Swing Speed vs. Bat and Batter Mass

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A topic of high current interest concerns the connection between performance enhancing drugs (PEDs) and home run production. This topic was addressed in a recent article by Tobin.[1] In analyzing this problem, a key point is the connection between PEDs and bat speed, or more specifically the connection between muscle mass and bat speed. We address that topic in this brief note.

In his book,[2] Adair argues that a plausible model for the relationship between bat speed v and bat mass m can be derived by assuming that the batter puts a fixed amount of energy E into the kinetic energy of the bat plus some fraction of the batter's mass M. Accordingly he writes the formula

$$v = \sqrt{\frac{2E}{m + \epsilon^2 M}}\,, \tag{1}$$

where ϵ^2 represents the fraction of the batter's mass that shares the kinetic energy with the bat. He notes that observations show that, roughly speaking, the kinetic energy going into the bat and batter are about equal, thereby placing sensible bounds on the value of ϵ^2. In the book he proposes $\epsilon^2 = 1/81$. With this value and for a 162-lb batter swinging a 2-lb (32 oz) bat, the bat has half of the available kinetic energy. For a 200-lb batter swinging a 2-lb bat, the bat has only about 45% of the kinetic energy. Adair goes on to suggest that the energy E provided by the batter is proportional to the muscle mass M_m of the batter, where $M_m = fM$ and f is the fraction of total mass that is muscle. For an athlete, f is about 0.5, meaning that half of the body mass is muscle. Therefore a new relationship can be written that explicitly shows the dependence of v on both m and M:

$$v = k\sqrt{\frac{fM}{m + \epsilon^2 M}}\,, \tag{2}$$

where k is a normalizing constant having the dimensions of velocity.

There is an alternate way to characterize the dependence of bat speed on m and M given by the formula[3]

$$v = k\left(\frac{fM}{m}\right)^n\,, \tag{3}$$

where k is again a normalizing constant with units of velocity. The exponent n is unknown from any first principles but can be determined from experiments. The best experiment that I know of is the batting cage study of Crisco and Greenwald[4] in which high-speed motion capture cameras were used to track both the baseball and several points along the bat throughout the swing and subsequent collision. From their analysis, they are able to determine the speed of the bat at the point of impact just prior to the collision. The study utilized college and semiprofessional batters, with bat weights in the range 28-31 oz. A summary of their swing speed data is shown in Fig. 1, where the angular velocity of the bat is plotted versus the moment of inertia (MOI) of the bat, both about the knob.[7] The curve is a least-squares fit to the data using a modified version of Eq. 3,[8] with the result n=0.28±0.04. Similar results have been obtained in other similar experiments.[5,6]

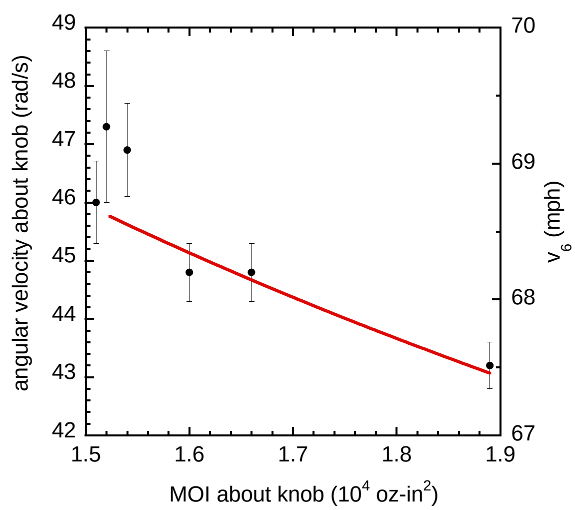

FIG. 1: Plot of the angular velocity of the bat about the knob just prior to impact versus the moment of inertia of the bat about the knob. Each point represents the angular velocity of a given bat, averaged over all impacts. For reference, the scale on the right is the equivalent bat speed for a 33-inch bat impacted 6 inches from the end of the barrel. The curve is a power-law fit of the form as in Eq. 3, with the best-fit exponent $n = -0.28 \pm 0.04$

It is very clear that Eq. 3 cannot be an accurate representation of the dependence of v on m for arbitrary m, since it clearly diverges for small m, unlike Adair's expression. However, I argue that Eqs. 2 and 3 are equivalent over some range of m. In the present context, equivalent means that, given the experimental value of n, there is some choice of ϵ^2 such that $(m/v)dv/dm$ is numerically the same for the two expressions. It is straightforward to derive the necessary expression:

$$\epsilon^2 = \frac{m}{M}\left(\frac{1}{2n} - 1\right). \tag{4}$$

If $m = \epsilon^2 M$, which is the case when half of the available energy goes into the bat, then $n = 0.25$. The extreme cases $m \ll \epsilon^2 M$ and $m \gg \epsilon^2 M$ correspond to $n = 0.5$ and 0, respectively. In the former case, none of the energy goes into the bat; in the latter, all the energy into the bat. Lacking any information about the batters used in the study, I simply assume M=190 lbs. With m=30 oz and $n = 0.28$, I find $\epsilon^2 = 1/129$, a value close to but somewhat smaller than the value 1/81 proposed by Adair. Adair's value would imply a smaller value, n=0.14, which does not appear to be consistent with the swing-speed data.

To address the issue of PEDs and bat speed, we now use Eq. 2 to estimate how an increase in muscle mass δM_m leads to a change in swing speed δv. The following expression is easily derived:

$$\frac{\delta v}{v} = \frac{1}{2}\frac{\delta M_m}{M_m}\left(1 - f\frac{\epsilon^2 M}{m + \epsilon^2 M}\right). \tag{5}$$

As a rough numerical estimate, suppose a batter chooses a bat such that $m \approx \epsilon^2 M$. Then with $f = 0.5$, the fractional change in v is 0.375 times the fractional change in M_m. For example, a 10% increase in muscle mass would lead to a 3.8% increase in bat speed. This result is not very sensitive to the precise value of ϵ^2. Consider the example where $M = 200$ lb and $m = 2$ lb (32 oz). Then a 10% increase in M_m leads to a 3.9% or 3.6% increase in bat speed for ϵ^2=1/129 or 1/81, respectively.

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[7] The experiments show that the instantaneous rotation axis of the bat just prior to the collision is very close to the knob.

[8] I ignore here the difference between the mass m and the MOI.