuth angles. The elevation angle is a measure of the vertical angle from the ground, while the azimuth angle is a horizontal measure. The angle of the beam is defined by the shape of the radiating signal or beam pattern. Figure 2.5 shows several typical patterns. A typical antenna used to track aircraft would use a pencil-beam pattern, seen in Figure 2.5a, where the beam width is about 1 or $2^{\circ}$ in both horizontal and vertical directions (12). This small angle gives accurate location measurements and great resolution of both azimuth and elevation angles. The pencil beam is typical in tracking radars and phased array radars (4).


Figure 2.5: Typical antenna beam patterns: (a) pencil beam; (b) fan beam; (c) stacked beams; (d) shaped beam

The fan-beam pattern as seen in Figure 2.5 b has a small angle in one direction compared to the other direction. In air surveillance radars using fan-beam patterns the azimuth angle may only be one degree, while the elevation angle may be between 4-10 times the azimuth angle (4). They are typically used in applications where the area being searched is a large volume of space. The narrow beam width in the azimuth angle gives accurate measurements with small search area, while the large angle in the elevation gives poor accuracy and a large search area. This would work well for 2-D tracking where the range and azimuth are needed. The same search area can be achieved with a stacked pencil-beam pattern seen in Figure 2.5c, but with added resolution in the stacked direction.

A fan-beam would not work well for tracking aircraft at high altitudes close to the radar. The detection of these targets would require a broad fan-beam, resulting in a very low gain. To
obtain a better detecting area at these parameters a shaped beam, like the one depicted in Figure 2.5d, would be used. A typical shaped beam employs a fan-beam pattern, modifying the gain in the elevation angle so that it is proportional to the square of the cosecant of the elevation angle (4).

There are two broad categories for classifying radar antennas, optical antennas and array antennas (12). Optical antennas are made up of antennas that work based on optical principles of wave propagation and include, mainly, reflector antennas and lens antennas. Reflector antennas are widely used for satellite dish television. For the purpose of this work, where a phased array antenna was used, optical antennas will not be discussed.

A phased array antenna is an antenna comprised of an array of smaller radiating elements with independent phase control. The array concept was originally introduced as a means for achieving a better control of the antenna aperture illumination (13). A typical array is depicted in Figure 2.6, showing identical elements spaced evenly in the x and y directions.


Figure 2.6: Planar phased array antenna with $4 \times 4$ elements.
This particular setup constitutes a planar array, although there are many different configurations. The system used for this work was similar to that depicted in Figure 2.6. In general all the previous information on individual antennas applies to array antennas. The major consideration when using array antennas is the spacing of the elements. To avoid multiple beams, or grating lobes, the elements need to be spaced a distance of $\lambda / 2$ apart (12). A grating lobe is of the same magnitude as the main beam and can cause ambiguities in the return signal. This can be seen in Figure 2.7, where the main beam is emitted in the $0^{\circ}$ direction and the grating lobe returns in the $180^{\circ}$ direction


Figure 2.7: Antenna showing side and grating lobes

## Section 2.5 Radar calculations

Once the signal is transmitted the radar waits for the return signal to determine a specific measurement of the object. The range or distance $(R)$ of the target is found by measuring the time $\left(T_{r}\right)$ it takes for the radar signal to travel to the target and return to the radar. The relationship is,

$$
\begin{equation*}
R=\frac{c T_{r}}{2} \tag{2.9}
\end{equation*}
$$

where the distance traveled is divided by two to take into account the trip to and from the object. The ability to measure range accurately under adverse weather conditions and at very long distances is one of the most important aspects of any radar. Ground-based radars used to
track aircraft can be made to track at distances only limited by their line of sight, generally 200250 nautical miles (nmi), at an accuracy of a few tens of meters (12).

The radar range (4) is useful to not only determine the maximum range of particular design, but also aids in the understanding of the factors affecting radar performance. The maximum range of a radar is defined by,

$$
\begin{equation*}
R_{\max }=\left[\frac{P_{t} G A_{e}}{(4 \pi)^{2} S_{\min }}\right]^{1 / 4} \tag{2.10}
\end{equation*}
$$

where $A_{e}$ is known as the effective area of the receiving antenna and is related to the physical area $A$ by the relationship $A_{e}=\rho_{a} A$. The term $\rho_{a}$ is called the antenna aperture efficiency. The minimum detectable signal $\left(S_{\text {min }}\right)$ is a statistical measure of the smallest measure of a return signal in the presence of noise. Equation 2.10 represents the simplest form of the radar equation and does not give an accurate measure of the maximum range. It does however show the relationship between design parameters of the radar and how they affect the performance of the radar.

The range of the object is only part of the information required to fully define the objects location in space. The second part needed are two angles of the object relative to the radar. There are two typical ways to calculate the angle based on measurements of either the amplitude or phase of the return signal. Both of which will be discussed using the monopulse processing technique, but emphasis will be given to the phase comparison technique because it was the technique used by the radar for this work. The term monopulse describes a way in which the location of an object is tracked by comparing signals received in two or more simultaneous beams emitted by a single pulse (4).

Using phase comparison monopulse principles a single pulse is emitted by a directive antenna and received by two separate antennas. These antennas are placed at a known distance from one another. The two antennas look in the same direction, and cover the same region of space. The amplitudes of the reflected signals are the same, but their phases are different. This is depicted in Figure 2.8 showing two antennas spaced a distance $d$ apart.


Figure 2.8: Phase-comparison monopulse in one angel coordinate
If the signal in Figure 2.8 arrives at an angle $\theta$ with respect to the ground, the phase difference in the signals received in the two antennas is,

$$
\begin{equation*}
\Delta \phi=2 \pi \frac{d}{\lambda} \sin \theta \tag{2.11}
\end{equation*}
$$

The phase difference occurs because the two waves are traveling at the same speed with the same frequencies, but one must travel slightly longer than the other. The measurement in the phase difference between the two signals provides the angle $\theta$ to the object. A setup with the antennas placed above one another provides an elevation angle, by adding another antenna to the left or right of one of these antennas provides the azimuth angle.

One factor in the accuracy of the measured phase change in equation 2.11 is the distance $d$ that the antennas are spaced. If $d$ is large compared to the diameter of the antenna then high sidelobes are produced causing ambiguities in the angle measurement (4). To limit the intensity of the sidelobes radars are typically designed so that $d$ is less than the diameter of the antennas.

A second technique used to determine the angle of a tracked object is called amplitude comparison monopulse. It is similar in principle to that of the phase comparison, but it compares the change of amplitude of reflected signals that are pointed in slightly different directions. Because the two beams are slightly displaced in angle, the amplitudes of the return signals are slightly different. This difference is proportional to the angular displacement (14). To increase the accuracy of the angle, several antenna beams can be used simultaneously, increasing the comparison between adjacent beams.

Another important aspect of locating and tracking an object is the object's speed. This is measured by taking advantage of the Doppler effect. This phenomenon can be described by the changing pitch of a fire truck as it travels toward or away from the listener. This phenomenon happens because the emitted sound waves of the fire truck as it is approaching the listener are compressing. That is, each successive wave crest is emitted from a position closer to the observer than the previous crest. Therefore the time between the arrivals of successive wave crests at the observer is reduced, causing an increase in the frequency. The opposite is also true for listeners where the fire truck is moving away.

The radar relies on the Doppler Effect to determine the speed and radial direction of objects. If an object is moving toward the radar, the frequency of the return signal is slightly
higher than the originating signal; and if it is moving away, the frequency is slightly lower (15). Knowing the range from the target to the radar, the total number of wavelengths for the full path of the signal is $2 R / \lambda$. Every wavelength equates to a phase change of $2 \pi$ radians, therefore the total phase change is,

$$
\begin{equation*}
\phi=2 \pi\left(\frac{2 R}{\lambda}\right)=4 \pi R / \lambda \tag{2.12}
\end{equation*}
$$

When the target is moving relative to the radar, $R$ will also be change resulting in a change of $\phi$. Differentiating equation 2.12 with respect to time results in the rate change of the phase, or the angular frequency,

$$
\begin{equation*}
\omega_{d}=\frac{d \phi}{d t}=\frac{4 \pi}{\lambda} \frac{d R}{d t}=\frac{4 \pi V_{r}}{\lambda}=2 \pi f_{d} \tag{2.13}
\end{equation*}
$$

where $V_{r}=d R / d t$ is the radial velocity and $f_{d}$ is the Doppler frequency shift. From equation 2.13 a relationship between the radial velocity and the Doppler frequency shift can be made,

$$
\begin{equation*}
f_{d}=\frac{2 V_{r}}{\lambda} \tag{2.14}
\end{equation*}
$$

where $V_{r}$ is a measure of the radial component of the objects velocity. This is illustrated in Figure 2.9.


Figure 2.9: Geometry of radar and target in deriving the Doppler frequency shift.

## Section 2.6 Radar types

There are two primary classifications of radars, pulse wave and continuous-wave radars. The majority of systems used today operate as pulse wave systems because of the ability to directly measure the range of the object being tracked. A pulse wave system is also more reliable at a longer distance, which is increasingly becoming a prominent design parameter.

Pulse radars that measure the Doppler frequency shift are called either moving target indication (MTI) or pulse Doppler radars. As the name sounds they work by emitting a signal from an antenna and wait for the reflected signal to return before emitting another pulse. The time in between pulses depends on the range of the target. Because the radar is never trying to receive a signal while it is also transmitting only one antenna is needed for both operations. This is compared to a system that is constantly emitting and receiving a signal called a
continuous-wave (CW) radar system (15). Because the system is constantly emitting a signal a separate antenna is needed to receive the reflected signal.

The major difference between the two systems is the ability to directly measure the range of an object. From equation 2.9, the ability to measure the range of an object depends on the ability to measure the time it takes for the signal to return to the radar. With a radar that is constantly emitting a signal, time cannot be measured. Therefore range information must be determined by some other method. While the manner in which range is determined by the system varies between a pulse and CW systems, the range performance for the same average transmitted power are approximately the same (16). It should be noted that CW Doppler radar cannot be converted to pulse radar by simply turning on and off the emitted signal to generate pulses.

## Section 2.7 Radar noise and filtering

Signal noise is inherent in any radar and is of concern when building and using a radar system. There are many different sources of noise experienced by different types of radars, but only three sources will be discussed in this work. Noise can come from within the system sometimes called noise temperature, which exists in a receiver and is due to resistive circuits in the unit (12). Another type of noise is referred to as clutter. Clutter is defined as unwanted echoes from the natural environment (4). The third type of noise experienced by a radar system is known as multipath reflections.

At microwave frequencies, the noise that competes with the reflected signal is usually generated from within the receiver itself. This internal noise is called noise temperature or
thermal noise and is generated by the random motion of thermally excited electrons (4). Below operating frequencies of about 6000 GHz , thermal noise is considered white noise. White noise is a random signal with a flat power spectral density $S(f)$ defined by,

$$
\begin{equation*}
S(f)=k T_{0} \tag{2.15}
\end{equation*}
$$

where k is Boltzmann's constant $\left(1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}\right)$ and $T_{0}$ is the system temperature ( K ) (6). This means that the signal contains equal power within a fixed bandwidth at any frequency. The noise power delivered to the receiver $\left(P_{n}\right)$ can be equated using,

$$
\begin{equation*}
P_{n}=K T_{0} B_{n} \tag{2.16}
\end{equation*}
$$

where $B_{n}$ is the bandwidth (12). This function is used to calculate the signal-to-noise ratio, which is used in radar design.

Unwanted reflections of the radar signal from the environment are classified as clutter. Clutter is divided in two different categories, surface clutter and volume clutter (4). Reflections of land and sea are classified as surface clutter and those associated with the environment are considered volume clutter. Surface clutter $\left(\sigma^{0}\right)$ is described mathematically as the ratio of the radar cross section $\left(\sigma_{c}\right)$ to the area occupied by the clutter $\left(A_{c}\right)(4)$,

$$
\begin{equation*}
\sigma^{0}=\frac{\sigma_{c}}{A_{c}} \tag{2.17}
\end{equation*}
$$

The symbol $\sigma^{0}$ is sometimes called the scattering coefficient, differential scattering cross section, normalized radar reflectivity, back scattering coefficient, or normalized radar cross section. Similarly, the clutter associated with volume clutter can be characterized by $\eta$, often called the reflectivity (4),

$$
\begin{equation*}
\eta=\frac{\sigma_{c}}{V_{c}} \tag{2.18}
\end{equation*}
$$

The reflectivity is a ratio of the radar cross section and the clutter that occupies a volume, $V_{c}$. Large clutter adds to the difficulty of detecting and tracking specific objects by masking their reflected signal. There are many ways to design around or minimize the effect clutter has on the operation of the radar.

Sometimes the target itself provides scatter, this is called glint. Glint is the angle noise experienced with objects of complex geometries such as aircraft (4). The reflections from glint further add to the difficulty in determining a correct target and then successfully tracking it. For the purposes of this work where a spherical object is being tracked glint is not apparent.

The final source of error discussed in this work is called multipath angle error or sometimes low-angle error and is due to the target's return signal reflecting off of other surfaces or objects on its way back to the radar. The geometry of multipath reflections is shown in Figure 2.10. This surface-reflected signal causes a significant error in the elevation measurement of the radar (12). The reflected signal undergoes a phase change because of the difference in path length of the direct path and the surface-reflected path. When the difference is $\lambda / 2$, the signals are additive and a larger signal is received by the radar causing a maximum interference (6). When the difference is $\lambda$ the two signals nearly cancel each other out. This results in large fluctuations of amplitude and phase of the return signal.


Figure 2.10: Geometry of multipath reflection where the reflection from the surface appears to the radar as an image below the surface.

The magnitude of the multipath error is largely dependent on the reflection coefficient of the reflecting surface, but it also depends on the angle between the direct path and the radar's horizon $(\theta)$ (17). Using equation 2.11 and assuming $d=10 \lambda$ (typical of 10 GHz radar), $\theta$ was calculated at different phase differences $(\Delta \phi)$. Then a phase error from multipath reflections of $\pi / 4(18)$ was added to $\Delta \phi$, and $\theta$ was re-calculated. The results of the difference between the vertical angle of the object by equation 2.11 and the vertical angle with the assumed multipath error can be seen in Table 2.2

Table 2.2: The effects of multipath error on vertical angle measurements

| $\Delta \phi$ | $\Theta$ (deg.) | $\Theta$ (deg.) w/error | $\mathrm{d} \theta$ (deg.) | Difference (\%) |
| :---: | :---: | :---: | :---: | :---: |
| $5 \pi / 2$ | 7.2 | 7.9 | 0.7 | 0.10 |
| $11 \pi / 2$ | 16.0 | 16.7 | 0.7 | 0.05 |
| $21 \pi / 2$ | 31.7 | 32.5 | 0.8 | 0.03 |


| $37 \pi / 2$ | 67.7 | 69.6 | 2.0 | 0.03 |
| :---: | :---: | :---: | :---: | :---: |

Table 2.2 illustrates how multipath error affects the elevation angle measurement of an object. Objects tracked at smaller angles to the horizon of the radar increase the effect of multipath error. Multipath error also affects the azimuth angle measurement, but the effects are more reliant on the signal strength of the direct signal compared to the signal strength of the surface reflected path. If the direct signal is stronger than the surface reflected path signal, then the ability for the radar to determine the correct signal is much easier and the error created by the multipath reflection is minimized. Regardless, a multipath signal will always be present when tracking objects close to the surface of the ground.

Detection of a reflected signal in the presence of noise is of concern when using a radar system. The ability to detect a weak reflected signal is limited by the amount of noise occupying the same part of the frequency spectrum as the reflected signal. A common application to overcome this problem is called threshold detection (4). A threshold is established based on the amplitude of the signal. If the signal breaks this threshold then an object is present. This leads to the idea of the probability of detecting an object in a noisy environment and the signal-to-noise-ratio (SNR).

The SNR is related to the radar equation (equation 2.10) (18) and can be represented in its simplest form by,

$$
\begin{equation*}
S N R=\frac{P_{t} G A_{e} \sigma}{(4 \pi)^{2} K T_{0} B F_{n} R_{\max }{ }^{4}} \tag{2.19}
\end{equation*}
$$

where $F_{n}$ is a noise figure and is defined as the measure of the noise out of a real receiver to that of an ideal receiver with only thermal noise. The minimum SNR is used in determining a
specified probability of detection and probability of false alarm. These parameters are important in radar design and analysis.

Noise from the environment or from the unit itself not only impedes the radar's ability to determine the presence of a target but also injects error in the tracking measurements of a system. Measurement error due to different sources is inherent in radar, but with careful evaluation and recognition the effects can be minimized. One particular way, providing that the statistical model is accurate, is with use of a Kalman filtering process (14). There are many techniques and different types of Kalman filters.

Published in 1960, R. E. Kalman described a recursive solution to the discrete-data linear filtering problem (19). Since then, the Kalman filter has evolved into many different forms including non-linear applications (20). The filter provides an efficient computational means to estimate the state of a process. The equations can be split up in to two groups; the time update equations and the measurement update equations (19). The time update equations can be described as predictor equations that project the current state forward as an estimate for the next time step. The measurement update equations provide feedback to incorporate into the predictor equations for the next prediction.

When being used with tracking software, the Kalman filter equations require specific details for the measurement error model, as well as a model for the target trajectory and the uncertainty of the trajectory (4). The accuracy of the final data depends on the models used by the filter, and in turn depends on the measurement quality, data rate, and sensor specific resolutions (18). Therefore determining the correct models to be used by the Kalman filter plays an important role in the quality of the filtered data.

## Section 2.8 Error in radar signals

The major limitation of the accuracy of any radar measurement is noise. While the understanding of the effect of noise is beneficial, quantifying noise allows the accuracy of radar measurements to be estimated. In this section the theoretical measure of error is outlined (4) and presented in the form of the root mean square (rms). It is assumed that the signal-to-noise ratio is large and that each measured parameter has independent errors. For simplification, it is also assumed that the radar waveform is a perfectly rectangular pulse as shown in Figure 2.11. This is of course not the case, but gives an accurate measure of the rms.


Figure 2.11: Measurement of time delay using the leading edge of the signal pulse.

The measurement of range is done by measuring the time it takes for the signal to make a full round trip path (4). The rms error is then,

$$
\begin{equation*}
\delta R=(c / 2) \delta T_{r} \tag{2.20}
\end{equation*}
$$

The error associated with measuring the time is based on the pulse of the signal. In Figure 2.11 a hypothetical rectangular pulse with associated noise is shown. The effect of noise is to shift
the time that the leading edge or trailing edge of the pulse crosses the threshold. This results in an error in the time measurement $\left(\Delta T_{R}\right)$. This error can be equated by comparing the slopes of both curves, this results in the following equation for the error (4) in the time of the signal and ultimately the error in the range measurement,

$$
\begin{equation*}
\delta T_{R}=\left(\frac{\tau}{2 B E / N_{0}}\right)^{1 / 2} \tag{2.21}
\end{equation*}
$$

where $E$ is the received signal energy, $N_{0}$ is the noise per unit bandwidth, and $B$ is the spectral bandwidth of the rectangular shaped filters.

The measurement of frequency in radar is of concern when determining an object's radial velocity. The actual calculation of $V_{r}$ from the measurement of the frequency is done by means of the Doppler frequency shift. The important aspect is that the rms error in the radial velocity is directly related to the rms error in the Doppler frequency $\left(\delta f_{d}\right)$ by,

$$
\begin{equation*}
\delta V_{r}=(\lambda / 2) \delta f_{d} \tag{2.22}
\end{equation*}
$$

Assuming a perfectly rectangular pulse of width $\tau$ the rms frequency error is

$$
\begin{equation*}
\delta f=\frac{\sqrt{3}}{\pi \tau\left(2 E / N_{0}\right)^{1 / 2}} \tag{2.23}
\end{equation*}
$$

where $E$ is the received signal energy and $N_{0}$ is the noise per unit bandwidth. Equation 2.23 illustrates the importance of the pulse width to the accuracy of the frequency measurement, showing a longer pulse width equates to a more accurate frequency measurement. The previous equations apply equally to a CW system in which the observation time is equivalent to the pulse duration (4).

A similar expression for the rms error in the angular measurement of a radar can be made. This expression is,

$$
\begin{equation*}
\delta \theta=\frac{\sqrt{3} \lambda}{\pi D\left(2 E / N_{0}\right)^{1 / 2}} \tag{2.24}
\end{equation*}
$$

where $D$ is the dimension of the antenna aperture and all other symbols have their stated meanings. The expressions for determining the rms values of range, radial velocity and angle are all similar even though the way in which each is measured is different. This shows that each measurement is ultimately related to the signal that was emitted and received.

## Section 2.9 Summary

This chapter provided a brief introduction to the operation of radar, including advantages and disadvantages to signals at different frequencies. Radars using frequencies lower than 30 MHz require large antennas and do not provide the needed resolution that is provided with higher frequencies.

Different transmitting radars were discussed. Radar systems using the CW method of transmitting, lack the long range accuracy provided when using a pulse wave system. The advantage of a CW system was the continuous collection of data, which allows for the measurement of things like spin rate.

Radars that calculate velocity do so by taking advantage of the Doppler frequency shift of moving objects. The elevation and azimuth angles are calculated by either comparing the difference between phase or amplitude of a return signal determined by at least two separate antennas. The monopulse phase comparison technique was discussed in detail. The technique relied on a single emitted wave being received by at least two separate antennas. The
difference in phase measured between the separate receiving antennas was proportional to the angle in which the signal was received.

A limiting factor when collecting data using radar is the susceptibility of radar signals to noise. Error from multipath reflections are important, and affect the elevation measurement. The magnitude and effect of the noise from multipath reflections is directly related to the elevation of the ball and ultimately the angle that the signal makes with the reflecting surface.

## Chapter 3 - TrackMan Radar

The following sections provide details about the company and unit used for this work. It defines operational parameters of the unit and proper setup procedures to help minimize noise in the signal. It explains the user interfaces and shows how they can be used to determine problems with the system setup or calibration. Finally, explanations of different data sets recorded and displayed by the radar are defined.

## Section 3.1 TrackMan background

TrackMan A/S is a Danish technology company that develops, manufactures, and sells 3D ball flight measurement equipment used in sports. They started developing a system for tracking golf balls in 2003 called a TrackMan radar or just TrackMan. The unit is used by professionals to analyze golf swings and determine efficiencies of ball club impacts. The golf unit is portable and can quickly and easily be setup, allowing for efficient data collection in short periods of time. The unit is setup on the ground behind the T-box pointing toward the player. As the player begins their swing the unit begins to track the club head. Once the ball is impacted the unit tracks the ball until the signal is lost or the ball hits the ground. It then displays information about the swing, ball contact, and ball trajectory. The data can be used by players to help improve their swing, and also by golf developers in measuring product information and performance. TrackMan A/S reports the accuracy of one particular golf model to be one foot at 100 yards or $.33 \%$ accurate.

More recently TrackMan A/S shifted their focus to developing a system for baseball and softball. The new unit reports data about both the pitch and hit simultaneously. These units are
used by many major league teams and different manufacturers in the baseball and softball community. It uses a similar setup as the golf unit except that it is positioned in the air on a tripod. The baseball unit also lacks the ability to track the bat. The differences in the geometry between a golf club and a bat make it difficult to.

The initial setup requires the radar to be placed vertically in the air maintaining a 3:1 ratio between the distance from home plate and the height of the unit. This is an optimal ratio determined by TrackMan A/S to optimize the field of view and minimize noise caused by multipath reflections. Positioning the radar higher in the air keeps the players on the field from blocking the line of sight to the ball.

Because multipath reflections are a product of low angle tracking, the positioning of the radar needs to be such that the return signal is of the greatest possible magnitude. This means positioning the radar such that the ball is at a minimal vertical and horizontal angle to the antennas in the radar, increasing the likelihood that the position of the ball during the pitch and hit will be directly in line with the emitted signal.

The 3:1 position ratio also increases the angle that the multipath reflection makes with the ground, as depicted in Figure 3.1. The multipath signal is represented by the dashed line shown for a hit at three locations in its trajectory. When the radar is closer to the ground the multipath signal makes a smaller angle when reflected off of the ground, resulting in a smaller angle at the antenna. The steeper angle of the multipath signal when the radar is raised in the air, results in a larger attenuation of the multipath signal making the effects on the actual signal much less.


Figure 3.1: Schematic showing how the height of the radar affects the angle that the multipath signal returns to the radar.

## Section 3.2 TrackMan II X

There are currently four different units in use for baseball and softball; TrackMan II X, TrackMan II Stadium, TrackMan III Stadium, and TrackMan III Portable. The model II X uses a constant starting range of the ball based on the pitching mound distance and the average stride of a pitcher. The model II Stadium and the model III's measure range based on an analysis of the frequency of the return signal. While there are several published ways to determine range from an analysis of the frequency of the return signal, the actual method was not disclosed by TrackMan A/S. Measuring the range for each hit, allows for a more accurate calculation of the
position of the ball. The TrackMan II X and TrackMan III Portable are portable versions that can quickly and easily be setup with only the use of a tape measure. The other versions require more precise setup and measurements for the calibrations portion and are not considered portable.

The unit used for this work was a TrackMan II X, and is depicted in Figure 3.2. It operates in the X-band frequency spectrum at about 10.5 GHz . It is a CW system with one emitting antenna and three receiving antennas. All the antennas are planar array antennas consisting of a $4 \times 4$ array.


Figure 3.2: Front view of the TrackMan II X used for this work.

The coordinate system of the radar uses the positive $x$-direction pointing toward the pitcher, level with the ground, positive y pointing upward, and positive z pointing to the catcher's right. Therefore a pitched ball is primarily moving in the negative x-direction. This can be seen in Figure 3.3. The field coordinate system can be seen starting at the tip of home
plate with the $X_{f}, Y_{f}$, and $Z_{f}$ axis pointing in the same direction as the radar coordinate system. The position of the ball when reported to the user is with respect to the field coordinate system.


Figure 3.3: Coordinate system of the radar and field.

The TrackMan II X uses a self-leveling system to keep the antennas pointed in an optimal angle of $-4^{\circ}$ tilted about the $Z_{r}$ axis. This four degree tilt allows the radar to be pointed slightly upward optimizing for tracking fly balls. The leveling system uses two legs on the bottom of the radar to also level about the $X_{r}$ axis so that the radar is level with the ground. The leveling legs can be seen in Figure 3.4.


Figure 3.4: Back view of the TrackMan II X used for this work

The radar is connected to a laptop using a standard USB connector. It has the capability to run on standard DC power or from a battery pack attached to the rear of the radar. The ports for the power and USB connections can also be seen in Figure 3.4. These connections are all water safe connections. In the event of a rain delay the USB connection should be removed from the laptop so that the laptop can be properly stowed, but the radar can remain out without fear of damage.

## Section 3.3 TrackMan interface and calibration

The interface program was developed by TrackMan A/S and allows for all aspects of a team including players and their defensive positions to be input by the user. Inputting the
location of the game, teams, players and player's positions is required. Once game and team information are input, the radar requires some basic calibration to orient itself with the field of play. The height of the radar from the ground to the middle of the " $K$ " on the face of the radar must be measured, along with the distance from the face of the radar to the tip of home plate. Finally, using a camera on the front of the radar, a picture of the batter's box area is taken like that shown in Figure 3.5. The user then moves a red indicator on the screen to the tip of home plate as shown in Figure 3.5. In the bottom left of the screen a close-up picture is used to accurately find the tip of home plate.


Figure 3.5: Calibration picture used to locate the tip of home plate.

This gives the radar a reference to determine its position in the field of play and allows the recorded data to be transformed to the coordinates of the field shown in Figure 3.3. This
means measuring each distance as accurately as possible while also aligning the radar in a straight line path toward the pitcher. In some instances it is not possible to place the radar directly in-line with the catcher and pitcher. The TrackMan II X then allows for an offset toward first or third base as described in Appendix A.

Pre-assigned settings have been defined by TrackMan A/S based on the type of game being tracked and must be selected by the user before tracking can begin. These settings change the initial range of the ball, and are changed using a program prompt on the desktop. Table 3.1 gives the three setups used for this work with assumed pitcher strides.

Table 3.1: Pitch distances and assumed pitcher strides for the radar's initial range assumptions for three different game settings

| Game Setting | Pitch distance $\left(\mathbf{d}_{\mathbf{p}}\right)$ | Pitcher's stride | Release locations |
| :--- | :--- | :--- | :--- |
| Baseball | $18.4 \mathrm{~m}(60.5 \mathrm{ft})$ | $1.8 \mathrm{~m}(6 \mathrm{ft})$ | $16.6 \mathrm{~m}(54.5 \mathrm{ft})$ |
| Slow pitch softball | $15.2 \mathrm{~m}(50 \mathrm{ft})$ | $1.1 \mathrm{~m}(3.5 \mathrm{ft})$ | $14.1 \mathrm{~m}(46.5 \mathrm{ft})$ |
| Fast pitch softball | $13.1 \mathrm{~m}(43 \mathrm{ft})$ | $1.5 \mathrm{~m}(5 \mathrm{ft})$ | $11.6 \mathrm{~m}(38 \mathrm{ft})$ |

If the release location of the actual pitch is $1.5-3 \mathrm{~m}(5-10 \mathrm{ft})$ closer to home then the assumed distance the radar will fail to recognize a pitch or resulting hit.

If the calibration portion of the setup or the range to the pitcher's release point were wrong the result would be an inaccurate measure of angle and distance of the ball. The error in the measured angle and distance is directly related to the error in calibration distances and/or error in initial range to the ball.

## Section 3.4 Reasons for radar not tracking

Even with the correct range, the radar will occasionally not track a pitch or more likely, track a pitch, but not track a resulting hit. There are several factors that may have this result. The first is that the hit was a ground ball. The radar was designed to stop tracking when the ball hits the ground, indicated by a drastic change in the trajectory. Because ground balls hit the ground very early in their trajectory the radar does not have time to track them.

The second reason a hit ball may not be tracked is because the ball leaves the field of view of the radar at the beginning of its trajectory. This is typical for foul ball type hits. The reported field of view is $\pm 45^{\circ}$ in the azimuth angle and $-15^{\circ}$ and $45^{\circ}$ in the elevation angle. The tilt of the unit $4^{\circ}$ provides a better field of view in the positive elevation angle.

The third reason for a hit ball not being tracked is due to the frequency or amplitude of the return signal not falling within a threshold of detection. In this case the radar cannot determine that a ball exists and therefore the tracking algorithm is suspended. This may be a product of noise in the environment or possibly a weak signal due to large distances or angles like that experienced during an infield fly ball. In either case, if the return signal does not fall within an undisclosed threshold the radar will not track.

There have been other instances where a well hit fly ball to the outfield is not tracked and it appears that it was due to the system running other executables such as leveling. There are also periodic losses in communication between the radar and the laptop which result in nothing being tracked. These instances appear to be rare and nearly $95 \%$ of the time the tracking runs as it should.

As stated above, the radar cannot track while it is performing other tasks, this includes determining information from a previously tracked ball. That is, the unit needs a window of time to finish its calculations before a subsequent pitch or hit can be tracked. This wait time varies between pitches and hits and is greatly affected by the trajectory of the ball. The typical wait time between successive pitches is 3-5 seconds, if a hit occurs, the wait time increases. For line drive hits where the ball is in the air for only one to one and half seconds the wait time is approximately 5-8 seconds. If the hit is a well hit fly ball to the outfield it is between 10 and 15 seconds. This may change slightly if the ball leaves the field of view of the radar because the unit will wait several seconds to determine if the ball came back into its field of view.

## Section 3.5 TrackMan data

Radar has an advantage over other tracking systems that use high speed cameras in that data is displayed for both pitches and hits seconds after the ball has landed. This is beneficial when the unit is being used to track games or for determining the performance of equipment. The data quantifies and completely defines both the pitch and hit. A typical data set for the pitch and hit displayed for the user would include; data about the release position and speed of the pitch, the location and speed of the pitch and hit at home plate, and the final landing position of the hit. A list and definition of all of the parameters displayed by the radar can be found in Appendix B.

Along with the data about the pitch and hit, data can also be entered in about the play. This data is entered in by the user on the user interface screen shown in Figure 3.6. The player and team information is also displayed so that an entire game can be tracked.


Figure 3.6: The tagging screen used to define the type and quality of both the pitch and hit.

Data about the pitch and hit is displayed on the right hand side of Figure 3.6 under "TrackMan Data." A close up of this data is shown in Figure 3.7. This data along with user input game data shown in Figure 3.8 fully defines a game and allows the system to keep track of balls, strikes, and the innings. If the batting order is input, the system will also automatically cycle through the batters.

| TrackMan Data |
| :--- |
| Pitch |
| Rel. Speed [mph] |
| Rel. Height [ft] |
| Vert. Rel. Angle [deg] |
| Spin Rate [rpm] |
| Spin Axis [deg] |
| Horizontal Break [in] |
| Perc. Horz. Break [in] |
| Vertical Break [in] |
| Plate Loc. Height [ft] |
| Plate Loc. Side [ft] |
| Hit |
| Exit Speed [mph] |
| Vert. Exit Angle [deg] |
| Horz. Exit Angle [deg] |
| Distance [ft] |

Figure 3.7: Portion of Figure3.6 showing the "TrackMan Data" section seen on the user interface.


Figure 3.8: Portion of Figure 3.6 showing the "Tagging" section seen on the user interface.

Once the tracking session is completed the pitch and hit data outlined above and defined in Appendix B, can be exported to a .csv file for editing. The actual data recorded by the radar, called the TrackMan raw data, is available by means of the TrackMan data file or .tmd file. These files can be exported to .xml format (using a separately licensed program called Workbench) that can be opened and edited using Microsoft Excel.

The TrackMan raw data file includes many measured details about the entire trajectory of a single pitch and hit including data that was unfiltered and data that was filtered (Kalman
filter with a modified time stamp). It also shows were the radar loses track of a ball, typically at the end of a hit when the ball has traveled sufficiently far from the radar. The portion of the data in the .xml file used for this work was the radial velocity of the ball ( $V_{r}$ ), the direction cosines $(\alpha, \beta$, and $\gamma)$ of $V_{r}$, and the initial range of the ball $(R)$. These parameters can be seen in Figure 3.9.


Figure 3.9: Relationship of raw $V_{r}, \alpha, \beta, \gamma$, and $R$.

The direction cosines, $\alpha, \beta$, and $\gamma$, are measured with respect to the axes defined by the radar. They include the $-4^{\circ}$ tilt of the radar about $Z_{r}$ and any rotation about the $Y_{r}$ axis. The range of the ball is defined as the distance from the ball to the radar as defined by the user during setup. The data depicted in Figure 3.9 is not filtered by the radar. A full description of the data reported from .xml files and the definitions can be seen in Appendix C.

## Section 3.6 TrackMan Workbench

The filtered and unfiltered data can be processed by the user using the Workbench. The Workbench is an application that allows for the analysis of individual data sets, provides lift and drag, and frequency spectrums like that shown in Figure 3.10. It gives the user the ability to change aspects of the game, including the ball dimensions, radar calibration distances, and weather conditions. This provides the ability to determine the effects they have on the reported data or to change the environment to reflect the current conditions of the game. The Workbench was primarily used to convert .tmd files into files that could be read by Microsoft Excel. This allowed for data along the entire trajectory of both the pitch and hit to be analyzed.


Figure 3.10: Frequency spectrum of the pitch and hit measured by the radar during a slow pitch tournament.

Figure 3.10 is a typical graph of the frequency spectrum of a pitch and subsequent hit. It shows several things about the pitch and hit recorded by the radar. The conversion of the speed from frequency can be seen on the two vertical axes in Figure 3.10. The strength of the
signal measured by the antennas is indicated by the color. A yellow color that fades to red/orange indicates a very strong signal. As the strength of the signal fades, the color on the graph fades to a green and then blue color.

The strong signal that that can be seen at a frequency of zero and remains from the start to the end of the data is a product of the background or ambient noise. This noise is constant and not changing and therefore has very little effect on the signal of the ball. The pitch of the ball is indicated by a strong negative frequency from time 0 to approximately 1.8 seconds. It starts with a quick peak in frequency at time 0 , which is the motion of the pitcher as seen by the radar. The pitch then travels along until it is impacted by the bat, which is indicated by a disturbance at approximately 1.8 seconds. The signal from the pitch does not attenuate along its path from the pitching mound to home plate, since the ball is approaching the radar and is strengthening.

There is then a large positive spike in the frequency just after the disturbance created by the movement of the bat and batter. This spike is the start of the tracking of the hit. The tracking continues until after four seconds when the tracking is suspended. The change in the color of the frequency of the hit indicates that the signal begins to attenuate as the ball travels further from the radar. There are also other disturbances at lower frequencies as the hit ball travels along its trajectory. These are most likely moving players in the infield or other objects in the field of view of the radar. There also appears to be a weaker signal that follows the same frequency path as the pitch and hit, but is opposite in sign. This is due to a mirror in the frequency and is easily determined by the radar therefore has no effect on the accuracy of the data.

The multipath reflection is very weak in Figure 3.10, but would show on the frequency spectrum as a weaker signal just above or below the actual signal. It has nearly the same change in frequency over time as the original signal, but a slightly different magnitude. These frequencies that are slightly higher or lower in magnitude make it difficult for the radar to determine the actual signal. This would result in the ball not being tracked or increased error. A spectrum with larger multipath signals can be seen in Figure 3.11.


Figure 3.11: An FFT of a hit, provided by TrackMan, showing oscillations in the frequency resulting from the spin rate of the ball and multipath reflections at the end of the hit.

The small oscillations in the weaker signal surrounding the beginning of the hit in Figure 3.11 have been determined by TrackMan A/S to be the result of the spinning ball. By analyzing this part of the signal the radar can then measure the spin rate of the pitch and hit ball. The data at the end of the hit in Figure 3.11 shows the effects of multipath signal. These signals are
strong compared to the actual signal of the hit therefore they don't adversely affect the reported data. They do however show that multipath signals are present and provide evidence that multipath signals are mixing with the signal of the ball and therefore would be affecting the accuracy of the data of the hit.

## Section 3.7 Summary

This chapter reviewed how the TrackMan II X, used for this work, operates and reports data. The unit operated in the x-band frequency spectrum at approximately 10.5 GHz . It uses four, $4 \times 4$ planar array antennas, one for emitting and three for receiving. The coordinate system of the radar was defined along with specific setups required by the radar to minimize effects of multipath errors.

The user interface and calibration screens were shown and the importance of the initial range of the radar was discussed. Because the unit used a fixed pitch release location to determine the initial range of the ball, three different pitch distance settings were created by TrackMan A/S. These pre-defined settings and their assumptions were shown for baseball, slow pitch softball, and fast pitch softball. If the proper setting was not used for tracking the result would be error in the angle and distance measurements made by the radar. The result of improper pitch settings chosen by the user may also result in the radar not tracking the pitch and subsequent hit.

The data recorded and displayed by the radar was explained, including how the Workbench application can be used to examine the full trajectory of both the hit and pitch. This
allows for the determination of noise in the radar signal, providing for a degree of confidence in the results.

## Chapter 4 - Radar Accuracy

Because the radar results are produced using proprietary algorithms, it was necessary to independently evaluate the data reduction process. This began by calculating the position of the ball for both the pitch and hit using unfiltered radar data. The accuracy of the data reported by the radar, including the pitch and hit speeds and associated angles at home plate and the landing point of hit balls were also determined.

## Section 4.1 Evaluation of radar data reduction with respect to position

The data recorded by the radar is filtered and smoothed before it is displayed to the user using unknown assumptions. The position of the ball determined by the radar was verified from the raw radar data.

The positions were calculated using $V_{r}, R$, and $\alpha, \beta$, and $\gamma$ throughout the trajectory of the pitch and hit. The raw data used to calculate the $x, y$, and $z$ positions can be seen in Figures 4.1-4.4.


Figure 4.1: Radial Velocity ( $V_{r}$ ) of both the pitch and hit.


Figure 4.2: The cosine of the angle measured about the x-axis of $V_{r}$ for both the pitch and hit.


Figure 4.3: The cosine of the angle measured about the $y$-axis of $V_{r}$ for both the pitch and hit.


Figure 4.4: The cosine of the angle measured about the z -axis of Vr for both the pitch and hit.

The data in Figures $4.1-4.4$ represents a single pitch and hit of an NCAA baseball. The pitch was projected with a two wheeled pitching machine at $28.7 \mathrm{~m} / \mathrm{s}(64.2 \mathrm{mph})$. The hit landed at $\mathrm{x}=109 \mathrm{~m}(357 \mathrm{ft})$ and $\mathrm{z}=8.5 \mathrm{~m}(27.9 \mathrm{ft})$, reaching a maximum height of $\mathrm{y}=17.0 \mathrm{~m}(55.8$ $\mathrm{ft})$ (coordinate system was defined in Figure 3.9). The pitch's $V_{r}$ seen in Figure 4.1 was multiplied by -1 to include it on a similar axis. In Figure 4.3 at $\mathrm{t}=0.72 \mathrm{~s}, \beta$ can be seen crossing zero, forming an angle of $90^{\circ}$ with the positive $y$-axis. This indicates the vertical location of the ball is at the same height as the radar. Figures 4.1-4.4 show typical trends of $V_{r}, \alpha, \beta$, and $\gamma$ for both pitched and hit baseballs.

In Figures 4.2-4.4 $\alpha, \beta$, and $\gamma$ (defined in Figure 3.9) include the $-4^{\circ}$ tilt about $Z_{r}$ and rotation about $Y_{r}$ that is present when the radar is not directly in line with pitching rubber and home plate. Therefore, to calculate the position of the ball with respect to the field coordinate system shown in Figure 3.3, $\alpha, \beta$, and $\gamma$ were transformed using the following rotation matrix (21),

$$
\left[\begin{array}{ccc}
\cos \theta \cos \varphi & \sin \theta & -\sin \varphi  \tag{4.1}\\
-\sin \theta & \cos \theta & 0 \\
\sin \varphi & 0 & \cos \varphi
\end{array}\right]\left[\begin{array}{l}
\alpha \\
\beta \\
\gamma
\end{array}\right]=\left[\begin{array}{l}
\alpha^{\prime} \\
\beta^{\prime} \\
\gamma^{\prime}
\end{array}\right]
$$

where $\theta$ represents rotation about $Z_{r}$ and $\varphi$ represents rotation about $Y_{r}$. The transformed coordinate system used to calculate the $x, y$, and $z$ positions of the ball can be seen in Figure
4.5.


Figure 4.5: Geometry and coordinate system of the radar and field.

In Figure 4.5, Position 2 represents the unknown position of the ball and Position 1 represents the previous or known position of the ball. At time $t=0$, Position 1 is determined from the initial range assumed by the radar, and the direction measured by the radar. The coordinates of Position 1 are reported with respect to the field coordinate system.

The location of Position 2 was found first by defining the line formed by the radar and the unknown position. A line in three dimensions is defined by (21),

$$
\begin{equation*}
x_{2}-x_{0}=A_{1} g, \quad y_{2}-y_{0}=A_{2} g, \quad z_{2}-z_{0}=A_{3} g \tag{4.2}
\end{equation*}
$$

where $<A_{1}, A_{2}, A_{3}>$ represent a non-zero vector parallel to the position vector, $\left(x 0_{0}, y_{0}, z_{0}\right)$ is the position of the radar with respect to the field coordinate system. The position of the radar in equation 4.2 represents a point on the line, and $g$ describes the location along the line for the unknown coordinates ( $x_{2}, y_{2}, z_{2}$ ). Equating the equations in 4.2, results in the symmetric equation for a line written as,

$$
\begin{equation*}
\frac{x_{2}-x_{0}}{A_{1}}=\frac{y_{2}-y_{0}}{A_{2}}=\frac{z_{2}-z_{0}}{A_{3}} \tag{4.3}
\end{equation*}
$$

where the unknown coordinates of Position 2 were $\left(x_{2}, y_{2}, z_{2}\right)$ and the coordinates ( $x_{0}, y_{0}, z_{0}$ ) were defined by the user during setup. The components of the parallel vector, $<A_{1}, A_{2}, A_{3}>$, were defined by the components of $V_{r 2}$ by,

$$
\begin{equation*}
A_{1}=\left(V_{r 2} \Delta t\right) \alpha_{2}{ }^{\prime}, \quad A_{2}=\left(V_{r 2} \Delta t\right) \beta_{2}^{\prime}, \quad A_{3}=\left(V_{r 2} \Delta t\right) \gamma_{2}^{\prime} \tag{4.4}
\end{equation*}
$$

Combining equations 4.3 and 4.4 results in the following equations for the unknown coordinates of the ball in Position 2.

$$
\begin{equation*}
\frac{x_{2}-x_{0}}{\left(V_{r 2} \Delta t\right) \alpha_{2}}=\frac{y_{2}-y_{0}}{\left(V_{r 2} \Delta t\right) \beta_{2}}=\frac{z_{2}-z_{0}}{\left(V_{r 2} \Delta t\right) \gamma_{2}} \tag{4.5}
\end{equation*}
$$

Equation 4.5 represent two equations and three unknowns. The third equation is obtained from an intersecting plane defined by the components of $V_{r 1}$. Figure 4.6 shows a magnified portion of Figure 4.5 illustrating how the ball moves from Position 1 to Position 2.


Figure 4.6: Magnified portion of Figure 4.5, showing a relationship between Positions 1 and 2.

Assuming that $V_{r 1}$ is constant over $\Delta t$ from Position 1 to Position 2, and Position 2 lies on a plane perpendicular to $V_{r 1}$ at $\mathrm{t}=\Delta \mathrm{t}$, an equation can be found describing the plane that intersects the line defined by equations 4.5.

The intersecting plane in Figure 4.6 was defined using (21),

$$
\begin{equation*}
0=a_{1}\left(x_{2}-x^{\prime}\right)+a_{2}\left(y_{2}-y^{\prime}\right)+a_{3}\left(z_{2}-z^{\prime}\right) \tag{4.6}
\end{equation*}
$$

where $<a_{1}, a_{2}, a_{3}>$ represent a vector that is normal to the plane and $\left(x^{\prime}, y^{\prime}, z\right)$ represent the coordinates of a point on the plane. The displacement vectors $u, v$, and $w$ seen in Figure 4.6 are defined by,

$$
\begin{equation*}
u=\left(V_{r 1} \Delta t\right) \alpha_{1}{ }^{\prime}, \quad v=\left(V_{r 1} \Delta t\right) \beta_{1}{ }^{\prime}, \quad w=\left(V_{r 1} \Delta t\right) \gamma_{1}{ }^{\prime} \tag{4.7}
\end{equation*}
$$

Adding the initial coordinates of Position 1, $\left(x_{1}, y_{1}, z_{1}\right)$, to $u, v$, and $w$ in equation 4.7 results in the coordinates of a point on the intersecting plane in the field coordinate system $\left(x^{\prime}, y^{\prime}, z\right)$. The normal vector, $\left\langle a_{1}, a_{2}, a_{3}\right\rangle$, is defined by the location of the ball at Position 1 in the field coordinate system. This results in the equation of a plane that passes through the coordinates of Point 2.

$$
\begin{equation*}
0=x_{1}\left(x_{2}-x^{\prime}\right)+y_{1}\left(y_{2}-y^{\prime}\right)+z_{1}\left(z_{2}-z^{\prime}\right) \tag{4.8}
\end{equation*}
$$

Solving equation 4.5 for $x_{2}$ and $z_{2}$ in terms of $y_{2}$ results in the following equations,

$$
\begin{gather*}
x_{2}=\left(\frac{y_{2}-y_{0}}{\left(V_{r 2} \Delta t\right) \beta_{2}{ }^{\prime}}\right)\left(V_{r 2} \Delta t \alpha_{2}^{\prime}\right)+x_{0}  \tag{4.9}\\
z_{2}=\left(\frac{y_{2}-y_{0}}{\left(V_{r 2} \Delta t\right) \beta_{2}{ }^{\prime}}\right)\left(V_{r 2} \Delta t \gamma_{2}^{\prime}\right)+z_{0} \tag{4.10}
\end{gather*}
$$

Substituting equations 4.9 and 4.10 into 4.8 and solving for $y_{2}$ results in the y coordinate of Point 2,

$$
\begin{equation*}
y_{2}=\frac{B\left(-x_{0} x_{1}+x^{\prime} x_{1}+y_{1} y^{\prime}+z_{1} z_{0}+z^{\prime} z_{1}\right)+y_{0} A x_{1}+y_{0} G z_{1}}{\left(A x_{1}+B y_{1}+G z_{1}\right)} \tag{4.11}
\end{equation*}
$$

where $A, B$, and $G$ represent $\left(V_{r 2} \Delta t\right) \alpha_{2}^{\prime},\left(V_{r 2} \Delta t\right) \beta_{2}{ }^{\prime}$, and $\left(V_{r 2} \Delta t\right) \gamma_{2}{ }^{\prime}$ respectively. From which $X_{2}$ and $z_{2}$ were calculated using equations 4.9 and 4.10.

A representative comparison of the positions calculated using equations 4.9-4.11 to those positions given by the radar can be seen in Figures 4.7 and 4.8.


Figure 4.7: Comparing the $x$ and $y$ positions of both the pitch and hit of a baseball reported by the radar and calculated from equations 4.9 and 4.11. The hit shown here traveled approximately $109 \mathrm{~m}(357 \mathrm{ft})$ in the x -direction and approximately 8 m ( 26 ft ) in the z - direction.


Figure 4.8: The $x$ and $z$ positions of both the pitch and hit described and seen in Figure 4.7.

Figures 4.7 and 4.8 show both the pitch and hit recorded by the radar. The data in

Figures 4.7 and 4.8 shows a strong correlation between the data calculated using equations 4.9

- 4.11, to that used from the radar. The error, $E_{p}$, between the radar positions and those calculated using equations 4.9-4.11 for the different coordinates can be seen in Figures 4.9-
4.11. Error was defined by,

$$
E_{p}=\frac{X_{2}-X_{1}}{\left(\frac{X_{2}+X_{1}}{2}\right)}
$$



Figure 4.9: $E_{p}$ between the radar x-coordinates and that calculated using equation 4.9.


Figure 4.10: $E_{p}$ between the radar y-coordinate and that calculated using equation 4.11.


Figure 4.11: $E_{p}$ between the radar z-coordinate and that calculated using equation 4.10.

The spikes in $E_{p}$ at the end of the pitch and beginning of the hit in Figures 4.9-4.11 were the result of small $\mathrm{x}, \mathrm{y}$, and z positions in the denominator used to calculate $E_{p}$. It is clear from examining $E_{p}$ in Figures 4.9-4.11 that the coordinates agree very well, on average they were within $1 \%$ of each other. The agreement between equations $4.9-4.11$ and the radar positions provided confidence in our understanding of the radar measurements and its data reduction process.


Figure 4.12: Positional data recorded by the radar for the pitch and hit shown in Figures 4.7 and 4.8.

Some interesting insights can be made about how noise affected the positional data by examining Figure 4.12. The positional data in Figure 4.12 is from the same pitch and hit shown in Figures 4.7 and 4.8. In Figure 4.12, at a time of 2.5 s , the $\mathrm{x}, \mathrm{y}$, and z positions begin to deviate from a continuous line. Because the deviation increases as the ball travels further from the radar the cause is attributed to the product of a weaker return signal being distorted by noise. Figure 4.12 also shows that the effect of noise impacts the $y$ and $z$ positions slightly more than the $x$. The effect of noise on position was explored for several different hits for both baseball
and softball with similar results as shown in Figure 4.12. By magnifying a portion of the data in Figure 4.7, the onset of noise becomes more apparent, as seen in Figure 4.13.


Figure 4.13: Portion of Figure 4.7 showing the effects of noise on the position of the ball.

The deviation of the data from a smooth line appears to begin at an x-position of 35 m and is characterized by a deviation of approximately 10 cm . The trend in Figure 4.12 of the effects of noise increasing as the ball travels further from the radar show that noise affects the $x, y$, and $z$ position of the ball more as the return signal becomes weaker. These phenomenon were not specific to this hit, but rather were seen continuously throughout the collected data.

The noise seen in Figures 4.7 and 4.8 did not seem to have a large impact on $E_{p}$ as shown in Figures 4.9-4.11. This indicated that even when noise was present the two methods determine similar $x, y$, and $z$ coordinates of the ball. But the accuracy of the coordinates is, nevertheless, sensitive to noise. To better determine what portion of the raw data is most affected by noise the graphs of $V_{r}, \alpha, \beta$, and $\gamma$ seen in Figures $4.1-4.4$ were examined.

The direction cosines shown in Figures 4.2-4.4 are plotted on the same scale in Figures

### 4.13 and 4.14.



Figure 4.14: Angles of $V_{r}$ for a baseball pitched with a pitching machine.


Figure 4.15: Angles of $V_{r}$ for a hit baseball.

The data in Figures 4.14 and 4.15 show that all of the angles are affected by noise for both the pitch and hit. Because all three angles are equally affected by noise, the noise is attributed to multipath reflections affecting the measurement of the azimuth and elevation angles of the return signal. The hit in Figure 4.15 represents a ball that landed within $\mathrm{z}=8 \mathrm{~m}$ of the pitcher line defined by a straight-line between the tip of home plate and the middle of the pitching rubber.

To determine how the noise from multipath reflections affected balls hit at larger azimuth angles (deviate further from the pitcher line), the angles of three different hit balls recorded by the radar on the same day of a slow pitch softball tournament were compared. The angle data can be seen in Figures 4.16 - 4.18 for hits with landing positions deviating further from the pitcher line.


Figure 4.16: Angle in degrees for a hit from a slow pitch tournament that landed $x=92.7 \mathrm{~m}(304 \mathrm{ft})$ along the pitcher line and $\mathrm{z}=44.3 \mathrm{~m}(145 \mathrm{ft})$ to the left of the pitcher line with an initial velocity of $39.2 \mathrm{~m} / \mathrm{s}(87.7 \mathrm{mph})$ herein referred to as set 1.


Figure 4.17: Angle in degrees for a hit from a slow pitch tournament that landed $x=87.1 \mathrm{~m}(285.7 \mathrm{ft})$ along the pitcher line and $\mathrm{z}=23.1 \mathrm{~m}(75.8 \mathrm{ft})$ to the left of the pitcher line with an initial velocity of $37.5 \mathrm{~m} / \mathrm{s}(83.9 \mathrm{mph})$ herein referred to set 2 .


Figure 4.18: Angle in degrees for a hit from a slow pitch tournament that landed $\mathrm{x}=71.5 \mathrm{~m}(234.5 \mathrm{ft})$ along the pitcher line and $\mathrm{z}=2.7 \mathrm{~m}(8.9 \mathrm{ft})$ to the left of the pitcher line with an initial velocity of $31.3 \mathrm{~m} / \mathrm{s}(70.0 \mathrm{mph})$ herein referred to as set 3 .

The angles for the hit shown in Figure 4.16, referred to as set 1, have the most noise while the angles in Figure 4.18, referred to as set 3 have less noise. The major differences between the data sets seen in Figures 4.16-4.18 are the distance the hits traveled left of the pitcher line before they landed. Set 1, seen in Figure 4.15 landed $z=44.3 \mathrm{~m}(145 \mathrm{ft})$ to the left of the pitcher line, while sets 2 and 3 in Figures 4.16 and 4.17 landed $z=23.1 \mathrm{~m}(75.8 \mathrm{ft})$ and $\mathrm{z}=2.7$ $\mathrm{m}(8.9 \mathrm{ft})$ to the left of the pitcher line. The hits all traveled over $71 \mathrm{~m}(233 \mathrm{ft})$ along the pitcher line before hitting the ground, at initial vertical angles of approximately $32^{\circ}$. Figure 4.19 shows the trajectories of sets 1,2 , and 3 with respect to the pitcher line.


Figure 4.19: Location of the hits described in Figures 4.15-4.17.

By comparing the positional data of three different hits projected to different horizontal locations of the field, a relationship between the effects of noise and the horizontal location of the ball have been shown. As a hit deviates left of a straight line path from the radar to home plate the signal loses its strength. This then allows the effects of multipath signals to interfere with the amplitude and phase of the return signal, creating noise in $\alpha, \beta$, and $\gamma$. The noise in the return signals were not present in the radial velocities of either the pitch or the hit seen in Figure 4.1. Since multipath reflections only affect the angle measurements (and not the change in frequency used to measure $V_{r}$ ) this shows the relatively large contributions of multipath error on radar noise.

## Section 4.2 Evaluation of radar reported velocity and acceleration

It was evident from the noise in the positional data shown that a piecewise derivative of position would not yield accurate velocities. In the following the positional radar data will be used in the filtered (TrackMan Kalman Filter) and unfiltered form.

The method for determining the velocities and accelerations for the ball included fitting a $3^{\text {rd }}$ order polynomial to the positional data using a least squares fit (Microsoft Excel). Differentiating this equation with respect to time resulted in the $x, y$, and $z$ components of velocity. A second derivative yielded the components of acceleration throughout the entire trajectory. The equation used to calculate the velocity of the ball was,

$$
\begin{equation*}
V_{x}(t)=\frac{d x(t)}{d t}, \quad V_{y}(t)=\frac{d y(t)}{d t}, \quad V_{z}(t)=\frac{d z(t)}{d t} \tag{4.13}
\end{equation*}
$$

The third order fit represented a constant jerk model for trajectories experiencing a constant change in acceleration. The velocities can be seen in Figures 4.20-4.22, where the blue line represents the filtered velocity data reported by the radar, and the red line is from equation 4.13. The data used to calculate the velocities was from the same pitch and hit shown in Figures 4.7 and 4.8.


Figure 4.20: Velocities in the $x$ direction calculated using equation 4.13 for a constant jerk model.


Figure 4.21: Velocities in the $y$ direction calculated using equation 4.13 for a constant jerk model.


Figure 4.22: Velocities in the $z$ direction calculated using equation 4.13 for a constant jerk model.

Figures 4.20-4.22 show the components of velocity for both the pitch and hit. The figures show a strong relationship between the velocities of the hit with an average $E_{p}$ of $0.4 \%$, $3.4 \%$, and $5.9 \%$ for $V_{X}, V_{y}$, and $V_{z}$ respectively. The pitch data compares equally well with the exception of $V_{y}$. The average $E_{p}$ for the pitch data was $0.1 \%, 16.9 \%$, and $5.8 \%$ for $V_{x}, V_{y}$, and $V_{z}$ respectively. The larger $E_{p}$ from the velocity of the pitch in the y-direction (16.9\%) is explained in Figure 4.23.


Figure 4.23: Y and Z coordinates of the pitch showing the deviation compared to the filtered TrackMan raw data and that calculated using equation 4.13.

Figure 4.23 shows that the derivative of the $3^{\text {rd }} \operatorname{Order}$ fit $(Y)$ curve will not match the derivative of the Filtered $Y$ curve. While it is not known which fit represents the actual location of the ball, to compare the radar computations of velocity to those calculated using equation 4.13, the filtered positional data will be used.

Filtered positional data from three different pitches and hits during one day of a slow pitch tournament were used to verify the radar computations. The trajectories for the different pitches and hits were described in Figures 4.16-4.18. The velocities and accelerations were calculated using equation 4.13. The noise for each trajectory can be seen in Figures 4.16-4.18. The average difference between the velocities and accelerations calculated using equation 4.13
and those calculated and filtered by the radar for the pitch and subsequent hit are presented in

Table 4.1.

Table 4.1: Average difference between velocities and accelerations calculated using equation 4.13 and those determined by the radar.

|  | Set 1 |  | Set 2 |  | Set 3 |  | Average |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Pitch | Hit | Pitch | Hit | Pitch | Hit | Pitch | Hit |
| $\Delta V_{X}(\mathrm{~m} / \mathrm{s})$ | 0.093 | 0.107 | 0.090 | 0.072 | $2 \mathrm{E}-4$ | 0.002 | 0.061 | 0.060 |
| $\Delta V_{y}(\mathrm{~m} / \mathrm{s})$ | 0.001 | 0.002 | 0.019 | 0.015 | $7 \mathrm{E}-5$ | 0.009 | 0.007 | 0.009 |
| $\Delta V_{Z}(\mathrm{~m} / \mathrm{s})$ | 0.029 | 0.033 | 0.024 | 0.019 | $3 \mathrm{E}-4$ | 0.003 | 0.018 | 0.018 |
| $\Delta A_{X}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ | 0.134 | 0.407 | 0.159 | 0.265 | 0.001 | 0.336 | 0.098 | 0.336 |
| $\Delta A_{y}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ | 0.007 | 0.008 | 0.016 | 0.110 | 0.023 | 0.232 | 0.015 | 0.117 |
| $\Delta A_{Z}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ | 0.139 | 1.478 | 0.034 | 0.011 | $2 \mathrm{E}-4$ | 0.110 | 0.058 | 0.533 |

Table 4.1 shows that velocities calculated from equation 4.13 are in excellent agreement with the radar when the filtered positional data is used. On average the components of velocity calculated using equation 4.13 are within $0.06 \mathrm{~m} / \mathrm{s}$ of the components of velocity reported by the radar. This shows that a $3^{\text {rd }}$ order polynomial fit is a good approximation of the algorithms` used by the radar.

The accelerations of the pitch determined by the radar agree better with those calculated with equation 4.13 than the hit. The larger difference between the accelerations of the hit compared to the accelerations of the pitch can be attributed to noise. Recall that the hit data seen in Set 1 experienced more noise than that seen in Set 3. The differences in acceleration in Table 4.1 labeled Set 1 are on average greater than those for Set 3. In the case of $\Delta A_{z}$ that difference is greater than a factor of ten. This excess noise may be affecting the ability for the radar to accurately calculate the acceleration of a hit ball.

## Section 4.3 Comparison of radar and high speed video at home plate

Much of the data reported by the radar concerns the pitch and hit near home plate. For this reason two high speed cameras were setup with a field of view of approximately 1.2 by 2.4 $\mathrm{m}(4$ by 8 ft$)$ around home plate. The cameras were positioned such that both the incoming pitch and subsequent hit could be recorded. Data was collected for 192 pitches and hits on the same day of a slow pith tournament in Oklahoma City, OK. The balls were a $30.5 \mathrm{~cm}(12 \mathrm{in})$, 52 COR, 300 lb compression Worth slow pitch softballs.

The cameras were positioned approximately $45^{\circ}$ from each other, 4.6 m above the ground and 4.6 m from home plate. The cameras were suspended in the air using jibs. They were calibrated using two 1.2 by 1.2 m panels with equally spaced markers arranged in 7 rows of 7 for $49,2.5 \mathrm{~cm}(1 \mathrm{in})$ dots on each panel. The fixture can be seen in Figure 4.24.Thirty frames were saved for each impact at 1000 fps . This included 15 frames prior to impact and 15 frames after impact. Figure 4.25 shows the orientation of the cameras and radar to the field. The calibration fixture was placed on the white lines near home plate as seen in Figure 4.25.


Figure 4.24: Calibration fixture used for high speed video data captured near home plate.


Figure 4.25: Oklahoma high speed camera field study camera orientation.

After the video was collected the ball was tracked in 3-D space using ProAnalyst 3D
Professional. The coordinates for the ball were fit to second order polynomial equations.

Differentiating with respect to time resulted in the velocity of the ball. The associated horizontal and vertical angles were calculated using the angles between the components of the velocity vectors of the ball.

The velocity reported by the radar was compared to the velocity obtained with the high speed video tracking, seen in Figure 4.26. The average difference in pitch speed between the two systems was $0.20 \mathrm{~m} / \mathrm{s}(0.45 \mathrm{mph})$ with a standard deviation of $0.9 \mathrm{~m} / \mathrm{s}(2.01 \mathrm{mph})$, this represents an average $E_{p}$ of $2.3 \%$. The average difference in hit speed was $-0.65 \mathrm{~m} / \mathrm{s}(-1.46$ $\mathrm{mph})$ with a standard deviation of $3.1 \mathrm{~m} / \mathrm{s}(6.86 \mathrm{mph})$, representing an average $E_{p}$ of $2.8 \%$.


Figure 4.26: Comparison of the speed of the ball reported by the radar to the speed calculated using high speed video tracking for both the pitch and the hit.

The horizontal and vertical angles associated with the velocities can be seen in Figures 4.27 and 4.28 respectively. The average difference in vertical angle of the pitch measured between the two systems was $2.1^{\circ}$ with standard deviation of $6.0^{\circ}$. The average difference in vertical angle of the hit between the two systems was $-3.5^{\circ}$ with standard deviation of $2.6^{\circ}$. The average difference of the horizontal pitch and hit angles measured by the two systems were $0.52^{\circ}$ and $-1.2^{\circ}$ respectively.


Figure 4.27: Comparison of the vertical angle of the ball reported by the radar to the angle calculated using high speed video tracking for both the pitch and the hit.


Figure 4.28: Comparison of the horizontal angle of the ball reported by the radar to the angle calculated using high speed video tracking for both the pitch and the hit.

Figure 4.27 suggests that a systematic offset in one of the systems was causing some error in the vertical angle. A least squares correlation line to the data in Figure 4.27 was found to be $y=0.994 x-2.9$. Because the radar levels itself vertically, the $2.9^{\circ}$ offset was most likely due
to the calibration fixture, used for the cameras, not being level with the ground. When $2.9^{\circ}$ was subtracted from the vertical angle of the hit measured by the video, the error between the two systems drops to $0.98^{\circ}$ from $3.5^{\circ}$. The corrected vertical angles can be seen in Figures 4.29.


Figure 4.29: Comparison of the corrected vertical angle of the pitches and hits shown in Figure 4.27.

The radar speeds of both the pitch and the hit at home plate correlate well (within 3\%) of the cameras. The horizontal angles of the pitch at home plate also correlate well (within $0.5^{\circ}$ ). The large average difference in the vertical angle of the pitch at home plate $\left(6^{\circ}\right)$, indicated an error in one of the two systems.

Examining possible pitch trajectories provides insight into which system more accurately measured the vertical angle. The pitch originated, on average, from a distance of $14 \mathrm{~m}(46 \mathrm{ft})$ from home plate, $0.46 \mathrm{~m}(1.5 \mathrm{ft})$ above the ground. The impact of the bat and ball took place $1.2 \mathrm{~m}(4 \mathrm{ft})$ in front of home plate. Therefore a strike zone was setup with the leading edge 12.8 $\mathrm{m}(42 \mathrm{ft})$ from where the ball originated. The strike zone was defined at $0.6 \mathrm{~m}(2 \mathrm{ft})$ from the ground and the top of the zone at $1.5 \mathrm{~m}(5 \mathrm{ft})$ from the ground; as shown in Figure 4.30.


Figure 4.30: Simulated trajectories of pitches.
Trajectories to reach the top, bottom, and center of the strike zone (following the assumed $0.6 \mathrm{~m}(2 \mathrm{ft})$ and $1.5 \mathrm{~m}(5 \mathrm{ft})$ limits and the maximum and minimum arc heights defined by ASA of $1.8 \mathrm{~m}(6 \mathrm{ft})$ and $3.05 \mathrm{~m}(10 \mathrm{ft}))$ are shown in Figure 4.30 and Table 4.2. These trajectories also represent pitches that would pass through the field of view of the cameras. Thus the speed of the pitch as it enters the strike zone must be between $10.7 \mathrm{~m} / \mathrm{s}(23.9 \mathrm{mph})$ and $14.6 \mathrm{~m} / \mathrm{s}(32.7 \mathrm{mph})$. In Figure 4.26 the radar speeds varied between $10.5 \mathrm{~m} / \mathrm{s}(23.4 \mathrm{mph})$ and $12.3 \mathrm{~m} / \mathrm{s}(27.6 \mathrm{mph})$, while the video pitch speeds varied between $9.2 \mathrm{~m} / \mathrm{s}(20.6 \mathrm{mph})$ and $14.1 \mathrm{~m} / \mathrm{s}$ ( 31.4 mph ). The faster speeds recorded by both the radar and video represent pitches that would pass through the strike zone, but the slower speeds recorded by the video create trajectories that would not pass through the strike zone and ultimately would not travel through the field of view of the cameras. This suggests that the radar is more accurate than the video in measuring pitch speed.

Table 4.2: Summary of trajectory data seen in Figure 4.28.

|  |  | Initial Conditions | Strike Zone Conditions |
| :--- | :---: | :---: | :---: |
| Trajectory 1 | Velocity (m/s) | $11.7 \mathrm{~m} / \mathrm{s}$ | $10.7 \mathrm{~m} / \mathrm{s}$ |
|  | Vertical Angle | $37^{\circ}$ | $-38^{\circ}$ |
| Trajectory 2 | Velocity (m/s) | $12.2 \mathrm{~m} / \mathrm{s}$ | $11.4 \mathrm{~m} / \mathrm{s}$ |
|  | Vertical Angle | $32.4^{\circ}$ | $-34^{\circ}$ |
| Trajectory 3 | Velocity (m/s) | $16.4 \mathrm{~m} / \mathrm{s}$ | $14.6 \mathrm{~m} / \mathrm{s}$ |
|  | Vertical Angle | $17.2^{\circ}$ | $-10.0^{\circ}$ |

Positional data in Figure 4.19 showed that noise affected the return signal more as the ball deviated left from the pitcher line. To examine if this affected the accuracy of the hit or pitch speeds measured by the radar, the difference in speed measured between the radar and video were plotted against the vertical and horizontal angles. This can be seen in Figures 4.31 and 4.32.


Figure 4.31: The difference between the measured velocities of the radar and cameras for both the pitch and hit shows no correlation to the horizontal angle of the ball.


Figure 4.32: The difference between the measured velocities of the radar and cameras for both the pitch and hit shows no correlation to the vertical angle of the ball.

The speeds and vertical angles have been corrected for the $2.9^{\circ}$ offset in both Figures 4.31 and 4.32. The data does not show a strong relationship between the differences in pitch or hit speed measured by the systems with respect to the horizontal or vertical angles. The data does show that on average the hit speeds measured by the radar were $0.47 \mathrm{~m} / \mathrm{s}(1.05 \mathrm{mph})$ higher than those measured by the video. The average difference in pitch speeds between the radar and the video was $0.2 \mathrm{~m} / \mathrm{s}$ ( 0.45 mph ).

## Section 4.4 Comparison of radar and Infrared video at home plate

A study was conducted to compare speeds and angles of the pitch and hit measured at home plate by radar with Infrared (IR) cameras. Data was collected for 264 pitches and hits over two days in a batting cage near Providence, RI. The pitch was projected using a two wheeled pitching machine with a fixed radar pitching range of $13.1 \mathrm{~m}(43 \mathrm{ft})$ from home plate (the actual
distance was not measured). The balls and bats were supplied by Dr. Trey Crisco from Brown University. Reflective tape was affixed to both the bats and baseballs for IR tracking. The collection and reduction of the data recorded by the IR cameras was done by Dr. Crisco's lab and supplied for use in this work.

Several cameras were setup throughout the cage and focused on the area surrounding home plate. The calibration was done using an "L" rod with reflective emitters affixed to it, similar to other studies using IR tracking (22). The radar was setup 4.1 m ( 13.5 ft ) behind the tip of home plate and $1.4 \mathrm{~m}(4.5 \mathrm{ft})$ above the ground, having no offset in the z direction.

The coordinate system of the IR cameras was arranged such that the positive $x$-axis pointed toward the pitcher, the $z$-axis pointed vertically upward, and the $y$-axis pointed to the catcher's left. The coordinates of the ball as a function of time were fit to a straight line for both the pitch and hit. Subsequently the incoming pitch and outgoing hit speeds were found from the slopes of the lines. The IR data was evaluated such that a ball-in error and ball-out error were calculated. This was done by calculating the standard error of the slope of the $x(t), y(t)$, and $z(t)$ linear fit to the ball coordinates. The average ball-in and ball-out error were .36 and 1.36 mph respectively.

The radar data was sorted to eliminate balls that were hit with vertical angles less than $6^{\circ}$ and greater than $40^{\circ}$. These hits were determined from an analysis of the frequency data be largely affected by noise. The hit and pitch speeds for both systems recorded on both days are shown in Figure 4.33.


Figure 4.33: Comparison of the pitch and hit speeds reported by radar and the Infrared Cameras for days one and two of the Providence, RI field study.

The data showed a linear relationship between the two systems. The average difference between the two systems for both days was $1.7 \mathrm{~m} / \mathrm{s}(3.7 \mathrm{mph})$ and $-0.17 \mathrm{~m} / \mathrm{s}(-0.37 \mathrm{mph})$ for the hit and pitch respectively. There is no discernible difference between the data collected during day one and two. There does appear to be a significant difference between the velocities of the hit speed between the two systems.

The corresponding horizontal and vertical angles, $\theta_{H}$ and $\theta_{V}$, associated with the hit are presented in Figures 4.34 and 4.35. The figures show a correspondence between the two systems, but not the expected 1:1 correlation.


Figure 4.34: Comparison of the horizontal angle of the hit reported by radar and the Infrared cameras for days one and two of the Providence, RI field study.


Figure 4.35: Comparison of the vertical angle of the hit reported by radar and the Infrared cameras for days one and two of the Providence, RI field study.

While the indoor batting cage produced a relatively noisy radar environment, Figure
4.36, this did not appear to affect the speed, Figure 4.37. Further, the differences between the
radar and IR cameras in Figures 4.31 and 4.32 appear to be systematic not random. A plausible explanation for the systematic error would be an error in the assumed 13.1 m pitcher distance.


Figure 4.36: Frequency spectrum of a pitch and hit recorded on day two of the field study in Providence, RI.


Figure 4.37: Velocities reported by the radar using TrackMan's filtering process (Radar) and calculated using equation 4.16.

To compare how noise in the unfiltered data affects the velocity of the ball, the velocity in Figure 4.37 was calculated using the unfiltered $V_{r}, R$, and $\alpha, \beta$, and $\gamma$. The coordinate system used to calculate the velocity can be seen in Figure 4.38.


Figure 4.38: Relationship between $V_{t}$ and $V_{r}$ used to calculate the velocity of the ball ( $V_{1-2}$ ) from the unfiltered radar data.

The tangential velocity, $V_{t}$, is calculated using,

$$
\begin{equation*}
V_{t}=(d \theta) R \tag{4.14}
\end{equation*}
$$

where $\mathrm{d} \theta$ is determined from the dot product between $\alpha^{\prime}, \beta^{\prime}$, and $\gamma^{\prime}$. The angle, $\varphi$, is determined from,

$$
\begin{equation*}
\varphi=\tan ^{-1} \frac{V_{t}}{V_{r_{1}}} \tag{4.15}
\end{equation*}
$$

The resulting component of the velocity of the ball is calculated from,

$$
\begin{equation*}
V_{1-2}=\frac{V_{r_{1}}}{\cos \varphi} \tag{4.16}
\end{equation*}
$$

The data in Figure 4.37, calculated using equation 4.16 agrees very well with the radar data showing that noise was not the contributing factor in the disassociated radar and IR data seen in Figures 4.34 and 4.35.

To show how error in the assumed ball release location (i.e. range) affected the resulting data, the distance of the radar was manipulated in the Workbench. The effect of range error is shown in Figures 4.39-4.41. Notice that the effect of range error increases with $\Theta_{h}$ and $\Theta_{V}$ as observed in Figures 4.34 and 4.35.


Figure 4.39: Error in velocity when the range was changed in the Workbench ${ }^{1}$.
${ }^{1}$ The initial assumed range, $R_{i}$, for the data shown in Figure $4.42-4.44$ was $27.6 \mathrm{~m}(90.5$ $\mathrm{ft})$. Error was added to $R_{i}$ in the Workbench from $-3 \mathrm{~m}(9.8 \mathrm{ft})$ to $+4 \mathrm{~m}(13.1 \mathrm{ft})$ at $1 \mathrm{~m}(3.28 \mathrm{ft})$ increments. The data was from several different hits projected at different $\theta_{H}$ and $\theta_{V}$ angles during a single day of a baseball field study.


Figure 4.40: Error in horizontal angle when the range was changed in the Workbench ${ }^{1}$.


Figure 4.41: Error in vertical angle when the range was changed in the Workbench ${ }^{1}$.

The error in the landing positions of the hits was also analyzed and a similar trend was found of increasing error in the $x$ and $z$ location of the landing point of a hit with increasing error in $R$. The maximum error in the x and z landing positions were was $5 \mathrm{~m}(16.4 \mathrm{ft})$ and $\pm 2 \mathrm{~m}$
( 6.56 ft ) respectively. Both maximum errors in the $x$ and $z$ landing positions occurred at the maximum positive $R$ error value of $4 \mathrm{~m}(13.1 \mathrm{ft})$.

The radar data in Figure 4.32 was recomputed, where the range was varied to minimize the difference between the radar and IR for $V, \Theta_{H}$, and $\Theta_{V}$. A range of $15.2 \mathrm{~m}(49.9 \mathrm{ft})$ was observed to minimize the difference. With the range correction, a constant offset of $1.7^{\circ}$ between the radar and IR for $\Theta_{V}$ was still apparent. This was likely due to the calibration of the IR, since the radar has an automatic vertical angle adjustment. With the range and vertical angle corrections applied, the data in Figures 4.33-4.35 was re-plotted in $4.42-4.44$.


Figure 4.42: Corrected velocities.


Figure 4.43: Corrected horizontal angles.


Figure 4.44: Corrected vertical angles.

The velocities in Figure 4.42 correlate very well, on average the two systems measure speed to within $0.2 \%$ of each other, at an average hit speed of $23.3 \mathrm{~m} / \mathrm{s}$ ( 52.2 mph ). The angles measured by radar and IR also correlate well, on average the horizontal angles were measured to within $0.6^{\circ}$ and the vertical angles were within $0.1^{\circ}$. To verify that any deviation in hit speeds was independent of $\theta_{H}$ and $\theta_{V}$ both were plotted on separate graphs in Figures 4.45 and 4.46.


Figure 4.45: The difference between the hit speed reported by the IR cameras and the radar versus the associated horizontal angle for both days of the Providence, RI field study.


Figure 4.46: The difference between the hit speed reported by the IR cameras and the radar versus the associated vertical angle for both days of the Providence, RI field study.

The data in Figures 4.45 and 4.46 used the corrected speeds and angles calculated from the corrected range in the Workbench. The data does not show a strong dependence of the error in hit speed on $\theta_{H}$ or $\theta_{V}$.

The two systems measure hit and pitch speeds to within $0.2 \%$ and $0.8 \%$ of each. This was better than the high speed cameras which measured speeds to within $2.8 \%$ of speed measured by the radar. The values of $\theta_{H}$ and $\theta_{V}$ recorded by the radar and the IR cameras measured to within $0.6^{\circ}$ and $0.1^{\circ}$ respectively. This was again better than the high speed video comparison where $\theta_{H}$ and $\theta_{V}$ agreed to within $1.2^{\circ}$ and $1.0^{\circ}$ respectively.

## Section 4.5 Accuracy of landing position of hit balls

The accuracy of the landing position of different types of hits was determined using high speed video tracking. This gave some insight into how well the radar determines the landing point of the ball in the presence of noise. This also provided a measure of how well the Kalman filter filtered positional data.

The cameras were Phantom V711 high speed cameras with 50 mm lenses, set to record at 400 fps . The data was collected inside the Kibbie Dome, a large indoor football facility at the University of Idaho, shown in Figure 4.47. This allowed for the collection of data without having to contend with weather conditions.


Figure 4.47: Kibbie Dome field study, showing the setup for the line drive hits.

The cameras were spaced $9.14 \mathrm{~m}(30 \mathrm{ft})$ apart at varying heights of $.914 \mathrm{~m}(3 \mathrm{ft})$ and $1.83 \mathrm{~m}(6 \mathrm{ft})$. They were positioned 35.4 m ( 116 ft ) from the calibration fixture seen in Figure 4.48. This provided for approximately a $21.3 \mathrm{~m}(70 \mathrm{ft})$ high by $30.5 \mathrm{~m}(100 \mathrm{ft})$ viewing area. The calibration fixture consisted of two $6.1 \mathrm{~m}(20 \mathrm{ft})$ square tarps placed side by side. The tarps had $25,15.2 \mathrm{~cm}$ ( 6 in ) dots equally spaced in 5 rows of 5 . The dots were made of adhesive backed vinyl and were placed on the tarp at known distances. The calibration fixtures were then raised in the air and oriented using scissor lifts.

The calibration fixtures determined a coordinate system that was transformed to the field coordinate system seen in Figure 3.3 with the origin at the initial position of the hit ball.


Figure 4.48: Two $6.1 \mathrm{~m}(20 \mathrm{ft})$ square tarps used as a calibration fixture for 3D motion tracking.

The large indoor facility allowed for multiple setups to measure different trajectories of hit balls. The balls were NCAA baseballs and were hit off of a baseball $T$ at two different setups. The first setup was designed for a batter to hit line drive balls so the baseball T was placed 22.9 m (25 yards) in front of the front edge of the landing area defined by the cameras. The second setup moved the baseball $T$ back so that it was approximately 68.6 m ( 75 yards) from the front edge of the landing area so the hit represented longer fly ball hits. A third setup projected baseballs using a pneumatic cannon at various angles. The cannon was positioned 68.6 m (75 yards) from the front edge of the landing area, at initial inclinations of $15^{\circ}, 30^{\circ}$, and $45^{\circ}$.

The first two setups, where a batter was used, the radar was placed on a tripod 2.7 m (9 $\mathrm{ft})$ above the ground, $8.2 \mathrm{~m}(27 \mathrm{ft})$ behind the starting position of the hit, in a straight line with the baseball T and the center of the landing area. The third setup, where the pneumatic cannon was used, the radar was positioned at the same height and distance as setups one and two, but was offset to the right at $0 \mathrm{~m}, 0.6 \mathrm{~m}(2 \mathrm{ft})$, and $1.5 \mathrm{~m}(5 \mathrm{ft})$. A total of 23 hits were recorded with a batter and 30 hits were recorded with a pneumatic cannon. Half of the balls tracked using setup three had no radar offset.

ProAnalyst 3D Professional was used to calibrate the positions of the cameras. ProAnalyst reported the error of the calibration to be a mean error of 2.72 cm ( 1.07 in ) with a maximum and minimum error of $8.94 \mathrm{~cm}(3.52 \mathrm{in})$ and $.61 \mathrm{~cm}(0.24 \mathrm{in})$ respectively. The quality of the video tracking was checked by positioning tracking points in ProAnalyst on each hash mark, over a 5 yard distance on the indoor football field. The average distances between the tracked hash marks were 0.90 m ( 0.99 yards) for an error of $0.01 \mathrm{~m}(0.39 \mathrm{in})$. The data in the x
and y plane for a typical hit can be seen in Figure 4.49 and the data in the $x$ and $z$ plane can be seen in Figure 4.50.


Figure 4.49: Typical data using high speed cameras for a trajectory in the $x-y$ plane.


Figure 4.50: Typical data using high speed cameras for a trajectory in the x-z plane.

Figure 4.49 shows a smooth transition between points along the entire trajectory of the hit, indicating a smooth track of the ball in the $x-y$ plane. This is not the case in Figure 4.50, showing a choppy transition between points in the x-z plane. In Figure 4.50, the z-location
jumps as much as $0.08 \mathrm{~m}(0.25 \mathrm{ft})$ from an average line. This is due to the relatively small angle between the cameras, $14.7^{\circ}$.

The ability to control the landing point of a hit or projected ball proved difficult. Only 12 hits landed in the field of view of the camera, five of which either had problems with the camera or radar tracking. Of the seven good hits five were from setup 2 , and two were from setup 3 with the radar at a zero offset. A plot of the $x$ and $z$ landing positions measured by the radar and the video tracking are shown in Figures 4.51 and 4.52 . Figure 4.51 also includes a comparison of the hit distance of several hits measured by the radar compared to that measured using a range finder. The range finder measured the distances from the start of the hit to the landing point. The range finder was accurate to $.08 \mathrm{~m}(1 / 32 \mathrm{in})$ (which was much less than our ability to visually identify the location). A total of 49 landing distances were measured, 23 measured for setup 2 and 26 measured using various offsets for setup 3


Figure 4.51: Comparison of the landing position of the ball measured with radar, video, and a range finder.


Figure 4.52: Comparison of the $z$-coordinate of the landing position of the ball measured with radar and video.

Nearly all the points fell on the correlation line indicating equal measure of the coordinates for all three systems. The average difference between the x and z coordinates of the landing positions measured with video and radar were $0.6 \mathrm{~m}(2 \mathrm{ft})$ and $0.4 \mathrm{~m}(1.3 \mathrm{ft})$ respectively at an average hit distance of $71.5 \mathrm{~m}(234.5 \mathrm{ft})$.

The average difference between the distance measured by the radar and the range finder for setup 2 for all 23 recorded hits was $-0.6 \mathrm{~m}(-1.82 \mathrm{ft})$ at an average hit distance of 81.4 $\mathrm{m}(267 \mathrm{ft})$. This equated to an average difference of hit distance between the range finder and the radar of $0.7 \%$. The negative sign indicates that on average, the radar measured 0.6 m (1.82 ft ) shorter than the range finder.

Balls projected using a pneumatic cannon at $15^{\circ}, 30^{\circ}$, and $45^{\circ}$ were also tracked with radar and measured with a range finder. This provided a measure of the ability of the radar to measure the hit distance of balls at different initial vertical angles. The radar had trouble tracking the final landing point of the balls projected at $45^{\circ}$, so this data was not used. The data
labeled "Radar/Range finder 15" and "Radar/Range finder 30" in Figure 4.51 compares the distance of the hit measured by the radar and the range finder for hits projected at $15^{\circ}$ and $30^{\circ}$ using the pneumatic cannon. The average difference between the radar's measured hit distance and those recorded by the range finder were $-3.8 \mathrm{~m}(-12.5 \mathrm{ft})$ and $-1.1 \mathrm{~m}(-3.5 \mathrm{ft})$ at angles of $15^{\circ}$ and $30^{\circ}$ respectively. This equated to $4.7 \%$ and $1.4 \%$ error at average hit distances of $81.1 \mathrm{~m}(266 \mathrm{ft})$ and $77.7 \mathrm{~m}(255 \mathrm{ft})$ for the respective angles. The balls projected at $15^{\circ}$ had on average $3 \%$ greater error than those projected at $30^{\circ}$ suggesting that noise affected the balls at lower angles more than those at higher angles. This is consistent with the example given for the effects of multipath error in Section 2.7. The $x-y$ positions of two different hits projected at $30^{\circ}$ and at $15^{\circ}$ can be seen in Figure 4.53 and the $x-z$ positions in Figure 4.54.


Figure 4.53: The effects of vertical launch angle on the noise experienced by two different hits.


Figure 4.54: The $x$ and $z$ positions of the hit seen in Figure 4.29.

The data in Figure 4.53 shows that hits with initial vertical angles of $30^{\circ}$ are less affected by noise when compared to hits with initial vertical angles of $15^{\circ}$. This is due to the trajectory of the ball creating angles at the antenna that intensify the effect of multipath error.

To determine how offsetting the radar affected the accuracy of the hit distance, the radar was offset $0.6 \mathrm{~m}(2 \mathrm{ft})$ and $1.5 \mathrm{~m}(5 \mathrm{ft})$ to the right of a straight-line defined in setup 3 . This represented typical offset distances used while tracking games where an obstruction prevents a setup in line with home plate and the pitching mound. Five balls were projected at a vertical angle of $30^{\circ}$ for each offset distance. The differences between the radar and the laser range finder at the different offsets can be seen in Figure 4.55.


Figure 4.55: Difference between hit distances measured with radar and a laser range finder for radar offsets of 0.6 m and 1.5 m .

The data in Figure 4.55 shows that when the radar is offset the accuracy of the hit distance does not change. The average difference between the hit distances measured by the radar and the range finder for all three offsets was -1.1 m (-3.5 ft)

## Section 4.6 Summary

This chapter considered the accuracy of data reported by the radar. By independently calculating the position of the ball it was found that the data reduction performed by the radar to determine position, velocity, and acceleration were within $1 \%$ of the independent calculations. This also showed that as balls begin to deviate from the pitchers line, the effects of noise on the position of the ball increase. It should be noted that multipath noise only affected the measured angles, $\alpha, \beta$, and $\gamma$, and had no effect on $V_{r}$.

A comparison of velocities measured at home plate by high speed video tracking and radar found that the two systems agree to within $2.3 \%$ and $0.7 \%$ for the pitch and hit respectively. The associated horizontal and vertical angles of the pitch agree to within $0.5^{\circ}$ and $2.1^{\circ}$ respectively. The associated horizontal and vertical angles of the hit agree to within $1.2^{\circ}$ and $1.0^{\circ}$ respectively. Velocities of the pitch and hit and associated angles were also measured with IR cameras and compared to radar. The pitch speeds agreed on average by $0.9 \%$, while the hit speeds agreed on average by $0.2 \%$. The measured horizontal and vertical angles of the hit agreed on average by $2.1^{\circ}$ and $0.1^{\circ}$ respectively.

It was determined from a comparison of radar data to IR data that an error in the assumed range of the radar impacted the accuracy of the hit velocity, associated angles, and landing position of the ball. At a maximum range error of $4 \mathrm{~m}(13.1 \mathrm{ft})$ the average error in the hit speed of the ball was $6.9 \%$.

Landing positions measured with high speed video and a range finder were also compared to radar. The $x$ and $z$ coordinates of the landing point of the ball measured by radar were within $0.6 \mathrm{~m}(2 \mathrm{ft})$ and $0.4 \mathrm{~m}(1.3 \mathrm{ft})$ respectively of those measured with video. These were measured at average hit distances of $71.5 \mathrm{~m}(234.5 \mathrm{ft})$. The hit distance of the ball measured with the range finder was within $0.6 \mathrm{~m}(1.82 \mathrm{ft})$ at an average hit distance of 81.4 m (234.5 ft). It was also determined that offsetting the radar from a straight-line between the pitcher and home plate did not affect the accuracy of the hit distance measured by the radar.

The effect of the ball trajectory was also shown to affect the amount of noise experienced by the radar. Balls hit at angles greater than $\pm 27^{\circ}$ in the horizontal direction experienced more noise than those hit at smaller angles. Balls hit at vertical angles of $15^{\circ}$
experienced more noise than those hit at $30^{\circ}$. It was determined that the excess noise experienced by balls hit a smaller vertical angles affected the accuracy of the landing point of the ball by $3 \%$.

## Chapter 5 - Lift and Drag

Lift and drag are necessary to create an accurate trajectory model or to determine the design effects on different types of balls. Because radar measures the speed of the ball throughout the trajectory it can also be used to find the aerodynamic forces. The following examines the ability of radar to calculate lift and drag of balls in flight.

## Section 5.1 Review of Lift and Drag

Lift can be defined as the aerodynamic force acting perpendicular to both the motion and spin vector $(\omega)$ of the ball (23). It points perpendicular to the axis of $\omega$. The force of lift was first studied by Isaac Newton, but later explained and credited to G. Magnus in 1852 which is now commonly known as the Magnus force or Magnus effect. Lift on a ball is thought to be a function of the ball's surface geometry, velocity, orientation, and spin rate. The force of lift has no effect on the velocity of the ball and only changes its direction. The coefficient of lift, $C_{L}$, can be defined by (24),

$$
\begin{equation*}
C_{L}=\frac{2 F_{L}}{\rho A V^{2}} \tag{5.1}
\end{equation*}
$$

where $F_{L}$ is the force due to lift, $\rho$ is the fluid density, $A$ is the cross sectional area of the ball, and $V$ is the velocity of the ball.

Similar to the force of lift, the force of drag is thought to be a function of surface roughness, velocity, orientation, and spin rate. The force of drag acts opposite the direction of the velocity vector so it is constantly retarding the motion of the ball. The coefficient of drag, $C_{D}$, can be defined by (24),

$$
\begin{equation*}
C_{D}=\frac{2 F_{D}}{\rho A V^{2}} \tag{5.2}
\end{equation*}
$$

where $F_{D}$ is the force of drag. A free body diagram of a typical hit ball is shown in Figure 5.1.


Figure 5.1: Free body diagram of a hit ball with backspin.

Lift and drag forces are strongly influenced by the flow type and behavior very close to the ball called the boundary layer. Laminar flow in the boundary layer is smooth and uniform occurring at low speeds, where turbulent flow occurs at higher speeds and is unpredictable. Prandtl described this flow and defined how its effects changed based on the characteristics of the object (25). The boundary layer starts with velocities on the surface of the ball equal to zero. Further away from the ball the velocity of the fluid reaches its free stream velocity, or the velocity of the fluid without obstructions (26). The reduction in the velocity of the fluid near the surface of the ball is due to the friction forces of the object and the viscous shear forces on the surface (24). This results in changes in pressures on the surface of the ball, affecting the lift and drag.

It is conventional to normalize velocity and air properties using the non-dimensional Reynolds number, $R_{e}$, defined by

$$
\begin{equation*}
R_{e}=\frac{v D}{V} \tag{5.3}
\end{equation*}
$$

where $D$ is the diameter of the ball and $v$ is the kinematic viscosity of air. As a ball travels along its trajectory it experiences three major aerodynamic drag regions. The regions are defined by the flow in the boundary layer and the location of the flow separation. The first region is defined by $R_{e}<3 \times 10^{5}$; in this region flow in the boundary layer is laminar until separation occurs at approximately $80^{\circ}$ from the stagnation point of the ball (27). This early separation causes lower pressures downstream, increasing $F_{D}(24)$. The second region, $3 \times 10^{5}<R_{e}<3 \times 10^{6}$, flow separates at $80^{\circ}$ and reattaches at $120^{\circ}$ from the stagnation point where the boundary flow is classified as turbulent. The reattaching in the boundary layer delays the flow separation of the fluid, causing a dramatic drop in the $F_{D}$. This region is referred to as the drag crisis and is characterized by a significant drop in the drag coefficient over a very small change in Reynolds number. In the final critical region, $R_{e}>3 \times 10^{6}$, flow starts turbulent and remains in this state until separation occurs just before $120^{\circ}$. This results in a slight increase in the drag because the pressure is lower than in the first region, but slightly higher than the drag crisis region.

Lift is also affected by flow in the boundary layer and is influenced by the rotation of the ball. As the ball begins to rotate, the velocity of the fluid near the edges of the ball deviate based on whether they encounter the leading or trailing edge of the spinning ball. On the edge of the ball where $\omega$ is working with the flow of the fluid, separation occurs closer to the stagnation point on the ball. On the other side, $\omega$ is working against the fluid flow creating a separation further down the ball. This imbalance in flow separation causes pressure differences
on either side of the ball creating a force that wants to push the ball perpendicular to the velocity. This is depicted in Figure 5.2 where $V$ is the velocity of the ball.


Figure 5.2: How lift is generated in the boundary layer from the rotation of a ball.

The geometry or surface roughness of the ball also plays a critical role in the aerodynamic forces experienced by the ball. This is especially apparent when looking at complex balls like that of a softball or baseball. The seams effectively change the surface roughness of the object and the overall drag that it experiences. Along with the different seam heights, the orientation, velocity, and spin rate play a major role on the lift and drag experienced by a ball in flight.

The effect of velocity on the drag coefficient has been explored by many researchers. It is well known, for instance, that the drag coefficient decreases as the ball's velocity increases. Achenbach explored this phenomenon with balls of varying roughness in a wind tunnel (28). He mounted a 20 cm diameter sphere to a 2 cm support pole inside the tunnel and measured forces using strain gauges attached to the supports. Data from his experiments can be seen in

Figure 5.3, where the data is characterized by the ball's roughness parameter, $\frac{k}{d_{s}}$, where $k$ was the height of the roughness element and $d_{s}$ was the diameter of the sphere.


Drag coefficient $c_{d} v s$. Reynolds number for a sphere. Parameter: surfsse roughness. - smooth (Achenbach 1972); $\times, k / d_{s}=25 \times 10^{-5} ; \nabla, k / d_{s}=150 \times 10^{-5}$; $\bigcirc, k / d_{s}=250 \times 10^{-5} ; \Delta, k / d_{s}=500 \times 10^{-5} ; \square, k / d_{s}=1250 \times 10^{-5}$.

Figure 5.3: Effects of surface roughness on $C_{D}$ reported by Achenbach (28).

His data showed that as surface roughness increased, the drag crisis initiated at lower speeds, increasing the overall drag coefficient. Over the region of $10^{5}<R_{e}<2 \times 10^{5}$ he found the drag to be within $0.1<C_{D}<0.5$.

While Achenbach's study is widely referenced, it does not take into account the spin rate of the ball. This was explored by Bearman and Harvey in 1976 on golf balls in a wind tunnel (29). The ball model was hung between two wires with the upper wire attached to a load cell to measure lift forces. A motor inside the model of the ball was used to impart rotation on the ball. Data was collected for constant wind tunnel velocities while varying the rotational speed of the ball. Their data for a conventional golf ball can be seen in Figures 5.4 and 5.5.


Figure 5.4: Data for the lift coefficient of a conventional golf ball at varying spin rates reported by Bearman and Harvey (29).

The study found that as angular velocity increased at a constant flow velocity both $C_{L}$ (Figure 5.4) and $C_{D}$ (Figure 5.5) increased. This trend is apparent in Figure 5.4 indicating a clear relationship between $\omega$ and $C_{L}$. The fluctuations in Figure 5.5 indicate that the relationship between $\omega$ and $C_{D}$ may not be as significant.


Figure 5.5: Data for the drag coefficient of a conventional golf ball at varying spin rates reported by Bearman and Harvey (27).

A similar study was done by Asai et al. using soccer balls in a wind tunnel (30). They found a linear trend between $\omega$ and $C_{D}$, but the data showed fluctuations in the measured drag coefficient at constant translational velocities. The lift coefficient increased very sharply from 0.1 to 0.25 over a spin parameter, $S$, increase of only 0.1 , defined as,

$$
\begin{equation*}
S=\frac{\omega R}{V} \tag{5.4}
\end{equation*}
$$

where $R$ is the radius of the ball. Alam et al. used wind tunnel data to measure lift and drag on tennis balls, comparing it to computational data gathered from Fluent 6.0 (31). Their data showed an increase in $C_{D}$ with increasing $\omega$, but found a significant amount of fluctuation between spin rates at the same speed.

More recently studies have begun to look at these forces in situ where the ball's speed or position is measured outside of a wind tunnel. A study involving ten high speed infrared cameras were used to track Major League Baseball's (MLB) over a distance of 15 feet (22). The balls were projected with a two wheeled pitching machine at translational velocities ranging
from $50-110 \mathrm{mph}$ and $\omega$ ranging from $1500-4500 \mathrm{rpm}$. The results for $C_{L}$ from the study can be seen in Figure 5.6.


Figure 5.6: Lift data for MLB baseballs from Nathan's study using infrared camera tracking (29).

Similar to previous studies (29)(30)(31), the data in Figure 5.6 shows an increasing lift coefficient with increasing spin factor. Similarly it shows from $0.0<S<0.15$ the lift coefficient increases rapidly, and then slows to reach a maximum $C_{L} \sim 0.35$ at $S^{\sim} 0.6$. The large amounts of scatter associated with $C_{D}$ did not allow for the data to be displayed.

Lift and Drag results have also been analyzed using PITCHf/x. The system recorded balls thrown by Boston Red Sox lefthander Jon Lester in a game against the Seattle Mariners at Safeco Field (Go Mariners!) (32). The PITCHf/x system utilizes two cameras around the field to track the position of the ball during its path from the pitching mound to home plate. The position data was fit to a so called 9-parameter fit (constant acceleration), from which the initial
position, velocity, and acceleration for each of the three dimensions was determined. From this the coefficients of lift and drag were determined.


Figure 5.7: Lift and drag coefficients for pitches during a MLB game calculated by Nathan using PITCHf/x data (32).

The data shown in Figure 5.7 shows no evidence of a drag crisis. It shows $0.35<C_{D}<0.5$ for pitch velocities between 70 and 90 mph . The PITCHf/x system did not measure spin therefore the data was not compared to $\omega$. The data in Figure 5.7 does show drag coefficient scatter of approximately 0.1 at similar velocities. The values for $C_{D}$ and $C_{L}$ were similar to values found in other studies (28) (33) (34).

Most recently, Kensrud projected spinning and non-spinning balls through still air, using light gates to measure the change in velocity (34) (33). The light gates were spaced at known distances to one another recording the time it took the balls to pass through each gate. The study looked at different balls at velocities ranging from $17.9 \mathrm{~m} / \mathrm{s}$ to $60.8 \mathrm{~m} / \mathrm{s}$.


Figure 5.8: Lift coefficient for different balls calculated by Kensrud from data recorded using light gates (33).


Figure 5.9: Drag coefficient for different balls calculated by Kensrud from data recorded using light gates (34).

Data calculated by Kensrud in Figure 5.8 shows a rapid increase in $C_{L}$ over a minimal increase in $S$ for different types of baseballs. This is similar to other studies of different balls (31) (30) (22). The data shown in Figure 5.9 shows an increasing $C_{D}$ with increasing $S$, but the
trend is only clear for two of the three balls shown. Kensrud's research also showed a relationship between the drag coefficient and the roughness and orientation of different balls. The data identified relatively large scatter in the drag of baseballs in comparison to smooth spheres. This comparison showed that the aerodynamic complexity of seams significantly changed the repeatability of $C_{D}$ by as much as 0.1 for rotating baseballs.

## Section 5.2 Lift and drag reported by the Workbench

The radar calculated lift and drag coefficients over the entire trajectory of the pitch and hit. The $C_{d}$ and $C_{L}$ were displayed as a function of time or velocity in the Workbench. A graph of $C_{d}$ and $C_{L}$ reported by the Workbench can be seen in Figure 5.10, showing typical lift and drag curve as a function of velocity. This data was analyzed for many different pitches and hits from fast pitch and slow pitch softball tournaments and from a baseball field study.


Figure 5.10: The lift and drag coefficient reported from TrackMan's raw data file.

The data in Figure 5.10 was from a slow pitch game recorded during the ASA Border Battle. The graph shows $C_{d}$ and $C_{L}$ for both the pitch and hit. The pitch was a typical under hand toss and the hit traveled $56 \mathrm{~m}(183.7 \mathrm{ft})$ into the outfield and $8.9 \mathrm{~m}(29.2 \mathrm{ft})$ toward third base. The pitch data starts at approximately $12 \mathrm{~m} / \mathrm{s}(26.8 \mathrm{mph})$, while the hit takes place at $V^{\sim} 32.5$ $\mathrm{m} / \mathrm{s}$ ( 72.7 mph ). The data displayed in Figure 5.10 showed some obvious problems for both the lift and drag coefficients.

The discontinuities in both $C_{L}$ and $C_{d}$ at the end of the trajectory of the hit create some obvious errors. This was the product of the Kalman filtering process estimating the position of the ball after the radar loses the tracking. This estimation of the position of the ball was based on the recorded trajectory data and a predefined trajectory model. This resulted in changes in the velocities and ultimately changes in the accelerations, affecting the calculated lift and drag forces.

Beyond the discontinuities in the data were the trends of the data. The starting value of $C_{L}$ for the hit appears to be a reasonable value, but then $C_{L}$ decreases to 0.03 and peaks at 0.56 . This trend would indicate that the ball stopped rotating midway through its trajectory and then began rotating again. Additionally, the hysteresis shown in the $C_{d}$ curve for the hit was very large, values of $C_{d}$ varied by as much as 0.23 at equal velocities. This problem was more visible in $C_{d}$ given for the pitch. There is also an apparent problem with the lift and drag coefficients for the pitch because both far exceed a value of one toward the end of the trajectory. This would indicate very large lift and drag forces which have not been seen in any previous studies.

These trends are common amongst different data sets for data displayed with the workbench. This indicates a problem in the way that the Workbench is calculating $C_{d}$ and $C_{L}$ for
both the pitch and hit. This maybe a problem of excess noise in the signal causing incorrect calculations of acceleration, like that discussed in Section 4.2. Therefore the lift and drag data produced by Workbench is not useful in examining the aerodynamic forces affecting a pitched or hit ball.

## Section 5.3 Fitting position data to calculate lift and drag

To determine a way to calculate lift and drag coefficients from positional data, trajectory models were used to simulate hits. Lift and drag forces were found from a time derivative of a least squares fit to the $x$ and $y$ coordinates of the ball. This resulted in the components of the velocity of the ball. A second derivative with respect to time of the positional components of the ball resulted in the components of acceleration. Figure 4.23 showed that a third order fit did not adequately describe the position data from the raw data file.

To examine the affect that the order of the polynomial fit played on the reported $C_{d}$ data, three different trajectory models were used to produce $x$ and $y$ positions. The position data calculated from the different models all used a 0.01 s time step. The models were based on different lift and drag models. The first model was based on $C_{d}$ and $C_{L}$ data collected by Kensrud using light gates (34) (33), referred to as, Kensrud data. The second, referred to as the constant lift trajectory model used $C_{d}=-.005(V)+0.65$ and $C_{L}=0.3$. The third model, referred to as the spinning trajectory model, used $C_{d}=-.005(V)+0.65$ and $C_{L}=.35(S)+.2$.

The models were used to simulate four different trajectories of a hit baseball. In each trajectory, the ball started at an initial position of $x=0 \mathrm{~m}$ and $\mathrm{y}=0.6 \mathrm{~m}(2 \mathrm{ft})$. The ball was
projected in a 2D x-y plane, as defined in Figure 3.9. The initial conditions of the four trajectories can be seen in Table 5.1.

Table 5.1: Initial conditions of the four trajectories used to determine proper polynomial fits.

|  | Trajectory 1 | Trajectory 2 | Trajectory 3 | Trajectory 4 |
| :--- | :--- | :--- | :--- | :--- |
| $V_{i}(\mathrm{~m} / \mathrm{s})$ | $35(78.3 \mathrm{mph})$ | $40.2(90 \mathrm{mph})$ | $22.4(50 \mathrm{mph})$ | $12.2(27.4 \mathrm{mph})$ |
| $\theta_{V}$ | $30^{\circ}$ | $12^{\circ}$ | $45^{\circ}$ | $32.4^{\circ}$ |
| $\omega(\mathrm{rpm})$ | 2000 | 1500 | 2500 | 600 |

The positions were then fit to polynomials ranging from second order to eighth order and differentiated to obtain velocity and acceleration. The angle between the velocity and the acceleration vectors was determined from their dot product. This angle was then used to calculate the acceleration opposing the velocity of the ball. After subtracting the effects of gravity, the result was the acceleration due to drag. Similarly, the acceleration due to lift was calculated by determining the acceleration normal to the velocity of the ball. Using the atmospheric temperature and pressure conditions $C_{d}$ and $C_{L}$ were calculated using equations 5.1 and 5.2. A piecewise derivative was also used to calculate $C_{d}$ and $C_{L}$ based on equations 4.11. The piecewise derivative was taken over a constant acceleration fit to 0.20 s of data, which resulted in 20 data points. This increment was determined to minimize the effects of noise on the data, but also provide sufficient accuracy in the calculated lift and drag coefficients. The different fits were compared using the Root Mean Square Deviation ( $R M S D$ ) defined by,

$$
\begin{equation*}
R M S D=\sqrt{\frac{\sum_{i}^{n}\left(X_{1, i}-X_{2, i}\right)^{2}}{n}} \tag{5.5}
\end{equation*}
$$

The graphs of $C_{d}$ and $C_{L}$ calculated using the different polynomial fits can be seen for Kensrud, the constant lift, and spinning models in Figures 5.11-5.13 respectively. Table 5.2 shows the associated RMSD values of $C_{d}$ and $C_{L}$ for the different fits to the four trajectories defined in Table 5.1. The RMSD values in Table 5.2 were characteristic of all three models, but are shown for the spinning trajectory model only.


Figure 5.11: $C_{d}$ and $C_{L}$ for different polynomial fits based on position data calculated using Kensrud trajectory model (34) (33).


Figure 5.12: $C_{d}$ and $C_{L}$ for different polynomial fits based on position data calculated using the constant lift trajectory model.


Figure 5.13: $C_{d}$ and $C_{L}$ for different polynomial fits based on position data calculated using the spinning trajectory model.

Table 5.2: RMSD values for the different polynomial fits to position from the four different trajectories.

|  | Trajectory 1 |  | Trajectory 2 |  | Trajectory 3 |  | Trajectory 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RMSD $C_{d}$ | RMSD $C_{L}$ | RMSD $C_{d}$ | RMSD $C_{L}$ | RMSD $C_{d}$ | RMSD $C_{L}$ | RMSD $C_{d}$ | RMSD $C_{L}$ |
| $8^{\text {th }}$ | $\mathbf{0 . 0 0 0 8}$ | 0.0010 | $\mathbf{0 . 0 0 0 4}$ | $\mathbf{0 . 0 0 0 6}$ | $\mathbf{0 . 0 0 1 5}$ | 0.0020 | $\mathbf{0 . 0 0 1 6}$ | $\mathbf{0 . 0 0 2 7}$ |
| $7^{\text {th }}$ | $\mathbf{0 . 0 0 0 8}$ | $\mathbf{0 . 0 0 0 9}$ | $\mathbf{0 . 0 0 0 4}$ | $\mathbf{0 . 0 0 0 6}$ | 0.0016 | $\mathbf{0 . 0 0 1 9}$ | $\mathbf{0 . 0 0 1 6}$ | $\mathbf{0 . 0 0 2 7}$ |
| $6^{\text {th }}$ | 0.0011 | 0.0050 | $\mathbf{0 . 0 0 0 4}$ | 0.0007 | 0.0034 | 0.0032 | 0.0017 | $\mathbf{0 . 0 0 2 7}$ |


| $5^{\text {th }}$ | 0.0039 | 0.0091 | 0.0016 | 0.0011 | 0.0044 | 0.0029 | 0.0023 | 0.0030 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $4^{\text {th }}$ | 0.0154 | 0.0154 | 0.0036 | 0.0054 | 0.0185 | 0.0168 | 0.0070 | 0.0103 |
| $3^{\text {rd }}$ | 0.0711 | 0.0240 | 0.0122 | 0.0208 | 0.0718 | 0.0457 | 0.0342 | 0.0121 |
| $2^{\text {nd }}$ | 0.1595 | 0.1110 | 0.0966 | 0.0286 | 0.2239 | 0.1651 | 0.1380 | 0.1415 |
| piecewise | 0.0014 | 0.0024 | 0.0013 | 0.0019 | 0.0038 | 0.0024 | 0.0018 | 0.0032 |

The RMSD values shown in Table 5.2 indicate that regardless of the trajectory of the ball, a higher order fit provided a better correlation between the trajectory model and the least squares fit. The seventh order fit to position is, on average, the best fit for position. Figures 5.11-5.13 shows that a third order polynomial fit, which represents a constant jerk model, creates a large hysteresis in the value of $C_{d}$ after the apex of the hit. This hysteresis is typical of $C_{d}$ values produced by the Workbench.

A possible reason for the improved correlation with higher order polynomials can be seen from examining the calculated velocities and accelerations. The $x$ and $y$ velocities calculated from the different polynomial fits can be seen in Figure 5.14. The corresponding accelerations can be seen in Figure 5.15.


Figure 5.14: Velocity in the $x$ and $y$ directions for different polynomial fits to the position calculated from the spinning trajectory model.


Figure 5.15: Acceleration in the $x$ and $y$ directions for different polynomial fits to the position calculated from the spinning trajectory model.

The higher order fits to position results in better descriptions of both velocity and acceleration. Simulated noise was added to the position data from the spinning and constant lift trajectory models to determine how noise would affect the subsequent $C_{d}$ and $C_{L}$ values. Noise was added to both the $x$ and $y$ values independently. Two cases were considered: a large $\pm 0.08 \mathrm{~m}(0.25 \mathrm{ft})$ and small $\pm 0.15 \mathrm{~m}(0.5 \mathrm{ft})$ noise range. The small range $( \pm 0.08 \mathrm{~m})$ represented the average difference between the radar, and a smooth fit as seen in Figure 4.7. The large range ( $\pm 0.15 \mathrm{~m}$ ) was less than the maximum difference ( 0.24 m ), but provided an error that was representative of the data recorded by the radar. The average $R M S D$ values between fits to the noisy data and the model can be seen in Tables 5.3 and 5.4.

Table 5.3: RMSD values for $C_{d}$ and $C_{L}$ calculated from the spinning and non-spinning trajectories with $0.15 \mathrm{~m}(0.5 \mathrm{ft})$ of noise.

|  | Non-spinning trajectory model |  | Spinning trajectory model |  |
| :--- | :--- | :--- | :--- | :--- |
|  | RMSD |  | RMSD |  |
|  | $C_{d}$ | $C_{L}$ | $C_{d}$ | $C_{L}$ |
| Piecewise | 982 | 104 | 148 | 187 |
| Second order | 0.1714 | 0.1249 | 0.1308 | 0.1101 |
| Third order | 0.0791 | 0.0306 | 0.0626 | 0.0266 |
| Fourth order | 0.0277 | $\mathbf{0 . 0 0 8 5}$ | 0.0208 | 0.0071 |
| Fifth order | $\mathbf{0 . 0 1 2 0}$ | 0.0273 | 0.0143 | $\mathbf{0 . 0 0 4 8}$ |
| Sixth order | 0.0360 | 0.0184 | $\mathbf{0 . 0 1 1 1}$ | 0.0100 |
| Seventh order | 0.0149 | 0.0352 | 0.0448 | 0.0293 |
| Eighth order | 0.3548 | 0.0831 | 0.2056 | 0.0513 |

Table 5.4: RMSD values for $C_{d}$ and $C_{L}$ calculated from the spinning and non-spinning trajectories with $0.30 \mathrm{~m}(1 \mathrm{ft})$ of noise.

|  | Non-spinning trajectory model |  | Spinning trajectory model |  |
| :--- | :--- | :--- | :--- | :--- |
|  | RMSD |  | RMSD |  |
|  | $C_{d}$ | $C_{L}$ | $C_{d}$ | $C_{L}$ |
| Piecewise | 487 | 93 | 108 | 111 |
| Second order | 0.1684 | 0.1255 | 0.1299 | 0.1104 |
| Third order | 0.0792 | 0.0303 | 0.0638 | 0.0357 |
| Fourth order | $\mathbf{0 . 0 3 9 1}$ | $\mathbf{0 . 0 1 4 1}$ | $\mathbf{0 . 0 1 9 6}$ | $\mathbf{0 . 0 2 3 1}$ |
| Fifth order | 0.0434 | 0.1064 | 0.0503 | 0.0658 |


| Sixth order | 0.0891 | 0.0967 | 0.0846 | 0.0641 |
| :--- | :--- | :--- | :--- | :--- |
| Seventh order | 0.0856 | 0.0667 | 0.0723 | 0.0658 |
| Eighth order | 0.1039 | 0.1349 | 0.1327 | 0.1800 |

When noise was added to the positions calculated from the trajectory models, the $R M S D$ values for the different fits increased. This can be seen for both trajectory models in Tables 5.3 and 5.4. The RMSD values in Tables 5.3 and 5.4 also show that when the piecewise constant acceleration fits to position were used to calculate $C_{d}$ and $C_{L}$, the effects of noise rendered the calculations useless. The $R M S D$ values in the above tables show that on average the use of a fourth or fifth order fit to position is best when calculating $C_{d}$ and $C_{L}$ in the presence of noise.

The data presented so far has dealt with trajectories of hits. To determine how the technique used to calculated $C_{d}$ and $C_{L}$ worked for pitch trajectories, pitches were simulated using the Kensrud trajectory model. The trajectory simulated an average pitch seen during the 2011 ASA slow pitch softball championship series, with an initial velocity of $12.2 \mathrm{~m} / \mathrm{s}(27.4 \mathrm{mph})$ at an initial vertical angle of 32.4 and 600 rpm of spin. Random noise of $\pm 3.05 \mathrm{~cm}(0.1 \mathrm{ft})$ was added to both the $x$ and $y$ positions. The noise represented the average noise seen for trajectories of pitches. Figures 5.16 and 5.17 show $C_{L}$ and $C_{d}$ for the simulated pitch with noise.


Figure 5.16: Values of $C_{L}$ for a simulated pitch with the Kensrud trajectory model with $3.05 \mathrm{~cm}(0.1 \mathrm{ft})$ of random noise added to the x and y positions separately.


Figure 5.17: Values of $C_{d}$ for a simulated pitch with the Kensrud trajectory model with $3.05 \mathrm{~cm}(0.1 \mathrm{ft})$ of random noise added to the x and y positions separately.

The $C_{d}$ and $C_{L}$ data seen in Figures 5.16 and 5.17 show in the presence of noise, none of the polynomial fits accurately describe the trend of $C_{d}$ and $C_{L}$. The data in Figures 5.16 and 5.17
were calculated using a time step of .01 seconds, which retuned $170 X$ and $Y$ positional points.
To determine how the number of points of position used to fit the polynomials affected the accuracy of $C_{d}$ and $C_{L}$, the time step was varied. The results can be seen in Table 5.5 and $C_{d}$ and $C_{L}$ with the lowest RMSD values are plotted in Figure 5.18 and 5.19.

Table 5.5: The effects of number of points on the accuracy of $C_{d}$ and $C_{L}$ calculated in the presence of $3.05 \mathrm{~cm}(0.1 \mathrm{ft})$ of noise.

| time <br> step <br> (s) | \# of <br> points |  | time <br> step <br> (s) | \# of <br> points |  | time <br> step <br> (s) | \# of <br> points |  | time <br> step <br> (s) | \# of <br> points |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.001 | 1479 |  | 0.002 | 760 |  | 0.004 | 392 |  | 0.01 | 170 |  |
|  | $C_{d}$ | $C_{L}$ |  | $C_{d}$ | $C_{L}$ |  | $C_{d}$ | $C_{L}$ |  | $C_{d}$ | $C_{L}$ |
| Order | $R M S D$ | $R M S D$ | Order | $R M S D$ | $R M S D$ | Order | $R M S D$ |  |  |  |  |
| $R M S D$ | Order | $R M S D$ |  |  |  |  |  |  |  |  |  |
| $R M S D$ |  |  |  |  |  |  |  |  |  |  |  |
| 8th | 0.275 | 0.148 | 8th | 0.081 | 0.088 | 8th | 0.383 | 0.956 | 8th | 1.226 | 1.653 |
| 7th | 0.084 | 0.085 | 7th | 0.082 | 0.089 | 7 th | 0.120 | 0.499 | 7th | 0.494 | 0.166 |
| 6th | 0.043 | 0.84 | 6th | 0.085 | 0.090 | 6th | 0.143 | 0.158 | 6th | 0.197 | 0.146 |
| 5th | 0.036 | 0.072 | 5th | 0.042 | 0.033 | 5th | 0.062 | 0.036 | 5th | 0.132 | 0.051 |
| 4th | $\mathbf{0 . 0 3 2}$ | $\mathbf{0 . 0 1 2}$ | 4th | 0.053 | $\mathbf{0 . 0 1 5}$ | 4th | 0.060 | $\mathbf{0 . 0 2 8}$ | 4th | 0.089 | 0.047 |
| 3rd | 0.041 | 0.023 | 3rd | $\mathbf{0 . 0 4 2}$ | 0.029 | 3rd | $\mathbf{0 . 0 3 0}$ | 0.040 | 3rd | $\mathbf{0 . 0 4 5}$ | $\mathbf{0 . 0 1 5}$ |
| 2nd | 0.099 | 0.046 | 2nd | 0.099 | 0.047 | 2nd | 0.100 | 0.046 | 2nd | 0.097 | 0.050 |



Figure 5.18: $C_{d}$ for the bolded fits shown in Table 5.5.


Figure 5.19: $C_{L}$ for the bolded fits shown in Table 5.5.

It can be seen from Table 5.5 and Figures 5.18 and 5.19 that as the number of points used to fit the position drop, so does the accuracy of $C_{d}$ and $C_{L}$. Therefore if the number of positional points is insufficient in the presence of noise $C_{d}$ and $C_{L}$ cannot be determined from a polynomial fit to position.

## Section 5.4 The effects of noise on $C_{d}$ and $C_{L}$ determined from radar data

$C_{d}$ and $C_{L}$ were calculated using position data from the TrackMan raw data files. The lift and drag coefficients were determined along the full trajectory of several different hits of baseballs and softballs. Pitches were not considered because of a lack of data points. The process used a fifth order polynomial fit to the positional data recorded for two different games and a field study. The field study was conducted in the University of Idaho in-door football facility (Kibbie Dome). A comparison of the lift and drag coefficients calculated using a piecewise constant acceleration fit and a fifth order polynomial fit to position can be seen in Figure 5.20 and 5.21.


Figure 5.20: Drag coefficient for a well hit ball recorded inside the Kibbie Dome


Figure 5.21: Lift coefficient for a well hit ball recorded inside the Kibbie Dome

The lift and drag data shown in Figures 5.20 and 5.21 are typical of well hit balls experiencing minimal noise. The data shows that the polynomial and piecewise fits produce
nearly identical lift and drag data that follow similar trends. The polynomial fit produces a cleaner curve and therefore will be used for the following lift and drag calculations.

Data was recorded for baseballs hit by collegiate level baseball players and pitched from a two wheeled pitching machine. The baseballs were NCAA approved and represented a raised seam style ball. Data for softballs was collected from men's and women's slow pitch games from the ASA Championship Series. The softball was a 0.52 COR, 300 lb compression flat seam slow pitch ball. The data was from several games during one day of the tournament. Weather Underground was used to determine the atmospheric conditions for the days (35).

To determine the amount of noise that was affecting the return signal, the velocity of the ball was calculated from the unfiltered TrackMan raw data. A noisy hit can be seen in Figure 5.22. The data was collected from a single pitch and hit recorded during an ASA Softball Championship series slow pitch game. The hit was projected at a vertical angle of $41^{\circ}$ and a horizontal angle of $19^{\circ}$ with a speed of $37.7 \mathrm{~m} / \mathrm{s}(84.3 \mathrm{mph})$. The initial $\omega$ was measured by radar to be 3800 rpm and the hit traveled $78.1 \mathrm{~m}(256.2 \mathrm{ft})$ into the outfield and $10 \mathrm{~m}(32.8 \mathrm{ft})$ to the third base side of the pitcher line.


Figure 5.22: Noise in the return signal of a pitch and hit calculated from equation 4.16, compared to radar smoothed and filtered velocity.

The noise can be seen affecting the velocity data calculated using equation 4.16 at the end of the pitch and over the entire hit. The pitch can be distinguished from the hit by the negative values of $V$, where the positive values of $V$ indicate the velocity of the hit. The large amounts of scatter for $V$ throughout the entire hit show that excessive amounts of noise affected the return signal. A clean signal that is nearly free of noise can be seen in Figure 5.23. The data from this pitch and hit was recorded six hours later during the same tournament. The hit was projected at a vertical angle of $36^{\circ}$ and a horizontal angle of $-3.6^{\circ}$ with a speed of 31.2 $\mathrm{m} / \mathrm{s}(69.8 \mathrm{mph})$. The initial $\omega$ was measured by radar at 2033 rpm and the hit traveled 72 m $(236.2 \mathrm{ft})$ into the outfield and $2.5 \mathrm{~m}(8.2 \mathrm{ft})$ toward first base.


Figure 5.23: Noise in the return signal of a pitch and hit calculated from equation 4.16, compared to radar smoothed and filtered velocity.

The $C_{d}$ and $C_{L}$ calculated from a fifth order fit, for the entire trajectory of the hit shown

Figures 5.22 (Noisy) and 5.23 (Clean) can be seen in Figures 5.24 and 5.25.


Figure 5.24: $C_{d}$ for data from Figures 5.22 and 5.23 showing the full trajectory of the hit.


Figure 5.25: $C_{L}$ for data from Figures 5.22 and 5.23 showing the full trajectory of the hit.

It is clear by examining the figures of $C_{d}$ and $C_{L}$ for both hits that noise impacted the coefficients. In Figure 5.24 the maximum value of $C_{d}$ for the noisy data reaches 0.7 , larger than
other $C_{d}$ data collected at similar velocities (Figure 5.9). There was also a larger than expected value for $C_{L}$ for the noisy data.

The $C_{d}$ and $C_{L}$ values from the clean data seen in Figures 5.24 and 5.25 showed a large hysteresis of $C_{d}$ and $C_{L}$. Some hysteresis for $C_{L}$ should be evident because of the decreasing $\omega$ as the hit traveled along its trajectory. A decrease in $\omega$ has been shown by many researchers to cause a decrease in $C_{L}$ (31) (30) (29) (33) (22). The large difference between $C_{d}$ at the same velocities, sometimes as much as 0.15 , seen in Figure 5.24 was not expected.

There would be a small decrease in $C_{d}$ because of a decrease in $\omega$, but the effects of $\omega$ on $C_{d}$ have been shown to be small. The apex of the hit for the clean data seen in Figure 5.24 occurs at $\mathrm{V}=15.3 \mathrm{~m} / \mathrm{s}$ and $C_{d}=0.44$. Shortly after this point the values of $C_{d}$ begin to decrease as the force of gravity causes the ball to begin to accelerate.

Curves were fit to the noise extremes to determine how different fits in Figure 5.23 affected the values of $C_{d}$ and $C_{L}$. The fits can be seen in Figure 5.26 and the associated $C_{d}$ and $C_{L}$ values can be seen in Figures 5.27 and 5.28.


Figure 5.26: Different fits to velocity of the hit data shown in Figure 5.23.


Figure 5.27: Drag Coefficient for the different fits seen in Figure 5.23.


Figure 5.28: Lift Coefficient for the different fits seen in Figure 5.23.

The data in Figures 5.27 and 5.28 shows that the noise experienced at the end of a hit does not have a strong bias. That is, the noise appears to fall equally above and below and average line determined by the radar.

## Section 5.5 $C_{d}$ and $C_{L}$ calculated from radar data

Since ball acceleration was obtained from the derivative of its velocity, the acceleration (and hence drag) was sensitive to noise in the measured data. The part of the ball flight with the most noise occurred at the beginning of the hit and after the apex of its trajectory. In the following, data prior to 0.5 s after impact and data after the apex were not considered, effectively minimizing the effect of signal noise on ball drag. The drag coefficient is shown for raised seam baseballs as a function of Reynolds number in Figure 5.29.


Figure 5.29: Coefficient of drag for raised seam baseballs of varying initial spin rates. Kensrud (32), $100<\omega<4000$; Radar 1, 3000 < $\omega$ < 4000; Radar 2, 2000 < $\omega$ < 3000; Radar 3, 1000 < $\omega$ < 2000.

The radar results correlate well with results from balls projected through still laboratory air (34). The range of $C_{d}$ was similar to previous studies for drag in the region $10^{5}<R_{e}<2 \times 10^{5}$ (28). While the $C_{d}$ decreased rapidly over a small change in $R_{e}$, it was not apparent if this represented a drag crisis. The decrease in drag does however occur at similar $R_{e}$ as previous studies where a drag crisis was observed (28). To establish the existence of a drag crisis, ball speeds outside the $R_{e}$ of game play are needed. This suggests that in comparison to other sports, such as golf, the drag crisis in baseball and softball may play a smaller role.

The magnitude of the change in the drag coefficient during the ball's flight, $\Delta C_{d}$, depended on the ball's spin rate. Balls with $\omega<2000$ saw $\Delta C_{d} \approx 0.05$, while with $\omega>2000$ saw $\Delta C_{d} \approx 0.1$. The differences in $\Delta C_{d}$ can be attributed to the trajectory of the ball. Baseballs with $\omega$ < 2000 rpm had lower trajectories than those hit with higher spin rates, resulting in shorter flight times, smaller changes in velocity, and therefore a smaller change in $C_{d}$.

The average $\omega$ of hit balls was 2086 and 1586 rpm for baseballs and softballs respectively. The slower $\omega$ of the softballs, relative to baseballs, is likely due to the higher rotational inertia of the larger diameter softball and the lower rotational inertia of the smaller diameter softball bat.


Figure 5.30: Coefficient of drag for flat seamed softballs and baseballs of varying initial spin rates. Kensrud (34), $100<\omega<4000$ (MLB baseballs); Radar 1, $2000<\omega<3500$ (softballs); Radar 2, $900<\omega<2000$ (softballs).

The drag coefficient for flat seamed softballs is shown in Figure 5.30 as a function of Reynolds number. Major League Baseball (MLB) Baseballs have a flat seam similar to softballs and are included in Figure 5.30 for comparison (34). The drag of the flat seamed softballs obtained from radar measurements in a game setting agreed with the drag of the MLB balls projected through still air in a laboratory setting. At $R_{e}=2 \times 10^{5}$ the flat seamed balls had $0.2<C_{d}$ $<0.3$, while for the raised seam balls $0.3<C_{d}<0.4$. This is similar to the trend seen by Achenbach that smoother balls experience a lower $C_{d}$ at similar Reynolds numbers. The larger
scatter observed in the drag for both types of hit balls compared to laboratory data may be due to ball orientation, environmental forces (wind), and spin rate.

The effect of spin on $C_{d}$ for raised seam and flat seam balls is shown in Figures 5.31 and 5.32, respectively. The speed of the baseballs ranged from 23 to $40 \mathrm{~m} / \mathrm{s}$ and 700 to 4000 rpm while the speed of the softballs ranged from 17 to $39 \mathrm{~m} / \mathrm{s}$ and 400 to 3800 rpm .


Figure 5.31: Drag coefficient of raised seam baseballs at varying velocities.


Figure 5.32: Drag Coefficient of flat seam softballs and baseballs (Kensrud) at varying velocities.

For individual hits the results show an increase in $C_{d}$ with an increasing $S$, which is consistent with a large change in $V$ compared to a small change in $\omega$. This is most apparent for trajectories with large $S$, where the flight time is longer and the change in $V$ is greater.

The effect of spin on $C_{d}$ for baseballs and softballs at constant translational velocity ( $V=$ $32 \mathrm{~m} / \mathrm{s}$ ) is shown in Figures 5.33 and 5.34, respectively.


Figure 5.33: Drag coefficient of raised seam baseballs at $V=32 \mathrm{~m} / \mathrm{s}$.


Figure 5.34: Drag coefficient of flat seam softballs at $V=32 \mathrm{~m} / \mathrm{s}$

When $V$ was held constant, the trend of increasing $C_{d}$ with incresing $\omega$ was not as apparent, which may also be observed in Figs. 5.27 and 5.28. Hits with high spin rates (4000 rpm) decay to about 2400 rpm at their appex [12]. Thus, in Figure 5.27, if $\omega$ had a strong effect on $C_{d,}$ the data labled Radar 2 would start where Radar 1 ends; and the data labeled Radar 2 would start where Radar 3 ends. Instead all the data started in nearly the same range of $C_{d,}$, illustrating the insensitivity of $\omega$ on $C_{d}$.

The affect of spin on $C_{L}$ was explored from the data sets outlined in Figures 5.29-5.30. The data was also filtered by the landing position to the first or third base side of the pitcher line. Because the spin axis was not known, it was difficult to determine the direction of the lift force. Filtering out any hit that deviated further from the pitcher line than $20 \mathrm{~m}(65.6 \mathrm{ft})$ in the
positive or negitive direction ensured that the spin axis was primarily perpendicular to the $y-z$ plane or the force of lift was in the y-z plane. The filtering process also took into account the acceleration in the z-direction filtering out any acceleration in the z-direction that was greater than $2 \mathrm{~m} / \mathrm{s}^{2}$ and any in-plane initial launch angle that was greater than $7^{\circ}$. The effect of spin on $C_{L}$ for raised seam and flat seam balls is shown in Figures 5.35 and 5.36 , respectively.


Figure 5.35: Lift coefficient of raised seam baseballs at varying velocities.


Figure 5.36: Lift coefficient of flat seam softballs and baseballs (Kensrud) at varying velocities.

It is apparent from Figure 5.36 that $\omega$ had an effect on the $C_{L}$ experienced by the ball. There appeared to be a steep increase in $C_{L}(0-0.2)$ over a small change in $S(0-0.1)$. This was consistent with findings of the effects of $\omega$ on $C_{L}$ found by Kensurd (33). The smaller values of $C_{L}$ at the same $S$ as Kensrud can be attributed to the direction of $C_{L}$ calculated for this work not being exclusively in the $x-y$ plane. Therefore if the spin vector of the ball is not in the $x-y$ plane the $C_{L}$ values in Figure 5.36 represent a component of the overall Magnus force. Because the tilt of the spin vector is not known, a correction for the direction of the Magnus force cannot be determined. The bi-linear trend observed in Figure 5.36 and observed elsewhere (31) (33) (22) was not observed in Figure 5.35.


Figure 5.37: Lift Coefficient of raised seam baseballs at $V=32 \mathrm{~m} / \mathrm{s}$.

The effect of spin on $C_{L}$ for baseballs and softballs at constant translational velocity ( $V=32 \mathrm{~m} / \mathrm{s}$ ) is shown in Figures 5.37 and 5.38 , respectively.


Figure 5.38: Lift Coefficient of flat seamed softballs at $V=32 \mathrm{~m} / \mathrm{s}$ and MLB Baseballs at varying velocities (22).

Similar to the data in Figure 5.33 the $C_{L}$ for raised seam baseballs in Figure 5.35 does not appear to be largely influenced by $\omega$. The effect of $\omega$ on $C_{L}$ for flat seamed softballs is much more pronounced. It is clear from the data in Figure 5.36 that a trend of increasing $\omega$ translates to an increasing $C_{L}$. This is a similar trend seen in Figure 5.6 from data gathered by Nathan (22).

Fits to the $C_{d}$ and $C_{L}$ values for flat seam softballs, seen in Figures 5.30 and 5.36, provides an average trend for lift and drag. These fits can be seen in Figures 5.39 and 5.40.


Figure 5.39: Log fit to drag data for flat seamed softballs seen previously in Figure 5.30.


Figure 5.40: Linear fit to lift data for flat seamed softballs seen previously in Figure 5.36.

Using the fits shown in Figures 5.39 and 5.40 several different trajectories were predicted using initial conditions measured by the radar for slow pitch softball data. The predicted trajectories determined from the fit data were compared to trajectories determined
from the radar and those from in-situ $C_{d}$ and $C_{L}$ values. The in-situ $C_{d}$ and $C_{L}$ values were calculated for the full trajectory of the hit from a fifth order polynomial fit to $x, y$, and $z$ positions. A typical hit can be seen in Figure 5.41.


Figure 5.41: Positions determined from lift and drag data obtained by radar.

The analyzed hits traveled primarily in the $x-y$ plane. The trajectories in Figure 5.41 show that when a fit to $C_{d}$ and $C_{L}$ was used the resulting x position of the landing point falls short by an average of $2.0 \mathrm{~m}(6.57 \mathrm{ft})$ from the landing point determined by the radar. When the in-situ $C_{d}$ and $C_{L}$ values were used the average x position of the landing point was $0.7 \mathrm{~m}(2.3 \mathrm{ft})$ longer than those determined by the radar. The data shows a linear relationship between $C_{d}$ and $V$ or $C_{L}$ and $S$ provides a poor predictor of the trajectory of a hit ball. The in-situ $C_{d}$ and $C_{L}$ values used in Figure 5.41 can be seen in Figures 5.42 and 5.43.


Figure 5.42: In-situ drag coefficient of a flat seam softball.


Figure 5.43: In-situ lift coefficient of a flat seam softball.

It is clear from the coefficients in Figure 5.42 and 5.43 that a large hysteresis creates a non-linear relationship. The hysteresis provides a change in the coefficients that is necessary to
predict the landing point of the hit. The hysteresis may be the result of spin decay or the change in acceleration of the ball.

## Section 5.5 Summary

This chapter has examined what forces affect a ball in flight and how different parameters of both the ball and the trajectory affect these forces. Using four different ideal trajectory models it was determined that a fourth or fifth order polynomial fit to the position of the ball could be used to successfully calculated $C_{d}$ and $C_{L}$. It was also determined that in the presence of noise, trajectories that had small amounts of positional data (less than 300 points) were less accurate at describing $C_{d}$ and $C_{L}$.

It was found that $C_{d}$ measured in game conditions was similar to that measured in a laboratory setting at the same $R_{e}$, providing relevance in the use of radar for determining aerodynamic forces. Due to the limited range of ball speeds occuring in play, a definitive drag crisis was not observed from the radar results. A drop in $C_{d}$ was found at a similar $R_{e}$ to other studies in which a drag crisis was observed. By comparing balls with different stitch heights, it was observed that balls with flat seams expierence $0.2<C_{d}<0.45$, and those with raised seams expierence $0.25<C_{d}<0.5$. Under the game conditions considered in this study, the drag coefficient was observed to decrease with increasing speed and was not strongly sensitive to the spin rate of the ball. In many cases the scatter in drag was greater than the effect of ball speed and spin rate.

Large amounts of scatter were also observed when calculating $C_{L}$ for both types of balls. A correlation of increasing $C_{L}$ with increasing $S$ was found for softballs. When $V=32 \mathrm{~m} / \mathrm{s}$, the
measured $C_{L}$ for flat seamed softballs increased from 0.0 to 0.2 in direct proportion to $S$. This trend was not as evident for the data recorded for the baseball. A linear relationship between $C_{d}$ and $V$ or $C_{L}$ and $S$ is incorrect and will underestimate the trajectory of a hit.

## Chapter 6 Game data

An evaluation of data collected during different games and field studies provides a measure of the accuracy of the radar. The slow pitch data included 4200 pitches and 1988 hits from different player abilities collected during 78 games. The fast pitch data includes 6631 pitches and 1004 hits from different levels of play collected during 120 games. Baseball data was recorded during a single day of a field study, in which 1395 pitches and 958 hits were recorded.

The pitches from the baseball study were projected using a two wheeled pitching machine setup 15.2 m ( 50 ft ) from home plate. The baseballs were Rawlings NCAA approved balls. Because the pitches starting vertical position (release height) and horizontal position (releases side) remained nearly constant, any deviation in either direction between consecutive pitches was a product of the radar's error. The release height and side as a function of time can be seen in Figures 6.1 and 6.2.


Figure 6.1: Release height measured by the radar for a single field study, where baseballs were pitched using a two wheeled pitching machine.


Figure 6.2: Release side measured by the radar for a single field study, where baseballs were pitched using a two wheeled pitching machine.

The drop in release height in Figure 6.1 at a pitch count of 1184 represents an adjustment of the pitching machine's height. The data used to determine the standard
deviation started at a pitch count of 300 and stopped at 900 for both the release height and side. The data in Figure 6.1 shows that the standard deviation of the height of a pitch was 0.04 $\mathrm{m}(0.13 \mathrm{ft})$. A maximum difference between consecutive points in the release height of 0.18 m $(0.59 \mathrm{ft})$, occurs at a pitch count of 423 . The release side in Figure 6.2 shows several adjustments of the pitching machine in the horizontal direction. The standard deviation in Figure 6.2 was $0.03 \mathrm{~m}(0.09 \mathrm{ft})$ with a maximum difference of $0.1 \mathrm{~m}(0.33 \mathrm{ft})$ occurring at a pitch count of 735 .

The release height and side was also examined for fast pitch pitches recorded during live game play. The data can be seen in Figures 6.3 and 6.4.


Figure 6.3: Release side recorded by the radar over 120 fast pitch games.


Figure 6.4: Release height recorded by the radar over 120 fast pitch games.

Because the starting point of a pitch delivered by a pitcher is dynamic the same conclusions cannot be made about the error in the radar based on the deviation in the pitch release height and side. The data in Figures 6.3 and 6.4 does shed some insight into the functionality of the radar. Groupings of release side data in Figure 6.3 show how different pitchers change the horizontal release location of their pitches. Figures 6.3 and 6.4 also show some seemingly random points that do not appear to fall in a particular grouping. These points can be better seen in Figure 6.5.


Figure 6.5: Starting pitch position for all pitches recorded during fast pitch games.

The random scatter in Figure 6.5 was determined to be the result of errant tracking by the radar. This was determined by analysis of the game data recorded by the user while tracking the game. In each instance when a large release height and side occurred, no pitch results were tagged by the user. This indicated that the data was not from a pitch, but rather an errant recording of data by the radar. This could be caused by a throw from the outfield to home plate, the radar tracking something other than a ball, or the tracking of a thrown ball other than the pitch. This happened 63 times out of 6631 recorded pitches.

Horizontal and vertical break can be seen in Figures 6.6-6.7, and provide insight into the game of fast pitch softball.


Figure 6.6: Effects of the axis of $\omega$ on the horizontal break of 6600 fast ball pitches $\left(0^{\circ}=\right.$ break down ward, $90^{\circ}=$ break to the left, $180^{\circ}=$ break upward, $270^{\circ}=$ break to the left).


Figure 6.7: Effects of the axis of $\omega$ on the vertical break of 6600 fast ball pitches ( $0^{\circ}=$ break down ward, $90^{\circ}=$ break to the left, $180^{\circ}=$ break upward, $270^{\circ}=$ break to the left).

Figure 6.6 shows that the radar is properly relating spin axis and horizontal break.

Pitches with $0^{\circ}<$ spin axis $<180^{\circ}$, break to the pitcher's left (negative) and those with $180^{\circ}<$
spin axis $<360^{\circ}$ break to the pitcher's right (positive). Figure 6.7 shows that the radar is not correctly calculating vertical break. Pitches with $270^{\circ}$ < spin axis $<90^{\circ}$ would have top spin and break downward (negative), and those with $90^{\circ}<$ spin axis $<270^{\circ}$ would have back spin and break upward (positive). The same data was analyzed for baseballs projected from a two wheeled pitching machine in Figures 6.8 and 6.9.


Figure 6.8: Effects of the axis of $\omega$ on the horizontal break of 1395 baseball pitches projected using a two-wheeled pitching machine ( $0^{\circ}=$ break down ward, $90^{\circ}=$ break to the left, $180^{\circ}=$ break upward, $270^{\circ}=$ break to the left).


Figure 6.9: Effects of the axis of $\omega$ on the vertical break of 1395 baseball pitches projected using a two-wheeled pitching machine ( $0^{\circ}=$ break down ward, $90^{\circ}=$ break to the left, $180^{\circ}=$ break upward, $270^{\circ}=$ break to the left).

Because the ball was projected using a two wheeled machine with the bottom wheel spinning faster than the top wheel the ball should break upward as seen in Figure 6.9. The same upward break is seen in Figure 6.8 with very minimal break to the left or right, typical of a pitching machine with the wheels stacked vertically. An analysis, conducted by Nathan (33), of 99 pitches measured by the PITCHf/x tracking system and pitched by lefthander John Lester, provides a comparison of the break data recorded by radar shown in Figures $6.6-6.9$. The pitches were categorized by Nathan based on the type of pitch;
I. 4-seam fastballs thrown nearly overhand. The velocity is in the range of $90-95 \mathrm{mph}$ with a spin axis of around $170^{\circ}$, corresponding to mainly backspin. The spin rate was 2000 rpm
II. Slider thrown with a spin axis of $240^{\circ}$ that breaks up and to the left as seen from the catcher's perspective. The velocity is in the range of $85-90 \mathrm{mph}$ with a spin rate of 800 rpm.
III. 4-seam fastball thrown with a $3 / 4$ arm angle, rather than overhand. The velocity was 90 mph with a spin rate of 2000 rpm . The spin axis was $135^{\circ}$ indicating a pitch that breaks up and to the right as seen from the catcher's perspective
IV. Curveballs with a spin axis of $330^{\circ}$, breaking down and to the left as seen from the catcher's perspective. The velocity was between 70-75 mph with a spin rate of 1500 rpm.


Figure 6.10: Horizontal ( dx ) and vertical ( dz ) break for MLB baseballs pitched during a single game from a single pitcher (33).

When the break data in Figures 6.6 and 6.7 are compared to data recoreded by Nathan it is clear that baseballs pitched by major league pitchers break much more then fast pitch softballs. Baseballs in the 4-seam fastball category broke 5 inches to the pitcher's left (catcher's right) and 15 inches upward. This compares to similar pitches in fast pitch that broke 2 inches to the pitcher's left (catcher's right) and 8 inches upward.

Some insight about the radar can also be gained by looking at hit data recorded for baseballs, slow pitch softballs and fast pitch softballs. The spin rate as a function of vertical hit angle can be seen in Figure 6.11-6.13.


Figure 6.11: Spin rate of all hits during a single baseball field study.


Figure 6.12: Spin rate of all hits recorded by radar during slow pitch softball games.


Figure 6.13: Spin rate of all hits recorded by radar during fast pitch softball games.

Figures $6.11-6.13$ show that the radar does not display $\omega$ above 3500 rpm or below 500 rpm to the user. While the radar still measures $\omega$ in these regimes TrackMan $\mathrm{A} / \mathrm{S}$ is unable to verify that they are correct and therefore they do not display them to the user. The data in Figures 6.11-6.13 also show a slightly linear dependence of $\omega$ on $\Theta_{V}$. Because of the smaller rotational inertia of baseballs, the slope of the linear fit to the data in Figure 6.11 is larger than the 30.5 cm (12 in) softball data (Figures 6.12 and 6.13). The difference in slopes in Figure 6.12
and Figure 6.13 was due to the smaller circumferential inertia of fast pitch bats. Figures 6.14 -
6.16 show a relationship between the vertical hit angle and the hit distance.


Figure 6.14: Dependence of hit distance on the initial vertical angle of the hit for fast pitch softballs.


Figure 6.15: Dependence of hit distance on the initial vertical angle of the hit for slow pitch softballs.


Figure 6.16: Dependence of hit distance on the initial vertical angle of the hit for NCAA Baseballs.

The hit data in Figure 6.14-6.16 show an optimal hit angle of about $30^{\circ}$. Out of the 1004 fast pitch hits in Figure 6.14, 22 were hit at $\theta_{V}= \pm 1^{\circ}$ at average hit distances $14.6 \mathrm{~m}(48 \mathrm{ft})$ with a standard deviation of $4.5 \mathrm{~m}(15 \mathrm{ft})$. Of the 1988 slow pitch hits in Figure $6.15,60$ were hit at $\theta_{V}= \pm 1^{\circ}$ at average hit distances $23.1 \mathrm{~m}(75.7 \mathrm{ft})$ with a standard deviation of $4.3 \mathrm{~m}(14 \mathrm{ft})$. Of the 958 baseball hits in Figure 6.16, 33 were hit at $\theta_{V}= \pm 1^{\circ}$ at average hit distances 28.6 m ( 93.7 ft ) with a standard deviation of $6.5 \mathrm{~m}(21.2 \mathrm{ft})$. Recall that the data for baseball hits was collected during a field study in which the batters were instructed to hit fly balls, therefore the number of hits at $\theta_{V}= \pm 1^{\circ}$ may not be a fair representation of the game of college baseball.

The data in Figures $6.14-6.16$ shows that hit baseballs travel, on average, further than either fast pitch or slow pitch softballs. This may be due to the smaller rotational inertia of the baseball providing a greater Magnus force, allowing baseballs to travel further at smaller vertical angles. To determine if hit speed plays a role in baseballs being hit further than
softballs, the frequency of batted ball speeds (BBS) are plotted for slow pitch, fast pitch, and baseball in Figures 6.17-6.19 respectively.


Figure 6.17: Histogram of 1004 batted ball speeds from fast pitch softball games.


Figure 6.18: Histogram of 1988 batted ball speeds from slow pitch softball games.


Figure 6.19: Histogram of 958 batted ball speeds from a baseball field study.

The data in Figures $6.17-6.19$ shows that the majority of baseballs hit, were at faster BBS than either fast pitch or slow pitch softball. This likely accounted for the baseballs in Figure 6.16 traveling further than the fast pitch (Figure 6.14) or slow pitch (Figure 6.15) softballs.

## Chapter 7 Summary

This work evaluated a Doppler radar system produced by TrackMan A/S called a TrackMan II X. The limiting factor for radar operating at low tracking angles like that seen in baseball and softball is the signal's susceptibility to noise caused by multipath reflections from the ground.

The effects of noise caused by multipath reflections were found to affect the measured angles determined from a phase comparison of the return signal. This noise created scatter in the reported position of the ball. It was determined that balls hit at horizontal angles greater than $\pm 27^{\circ}$ experienced more noise, but it was not determined if this excess noise caused any error in the reported data. When compared to high speed video, the landing point of balls hit at vertical angles of $15^{\circ}$ deviated by $3 \%$ more than those hit at $30^{\circ}$.

It was also determined that when the assumption of the initial range of the ball was incorrect, the accuracy of the velocity, horizontal and vertical angles, and the coordinates of the landing point of the ball were affected. When the initial range assumption was shorter than the actual range by $4 \mathrm{~m}(13.1 \mathrm{ft})$; the velocity, horizontal angle, vertical angle, $x$ position, and z position of the hit deviated by as much as $30.9 \%$ from the actual values.

When compared to high speed video, the radar measured the velocity of both the pitch and hit at home plate to within $2.3 \%$ and $0.7 \%$ respectively. The associated horizontal and vertical angles of the pitch and hit were measured to within $0.5^{\circ}, 2.1^{\circ}, 1.2^{\circ}$, and $1.0^{\circ}$ respectively. The pitch and hit velocities reported by the radar at home plate were also compared to IR video tracking and found to be within $0.9 \%$ and $0.7 \%$ respectively. The horizontal and vertical angles of the hit at home plate agree to within $2.1^{\circ}$ and $0.1^{\circ}$.

Using a trajectory model to simulate pitches and hits, it was concluded that the radar more accurately measured speed and angle compared to high speed video tracking. Conversely the radar and IR videos had comparable accuracy.

Using a range finder the hit distance measured by the radar was found to be accurate to within $0.68 \%$ for 23 recorded hit balls. Balls projected at vertical angles of $30^{\circ}$ were tracked $3 \%$ more accurately then those projected at $15^{\circ}$. When the vertical angle was held constant at $30^{\circ}$ the accuracy of the hit distance was not affected by a horizontal offset of the radar.

Game data was used to show that the horizontal and vertical release positions of a pitch determined by the radar deviated by an average of $0.04 \mathrm{~m}(0.12 \mathrm{ft})$ and $0.02 \mathrm{~m}(0.05 \mathrm{ft})$ respectively. Game data was also used to show that radar can be used to calculate aerodynamic forces of hit balls. For baseballs, $0.3<C_{d}<0.5$, and for softballs $0.25<C_{d}<0.4$ in the Reynolds region, $1 \times 10^{5}<\operatorname{Re}<2 \times 10^{5}$. It was determined that $\omega$ had a minimal effect on $C_{d}$ for both types of balls. Coefficients of lift were also calculated and were found to depend on $\omega$, but were found to be smaller in magnitude than measured by others at similar $S$. The smaller $C_{L}$ values were attributed to uncertainty in the spin axis and therefore the direction of $C_{L}$.

## Section 7.2 Future Work

To use radar to determine aerodynamic forces, $\omega$ measured by the radar must be analyzed for accuracy. This includes determining an accurate spin decay model and an analysis of the effects of $\omega$ on $C_{d .}$. Because the $C_{L}$ measured in this work for NCAA baseballs was low compared to other studies, more time should be spent measuring $C_{L}$ for raised stitched balls. An accurate model of $C_{d}$ and $C_{L}$ as functions of $V$ and $S$ will allow for future ball trajectory
models to be accurate. This includes analyzing lift and drag for different trajectories, hit at different initial vertical and horizontal angles, speeds, spin rates, and at different spin axis.

A measure of the accuracy of the data about the pitch that is recorded as the ball leaves the pitchers hand would provide a complete accuracy analysis. This would provide confidence in the game data and allow for a quantifiable look at the games of softball and baseball including; the dependence of $\omega$ on the vertical and horizontal hit angles, dependence of horizontal and vertical release angle on break, deviation of release height and release side on the skill level of the pitcher. This would also include tracking the skill level of different players to determine its effect on the recorded data. An accurate analysis of the actual stride length of different players for slow pitch and fast pitch softball and baseball would also be benefit the accuracy of the reported data.

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## Appendix A: Different setups used for tracking games

The optimal setup for the radar is directly behind home plate in-line with the center of the pitching mound. The easiest way to accomplish this is by using a tri-pod or a bracket affixed to the pole that holds the backstop netting. Because some foul balls may be directed back toward the radar should be on the fan side of the backstop netting. To orient the radar so that it is not rotated about the $y$-axis defined by the radar, the user should use the calibration screen in Figure 3.3. The red indicator on the calibration screen is located in the middle of the screen and represents the center line of the $y$-axis. If the indicator needs to be moved either to the left or right during the calibration setup then the radar is slightly rotated about the $y$-axis. The slight rotation is not detrimental to operation, but a stronger return signal will result if the radar has not rotation about the $y$-axis.

Often the pole that holds the backstop netting will prevent the radar from being directly in-line with the pitcher's mound. In this case a slight offset is required. The radar should be offset to either the first base or third base side of the pole, making sure to carefully measure the offset. The radar should be offset enough so that the field of view of the radar is not impeded by the pole. The reflection from the pole will affect the accuracy of the data.

In some instances the backstop is made of chain link. In these cases, the radar will not work properly on the fan side of the chain link. The radar must be setup on the field side in order to record data. For this purpose a protective screen should be setup around the radar to prevent errant throws or foul balls impacting the radar. The screen must not contain any highly reflective materials and no cross member should impede the field of view of the radar. A simple
design used pvc piping for the structural supports and netting for the screen. This setup was used in Virginia during the 2011 DIII tournament and can be seen in Figures 0.1 and 0.2


Figure 0.1: Front view of radar mounted to a pole on the field side of the backstop. Hey Rich!


Figure 0.2: Side view of radar mounted to a pole on the field side of the backstop.

The setup of the radar requires the height of the radar to be $1 / 3$ of the measured distance from the tip of home plate as discussed in Chapter 3 of this work. In some cases the distance from the tip of home plate was such that, achieving the correct height required additional supports. That is, the height of the tripod was not tall enough, nor was there a central pole to mount the clamps that held the radar. This was the case in a Louisville Slugger field study in California. In order to raise the radar high enough to reach the $1 / 3$ ratio a table was brought in. This can be seen in Figure 0.3. Positioning the radar this high in the air caused some instability. For this reason this setup would not be optimal on a windy day.


Figure 0.3: Using tables to achieve the required height for the radar.
The majority of the data collected for this work was collected at the ASA Hal of Fame stadium in Oklahoma City, OK. There were four different fields at the complex, all of which were used to collect data. The setup on the stadium field had the radar clamped to a pole, which was centered behind home plate. The distance from home plate to the radar was 30 ft . This required the radar to be 10 ft in the air. The radar was clamped to the center pole, which
required a 10 inch offset toward third base. This provided a better field of view for the left outfield, which was primarily where hit balls from right handed batters were projected.


Figure 0.4: View from behind the back stop of ASA Stadium Field. The center pole was used to setup the radar.
ASA field 3 was used to collect data for the high speed camera field study described in section 4.4. It can be seen in Figure 4.28. The radar was setup 29.5 ft from the tip of home plate and 9.8 ft vertically upward. Because the field did not have a central pole to attach the radar to, a tri pod was used. The setup was on the fan side of the back stop to prevent it from being damaged by a foul ball.

ASA field 4 was used to collect game data of different slow pitch tournaments. It was a similar setup to that described for ASA field 3 . The radar was 29.67 ft behind the tip of home plate and 9.9 feet vertically upward. A tripod was also used because there was not a central
pole. Both fields 3 and 4 had nylon netting for back-stops, therefore the radar could be setup behind the netting. Field 2 did not have nylon netting. The backstop was chain link fence and for this reason Field 2 was never used.

## Appendix B: TrackMan Baseball Glossary of terms

Terms are listed in the order in which they appear on the spreadsheet output from the TrackMan baseball game tracking software.

1. Pitch number = sequential ordering of all pitches in the game.
2. Time $=$ Time of release of ball from the pitcher's hand (local time).
3. Pitcher = Name of the pitcher.
4. P. Dexterity $=$ Handedness of the pitcher.
5. Pitcher Team = Team that the pitcher plays for.
6. Batter = Name of the batter.
7. B. Dexterity $=$ Side of the plate the batter is hitting from.
8. Batter Team = Team that the Batter plays for
9. Rel. Speed = Speed of the pitched ball when it leaves the pitcher's hand (mph)
10. V. Rel. Angle = Initial vertical direction of the pitched ball when it leaves the pitcher's hand, reported in degrees. Positive numbers indicate the ball is moving vertically upward, while negative numbers mean the ball is moving vertically downward. (degrees)
11. H. Rel. Angle = Initial horizontal direction of the pitched ball when it leaves the pitcher's hand, reported in degrees. Positive numbers indicate the ball is moving to the right, while negative numbers mean the ball is moving to the left from the pitcher's perspective. (degrees)
12. Spin Rate $=$ The amount of spin on the ball as it leaves the pitcher's hand reported in revolutions per minute. (revolutions per minute)
13. Spin Axis = The direction the ball is spinning reported in degrees of tilt.
a. A ball thrown with a spin axis of $0^{\circ}$ has pure top spin
b. A ball thrown with a spin axis of $180^{\circ}$ has pure back spin.
c. A ball thrown with a spin axis of $90^{\circ}$ has the leading edge of the spin vector pointing to the left, creating a Magnus force to the left.
d. A ball thrown with a spin axis of $270^{\circ}$ has the leading edge of the spin vector pointing to the right, creating a Magnus force to the right.
14. Rel. Height = The height above the plate at which the pitcher releases the ball. (feet)
15. Rel. Side $=$ The distance from the center of the pitcher's mound at which the pitcher releases the ball. Positive numbers indicate a release side to the right of the pitcher and negative numbers indicate to the left. (feet)
16. Horz. Break = Indicates how much the pitch broke to the left or right of a straight line path to home plate. Positive numbers indicate the ball breaking to the right from the pitchers perspective and negative indicate a break to the left. (inches)
17. Vert. Break = How much the pitch deviates vertically from a straight line path to home plate. A negative value indicates a vertically downward break and positive indicates an upward break. (inches)
18. Plate Loc. H. = Plate location height, or the height of the pitched ball as it crosses the front of the plate. (feet)
19. Plate Loc. Side $=$ Plate location side, or the distance from the center of the plate of the pitched ball as it crosses the front of home plate. Negative numbers are to the catcher's left and positive numbers are to the catcher's right. (feet)
20. Zone Speed = Speed of the pitched ball as it crosses the front of home plate. (mph)
21. Vert. Appr. Angle = Vertical Approach Angle, or the vertical angle the trajectory of the pitched ball makes as the pitch crosses the front of home plate. (degrees)
22. Horz. Appr. Angle = Horizontal approach angle, or the azimuth angle that the pitched ball makes as it crosses home plate. A positive angle indicates that the ball is moving from left to right from the pitcher's perspective, while a negative angle indicates the pitched ball is moving from the right to left from the pitcher's perspective. (degrees)
23. Zone Time = The amount of time from the pitcher's release until it crosses the front of home plate. (seconds)
24. Exit Speed = The speed of the batted ball. (mph)
25. V. Exit Angle = Vertical exit angle, or the elevation of the ball as it leaves the bat. A positive number indicates the batted ball is moving positively upward. (degrees)
26. H. Exit Angle $=$ Horizontal exit angle, or azimuth angle of the batted ball. A negative number indicates a hit ball traveling to the third base side of a straight line from home plate to the center of the pitcher's mound. (degrees)
27. Spin Rate $=$ The amount of spin on the ball as it leaves the bat, reported in revolutions per minute. (revolutions per minute)
28. Ver. Spin Axis $=$ The direction the ball is spinning reported in degrees of tilt.
a. A ball thrown with a spin axis of $0^{\circ}$ has pure top spin
b. A ball thrown with a spin axis of $180^{\circ}$ has pure back spin.
c. A ball thrown with a spin axis of $90^{\circ}$ has the leading edge of the spin vector pointing to the left, creating a Magnus force to the left.
d. A ball thrown with a spin axis of $270^{\circ}$ has the leading edge of the spin vector pointing to the right, creating a Magnus force to the right.
29. Carry = The distance the ball travels as it hits the ground or is caught. (feet)
30. Bearing $=$ An azimuth measurement of the location on the field where the hit ball either hit the ground or was caught. A bearing of $0^{\circ}$ is line with home plate and the center of the pitcher's mound. A positive bearing indicates the hit ball landed to the first base side of this straight line. (degrees)
31. Total Pitch = Same as Pitch number (\# 1)
32. Pitcher's Set = Indicates whether the pitcher is throwing from the windup or set position, as tagged by the user.
33. Inning = The Inning the pitch was thrown.
34. Top/Bottom = Indicates which team is batting, the home team always bats from the bottom of the inning.
35. Outs = Indicates the number of outs before the pitch was thrown.
36. Balls = Indicates the number of balls in the count before the pitch was thrown.
37. Strikes = Indicates the number of strikes in the count before the pitch was thrown.

The majority of this work relied on data exported to the .xml files using the Workbench. The following gives descriptions of the.$x m l$ files opened in Microsoft Excel. The data is all reported in SI units.

| $\underline{1}$ | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | name | visible | name2 | value ${ }^{\text {v }}$ | type - | description |
| 2 | StrokeConfiguration |  | Version | 300 | u32_t | Version descriptor (v3.00) |
| 3 | StrokeConfiguration |  | Date | 9/10/2011 | string |  |
| 4 | StrokeConfiguration |  | Time | 8:16:01 | string |  |

Figure 0.5: Date and time in .xml file
The date and time that the file was recorded by the radar is shown in Figure 0.5.


Figure 0.6: Ball and environment settings.
Figure 0.6 shows the ball setting used by the Workbench to determine aerodynamic properties
of the ball. The figure also shows the environmental settings used by the Workbench, including
temperature, pressure, and humidity. Much of the data seen in Figure 0.6 was originally used
for tracking golf balls and golf clubs and is not used in any calculations for baseballs or softballs.

| 32 | TrackingConfiguration | TeePosition | point3d $x$ | 4.11304 |
| :--- | :--- | :--- | :--- | ---: | ---: |
| 33 | TrackingConfiguration | TeePosition | point3d $y$ | -1.3716 |
| 34 | TrackingConfiguration | TeePosition | point3d $z$ | -0.120351 |
| 35 | TrackingConfiguration | TargetPosition | point3d $x$ | 22.5455 |
| 36 | TrackingConfiguration | TargetPosition | point3d $y$ | -1.3716 |
| 37 | TrackingConfiguration | TargetPosition | point3d $z$ | -0.659702 |

Figure 0.7: Position of home plate relative to the radar and starting point of the pitch.
Figure 0.7 shows the user defined position of home plate relative to the radar and assumed
starting point of the pitch in meters. The position of home plate relative to the radar is seen in
rows $32-34$ in meters. The initial position of the pitch is shown in rows $35-37$ in meters.

|  | t | vr | $x$ | $y$ | 2 | xResidual | yResidual | zResidual | snr1 | snr2 | sn53 | snrcalc | alpha | beta | gamma | range |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 216 | 0 | -20.859 | 16.6 | 1.137 | 0.666 | 0 | 0 |  | 048.864 | 49.163 | 49.759 | 49.163 | 0.99669 | $-0.08129$ | 0.00291 | 20.7264 |
| 217 | 0.0043 | -21.158 | 16.51 | 1.128 | 0.658 | 0 | 0 |  | 055.01 | 55.641 | 56.368 | 55.641 | 0.9966 | $-0.08178$ | 0.00266 | 20.6368 |
| 218 | 0.0085 | -21.165 | 16.42 | 1.129 | 0.656 | 0 | 0 |  | 058.905 | 59.777 | 60.578 | 59.777 | 0.99665 | $-0.08175$ | 0.02269 | 20.5471 |
| 219 | 0.0127 | -21.159 | 16.331 | 1.141 | 0.652 | 0 | 0 |  | 61.369 | 62.439 | 63.324 | 62.439 | 0.99669 | $-0.08125$ | 0.00261 | 20.4574 |
| 220 | 0.017 | -21.138 | 16.242 | 1.164 | 0.643 | 0 | 0 |  | 62.992 | 64.184 | 65.158 | 64.184 | 0.99678 | $-0.08017$ | 0.00231 | 20.3678 |
| 221 | 0.0213 | -21.113 | 16.153 | 1.197 | 0.634 | 0 | 0 |  | 64.256 | 65.404 | 66.407 | 65.404 | 0.99691 | $-.07857$ | 0.002 | 20.2781 |
| 222 | 0.0255 | -21.099 | 16.063 | 1.232 | 0.626 | 0 | 0 |  | 65.376 | 66.246 | 67.172 | 66.246 | 0.99704 | $-0.07689$ | 0.00175 | 20.1884 |
| 223 | 0.0298 | -21.102 | 15.974 | 1.258 | 0.622 | 0 | 0 |  | 66.231 | 66.624 | 67.387 | 66.624 | 0.99713 | $-0.07564$ | 0.00168 | 20.0988 |
| 224 | 0.034 | -21.114 | 15.884 | 1.269 | 0.624 | 0 | 0 |  | 66.598 | 66.414 | 66.978 | 66.414 | 0.99717 | $-0.0751$ | 0.00195 | 20.0092 |
| 225 | 0.0382 | $-21.119$ | 15.795 | 1.267 | 0.631 | 0 | 0 |  | 066.367 | 65.678 | 66.086 | 65.678 | 0.99716 | $-0.07521$ | 0.00245 | 19.9196 |
| 226 | 0.0425 | -21.113 | 15.705 | 1.256 | 0.64 | 0 | 0 |  | 65.639 | 64.805 | 65.225 | 64.805 | 0.99712 | $-0.0758$ | 0.00302 | 19.8301 |
| 227 | 0.0468 | -21.098 | 15.615 | 1.247 | 0.639 | 0 | 0 |  | 064.782 | 64.347 | 64.948 | 64.347 | 0.99708 | $-.07629$ | 0.00315 | 19.7406 |
| 228 | 0.051 | -21.085 | 15.526 | 1.258 | 0.627 | 0 | 0 |  | 64.394 | 64.567 | 65.266 | 64.567 | 0.99712 | $-0.07577$ | 0.00266 | 19.6511 |
| 229 | 0.0553 | -21.072 | 15.437 | 1.284 | 0.616 | 0 | 0 |  | 064.796 | 65.262 | 65.828 | 65.262 | 0.99722 | $-0.07446$ | 0.00223 | 19.5617 |
| 230 | 0.0595 | -21,046 | 15.348 | 1.305 | 0.613 | 0 | 0 |  | 065.711 | 66.157 | 66.448 | 66.157 | 0.9973 | $-0.07338$ | 0.00222 | 19.4723 |
| 231 | 0.0637 | -20.988 | 15.258 | 1.321 | 0.618 | 0 | 0 |  | 066.662 | 67.092 | 67.117 | 67.092 | 0.99736 | $-0.07262$ | 0.00264 | 19.383 |
| 232 | 0.068 | -20.949 | 15.169 | 1.336 | 0.629 | 0 | 0 |  | 67.265 | 67.926 | 67.768 | 67.926 | 0.99741 | $-.07185$ | 0.00333 | 19.2937 |
| 233 | 0.0723 | -20.93 | 15.079 | 1.354 | 0.639 | 0 | 0 |  | 67.317 | 68.493 | 68.253 | 68.493 | 0.99748 | $-0.0709$ | 0.00404 | 19.20 |
| 234 | 0.0765 | -20.932 | 14.989 | 1.378 | 0.648 | 0 | 0 |  | 066.84 | 68.7 | 68.478 | 68.7 | 0.99756 | $-0.06965$ | 0.00664 | 19.1152 |
| 235 | 0.0808 | -20.943 | 14.9 | 1.409 | 0.654 | 0 | 0 |  | 066.14 | 68.56 | 68.436 | 68.56 | 0.99767 | $-0.06805$ | 0.00513 | 19.0261 |
| 236 | 0.085 | -20.951 | 14.811 | 1.445 | 0.659 | 0 | 0 |  | 065.651 | 68.144 | 68.146 | 68.144 | 0.9977 | $-0.06614$ | 0.00554 | 18.937 |
| 237 | 0.0892 | -20.95 | 14.7 | 1.482 | 0.663 | 0 | 0 |  | 065.522 | 67.573 | 67.675 | 67.573 | 0.99792 | $-0.06413$ | 0.00592 | 18.84 |

Figure 0.8: Unfiltered initial data recoded by the radar.

The unfiltered data initially recorded by the radar is seen in Figure 0.8. From columns left to right; the unaltered time step, radial velocity, x position of the ball, y position of the ball, z position of the ball. The next seven columns were not defined by TrackMan and were not used in the work. The last three columns are the measured direction cosines of the radial velocity vector; alpha, beta, and gamma. The last column represents the range of the ball measured from the center of the radar.

|  | 134 | V135 | VIW | X36 | Y37 | 238 | vx | w | V2 | ax | ay | 32 | d | cd | spin | SpinRatioRelativeToWind | Segind | type39 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 590 | 0 | 0-21.052 | 21.438 | 16.599 | 1.231 | 0.758 | -21.194 | 3.102 | -0.89 | 2.881 | -9.773 | 0.615 | 0.0826 | 0.2747 | 733.614 | 0.1738022 |  | 0 Measured |
| 591 | 0.01 | $1-21.09$ | 21.394 | 16.388 | 1.262 | 0.749 | -21.164 | 3.004 | -0.896 | 2.966 | -9,798 | 0.597 | 0.0795 | 0.275 | 733.7207 | 0.1741805 |  | 0 Measured |
| 592 | 0.02 | 2-21.09 | 21.352 | 16.176 | 1.291 | 0.74 | -21.134 | 2.906 | 0.902 | 2.951 | -9.823 | 0.578 | 0.0764 | 0.2752 | 733.8271 | 0.1745557 |  | 0 Measured |
| 593 | 0.03 | - 21 | 21.309 | 15.965 | 1.32 | 0.731 | 21.104 | 2807 | 0.908 | 2.936 | -9.848 | 55 | 0.0734 | 0.2754 | 733.9335 | 0.174927 |  | 0 Measured |
| 594 | 0.04 | 4-21.081 | 21.267 | 15.754 | 1.347 | 0.722 | 21.074 | 2.70 | 0.913 | 2.922 | -9.872 | 0.541 | 0.0705 | 0.2756 | 734.0399 | 0.1752881 |  | 0 Measured |
| 595 | 0.05 | 5-21.063 | 21.226 | 15.543 | 1.374 | 0.713 | -21,045 | 2.61 | -0.918 | 2.907 | -9.897 | 0.522 | 0.0677 | 0.2758 | 734,1462 | 0.175624 |  | 0 Measured |
| 596 | 0.06 | -21 | 21.186 | 15.333 | 1.4 | 0.703 | 21.016 | 2.5 | 0.924 | 2892 | -9.922 | -0.504 | 0.0649 | 0.2759 | 734.2526 | 0.176023 |  | Measured |
| 597 | 0.07 | 7-21.00 | 21.146 | 15.123 | 1.424 | 0.694 | -20.987 | 2.412 | -0.929 | 2.878 | 8-9.945 | 0.486 | 0.0623 | 0.2759 | 734.3589 | 0.1763818 |  | 0 Measured |
| 598 | 0.08 | - 20. | 21.106 | 14.91 | 1.48 | 0.685 | 20.559 | 2.31 | 0.933 | 2.863 | -9.969 | -0.468 | 0.0598 | 0.2 | 734,4653 | 0.1767668 |  | 0 Measured |
| 599 | 0.09 | - 20.937 | 21.068 | 14.704 | 1.47 | 0.675 | -20.93 | 2.212 | 0.938 | 2.849 | -9,993 | -0,45 | 0.0575 | 0.276 | 734.5717 | 0.177088 |  | 0 Measured |
| 600 | 0.1 | -20.90 | 21.029 | 14.49 | 1.492 | 0.666 | 20.902 | 2.11 | 0.942 | 2.834 | 34-10.017 | -0.432 | 0.055 | 0.2759 | 734.678 | 0.1774659 |  | 0 Measured |
| 601 | 0.11 | -20.87 | 20.95 | 14.286 | 1.513 | 0.657 | 20.873 | 2.012 | 0.946 | 2.82 | 32-10.041 | 14 | 0.05 | 0.2758 | 734.784 | 0.1777818 |  | 0 Measured |
| 602 | 0.12 | $12-20.84$ | 20.954 | 14.077 | 1.532 | 0.647 | 20.845 | 1.911 | 0.951 | 2.806 | 006-10.065 | 0.396 | 0.0516 | 0.2757 | 734.8807 | 0.1781232 |  | 0 Measured |
| 603 | 0.13 | $3-20.81$ | 20.918 | 13.669 | 1.551 | 0.638 | 20.817 | 1.811 | 0.954 | 2.792 | -10.088 | 0.378 | 0.0502 | 0.2756 | 734.9971 | 0.1784609 |  | 0 Measured |
| 604 | 0.14 | 4-20.79 | 20.882 | 13.661 | 1.569 | 0.628 | -20.789 | 1.71 | -0.958 | 2.778 | 78-10.112 | -0.361 | 0.049 | 0.2754 | 735.1035 | 0.1787949 |  | 0 Measured |
| 605 | 0.15 | 5 -20.77 | 20.846 | 13.45 | 1.585 | 0.618 | 20.762 | 1.608 | -0.662 | 2.764 |  | 0.343 | 0.0481 | 0.2751 | 735.2099 | 0.1791251 |  | 0 Measured |
| 606 | 0.16 | 6-20.74 | 20.81 | 13.246 | 1.601 | 0.609 | $-20.734$ | 1.507 | -0.965 | 2.75 | 75-10.158 | 0.326 | 0.045 | 0.2749 | 735.3163 | 0.1794514 |  | 0 Measured |
| 607 | 0.17 | 7-20.71 | 20.77 | 13.039 | 1.615 | 0.599 | $-20.707$ | 1.405 | -0.968 | 2.736 | -366-10.181 | 0.309 | 0.0473 | 0.2745 | 735.4226 | 0.1797736 |  | 0 Measured |
| 608 | 0.18 | , 8-20.68 | 20.743 | 12.832 | 1.629 | 0.589 | $-20.679$ | 1.303 | -0.971 | 2.722 | 22-10.204 | 0.291 | 0.0475 | 0.2742 | 735.529 | 0.1800919 |  | 0 Measured |
| 609 | 0.19 | -20.661 | 20.71 | 12.625 | 1.641 | 0.58 | -20.652 | 1.201 | -0.974 | 2.708 | -10.227 | -0.274 | 0.048 | 0.2738 | 735.6354 | 0.1880059 |  | 0 Measured |
| 610 | 0.2 | $2-20.639$ | 20.678 | 12.419 | 1.653 |  | -20.625 | 1.099 | -0.977 | 2.695 | -055-10.249 | -0.257 | 0.0488 | 0.2733 | 735.7418 | 0.1807156 |  | 0 Messured |
| 611 | 0.21 | - -20.613 | 20.646 | 12.212 | 1.663 | 0.56 | -20.598 | 0.996 | -0.979 | 2.681 | - 10.272 |  | 0.0499 | 0.2728 | 735.8881 | 0.1810202 |  | 0 Measured |

Figure 0.9: Filtered data.

Figure 0.9 shows the data that has been filtered and smoothed by TrackMan. The data includes; the time step, radial velocity, magnitude of velocity, $x, y$, and $z$ positions, components of velocity, components of acceleration, lift coefficient, drag coefficient, spin rates, and if the data was actually measured by the radar or calculated. This data has had the time step altered to . 01 s. The positions, components of velocity, and components of acceleration have all been filtered.

