

A Model of the Bat's Vertical Sweetness Gradient

David G. Baldwin
HylaPark, Inc., 6548 Via Barona, Carlsbad, CA, USA 92009

A. Terry Bahill
*Systems and Industrial Engineering Dept., University of Arizona,
Tucson, AZ, USA 85721-0020*

ABSTRACT: Sweetness is defined as the probability of attaining a base hit. The sweetness of a specific bat/ball collision is found by determining the ball's vertical launch characteristics, the range and hang time or ground time of the batted ball and, finally, the probability of batter success. An example is presented to demonstrate feasibility and how the model might be used.

INTRODUCTION

Warren Spahn summed up his pitching strategy with a pithy aphorism - "Hitting is timing. Pitching is upsetting timing." The great coach Johnny Sain advised his pitchers to avoid showing batters consecutive pitches of the same speed. The most difficult part of the batter's task is judging the speed of the pitch and this must be done early in its flight (Adair, 2002; Bahill and Baldwin, 2004). Once the batter begins the swing, the bat's momentum precludes alteration of the timing or height of the swing.

The batter must determine pitch speed in order to estimate the ball's height at impact because the ball is in a free fall throughout its flight. Misjudgment of the ball's speed results in a vertical error in the swing (e.g., underestimation of pitch speed results in anticipation that the potential contact point will be lower than its actual location). The vertical offset between the trajectory line of the ball's center and the trajectory line of the transverse center of the bat is a major factor determining the vertical launch characteristics of the ball immediately after collision with the bat.

How is this offset related to the batter's probability of success? We describe a bat/ball collision model relating offset and several other collision parameters to the behavior of the batted ball, which, in turn, is associated with a success probability function for the batter. Batter *success* is defined as the attainment of a base hit and *sweetness* (a bat property) is the probability of success for a given bat/ball collision.

DESCRIPTION OF THE MODEL

Baseball and softball batters swing a narrow cylinder with the axis more or less parallel with the ground. Thus, the transverse curvature of the bat's face (hitting

surface) is a vertical curvature. In combination with the vertical offset of the bat and ball trajectories, this vertical curvature strongly influences the ball's vertical launch velocity, angle, and spin rate. These launch characteristics can be included in a vector describing a specific point on the bat's face; a vector field can specify the launch characteristics of all the points on the face. Each vector determines the batted ball's behavior – the distance it travels in the air until it first strikes the ground (*range*), how long it stays in the air (*hang time*), and, for ground balls, the time taken for the ball to reach the positional arc of infielders (*ground time*).

The set of success probabilities associated with a specific vertical arc on the bat's face is called the *vertical sweetness gradient* of that arc. The face's vector field represents sweetness gradients in both the longitudinal (horizontal) and transverse (vertical) dimensions of the bat. However, we restrict our current discussion to vertical collision considerations and the radial placement of the ball in play in fair territory.

Figure 1 illustrates the initial vertical configuration of the bat and ball at the instant of collision. The initial parameters of the collision are:

- (1) Initial velocity vector of bat's contact point ($\mathbf{v}_{\text{bat}, 0}$).
- (2) Initial normal component of the bat's velocity vector ($\mathbf{v}_{\text{bat}, 0, n}$).
- (3) Initial tangential component of the bat's velocity vector ($\mathbf{v}_{\text{bat}, 0, t}$).
- (4) Initial velocity vector of ball ($\mathbf{v}_{\text{ball}, 0}$).
- (5) Initial normal component of the ball's velocity vector ($\mathbf{v}_{\text{ball}, 0, n}$).
- (6) Initial tangential component of the ball's velocity vector ($\mathbf{v}_{\text{ball}, 0, t}$).
- (7) Offset distance (D) from the ball's center perpendicular to the trajectory plane of the bat's transverse center
- (8) Vertical angle (θ) between the line connecting ball and bat centers (line of centers) and the horizontal plane ($z = 0$).
- (9) Vertical angle (γ) between the horizontal plane and the ball's trajectory plane..
- (10) Vertical angle (ψ) between the horizontal plane and the bat's trajectory plane.
- (11) Mass of bat (m_{bat}).
- (12) Mass of ball (m_{ball}).
- (13) Radius of bat at contact point (r_{bat}).
- (14) Radius of ball (r_{ball}).
- (15) Coefficient of restitution (CoR) of the bat/ball impact.
- (16) Coefficient of friction (μ) of the bat/ball impact.
- (17) Angular velocity ($\omega_{\text{ball}, 0}$) of the pitch.

The characteristics m_{ball} , r_{ball} , CoR, and μ will be considered constants. The values of CoR and μ must be derived empirically.

The resultant vectors and angles used to calculate launch velocity and angle are:

- (1) Resultant velocity vector of bat's contact point ($\mathbf{v}_{\text{bat}, R}$).
- (2) Resultant normal component of the bat's velocity vector ($\mathbf{v}_{\text{bat}, R, n}$).
- (3) Resultant tangential component of the bat's velocity vector ($\mathbf{v}_{\text{bat}, R, t}$).
- (4) Resultant velocity vector of ball ($\mathbf{v}_{\text{ball}, R}$). This is called the "launch velocity".
- (5) Resultant normal component of the ball's velocity vector ($\mathbf{v}_{\text{ball}, R, n}$).
- (6) Resultant tangential component of the ball's velocity vector ($\mathbf{v}_{\text{ball}, R, t}$).

- (7) Vertical angle (φ) between the line of centers and $\mathbf{v}_{\text{ball, R}}$.
- (8) Vertical angle (λ) between the horizontal plane ($z = 0$) and $\mathbf{v}_{\text{ball, R}}$. This is called the “launch angle”.

We also calculate the vertical launch spin rate ($\omega_{\text{ball, R}}$) and specify air density (ρ).

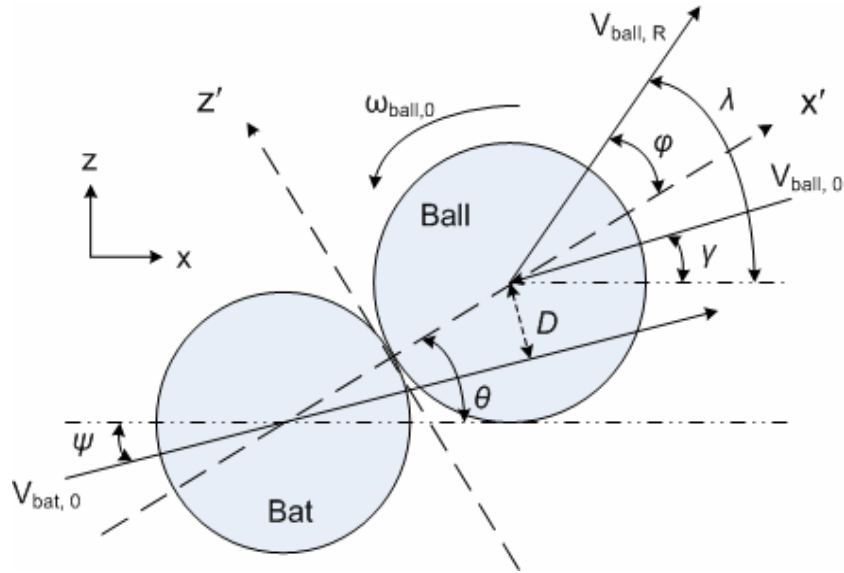


Fig. 1. Initial vertical configuration of the bat/ball collision.

The model is set in the x - z plane of a coordinate system with origin at the contact point, three feet in front of the vertex of home plate and at a height of three feet. The positive z axis points upward, positive x axis points toward the pitcher, and positive y axis points out of the plane (Sawicki, Hubbard, and Stronge, 2003). In Fig. 1, the x - z plane is reoriented so the x' axis lies along the bat/ball line of centers and the z' axis is tangential to the ball/bat contact point. Angular velocity is positive for pitch topspin and for batted ball backspin. D is positive if the bat undercuts the ball. The angle of descent (γ) of an average fastball is about ten degrees (Adair, 2002; Watts and Bahill, 2000) and batters generally uppercut the ball (ψ) at ten degrees or more.

BALL'S LAUNCH VELOCITY, ANGLE, AND SPIN RATE

The ball's launch parameters are calculated by decomposing the initial velocities of the bat and ball into their normal and tangential components at the point of contact. These velocities are used with the principles of conservation of momentum and conservation of energy to yield resultant normal and tangential velocities for the ball, which are then used to calculate the launch velocity of the ball and the angles φ and λ . The batted ball angular velocity is calculated from the normal and tangential linear velocities of the ball and bat and the ball's initial angular velocity.

The collision of two partially elastic bodies with friction is described by numerous authors (e.g., Goldsmith, 1960). The first step in building the model is to calculate θ and, from this, the normal and tangential components of the initial velocity vectors. The vertical angle of the line of centers, $\theta = \psi + \sin^{-1}(D / (r_{\text{bat}} + r_{\text{ball}}))$. The initial velocity components of the ball are $\mathbf{v}_{\text{ball}, 0, n} = \mathbf{v}_{\text{ball}, 0} \cos \theta$ and $\mathbf{v}_{\text{ball}, 0, t} = \mathbf{v}_{\text{ball}, 0} \sin \theta$. The initial velocity components of the bat are $\mathbf{v}_{\text{bat}, 0, n} = \mathbf{v}_{\text{bat}, 0} \cos \theta$ and $\mathbf{v}_{\text{bat}, 0, t} = \mathbf{v}_{\text{bat}, 0} \sin \theta$.

The resultant normal velocity of the ball (Bahill and Watts, 2000) is

$$\mathbf{v}_{\text{ball}, R, n} = \mathbf{v}_{\text{ball}, 0, n} - (1 + \text{CoR}) [(m_{\text{bat}} \mathbf{v}_{\text{ball}, 0, n} - m_{\text{bat}} \mathbf{v}_{\text{bat}, 0, n}) / (m_{\text{ball}} + m_{\text{bat}})]$$

Calculation of relative tangential velocity, resultant angular velocity of the ball, and final launch angle, λ , is described by Sawicki, et al. (2003) and by Watts and Baroni (1989). In these models, friction acts in the direction opposite to the slip of the ball. If friction is large enough, it halts the relative tangential velocity (the combined velocities of bat and ball surfaces relative to the contact point). When this occurs, slippage ceases, the ball sticks to the bat, and the ball begins to roll, contributing to the launch angular velocity. These models account for bat recoil and assume conservation of linear and angular momentum for tangential ball and bat motions. Both models ignore deformation of the ball during collision (they assume it remains a perfect sphere).

RANGE, HANG TIME, AND GROUND TIME

The launch velocity, angle, and spin rate are the input data into equations (Watts and Bahill, 2000) to calculate the batted ball's range and hang time. The vertical distance traveled by the batted ball (without regard to lift or drag) is $z = \mathbf{v}_{z0} t - 0.5 g t^2$, where \mathbf{v}_{z0} is the vertical velocity of the ball, t is the hang time, and g is the acceleration rate of gravity at the surface of the earth (32.17 ft/sec²). The horizontal distance traveled (again ignoring lift and drag) is $x = \mathbf{v}_{x0} t$, where \mathbf{v}_{x0} is the horizontal velocity component. The vertical rotation of the ball creates a Magnus force acting vertically perpendicular to the trajectory. This force tends to lift the ball (if backspin) or depress the ball (if topspin). It is calculated as $F_{\text{lift}} = 0.5 \rho \pi \omega r_{\text{ball}}^3 \mathbf{v}_{\text{ball}, R}$, where ρ is the air density. Friction of the ball passing through the air is a drag force acting directly counter to the trajectory. This force is calculated as $F_{\text{drag}} = 0.25 \rho \pi r_{\text{ball}}^2 \mathbf{v}_{\text{ball}, R}^2$.

Ground time is not calculated for this paper. It will be modeled by using the launch angle to find the angle of incidence on the first bounce. The incidental horizontal and vertical velocity components and launch spin rate then will be used to generate the bounce velocity, angle, and spin rate. An aerodynamics model will be used to find the flight characteristics between bounces, including the incidental angle on the subsequent bounce. Note that here CoR and μ will have values different from those for the bat/ball collision. As ω usually represents topspin on ground balls, angular velocity contributes to linear horizontal velocity and vice versa. If μ is large enough to overcome the combined angular and horizontal velocities, slippage stops and rolling begins.

In our model, drag and lift coefficients are independent of Reynold's number. Therefore, drag and lift are only approximations.

BATTING SUCCESS PROBABILITY FUNCTION

The characteristics of a batted ball can be associated with probability of success through a step function based on the potential of defensive players to prevent a base hit. Four kinds of batted ball behavior are represented in the model:

- (1) Flies (range > 130 ft, hang time > 1.0 sec).
- (2) Pop-ups (range <= 130 ft, hang time > 1.0 sec).
- (3) Line drives (range >= 115 ft, hang time <= 1.0 sec).
- (4) Grounders (range < 115 ft, hang time <= 1.0 sec).

Either an infielder or outfielder might catch a fly ball, depending on range and hang time. All pop-ups are caught by an infielder, pitcher, or catcher so the probability of success is zero. Only infielders catch line drives and grounders.

Each batted ball is associated with defensive coverage formulated as a function of time. A defensive player prevents a base hit if he or she can reach the ball during hang time or ground time. To determine coverage, we positioned outfielders and infielders on two arcs – the *outfield arc* with a radius of 300 ft and the *infield arc* with a radius of 115 ft. The outfield arc is divided into thirds and the infield arc into quarters, with a player positioned at the center of each arc segment. For example, the outfield arc has a length of 471.3 ft (300×1.571) so it is divided into three segments each of which is 157.1 ft long. The right fielder, then, is positioned 300 ft from home, 78.55 ft from the right field foul line, and 157.1 ft from the center fielder. The batted ball's range yields a *range arc* with length equal to 1.571 times range (angle in radians times radius).

On fly balls, each player's position is the center of an ellipse representing defensive coverage by the player (a fly ball is illustrated in Fig. 2). Hang time determines ellipse dimensions for a specific batted ball. Probability of a base hit is the proportion of the range arc that is not overlapped by ellipses.

If a line drive or grounder passes the infield arc without encountering an infielder, it is considered a base hit. Therefore, only infielders' lateral movements provide coverage. Success is the proportion of range arc (on line drives) or infield arc (on grounders) not covered by infielders. In this model, batters do not beat out infield hits and pitchers do not catch line drives or grounders.

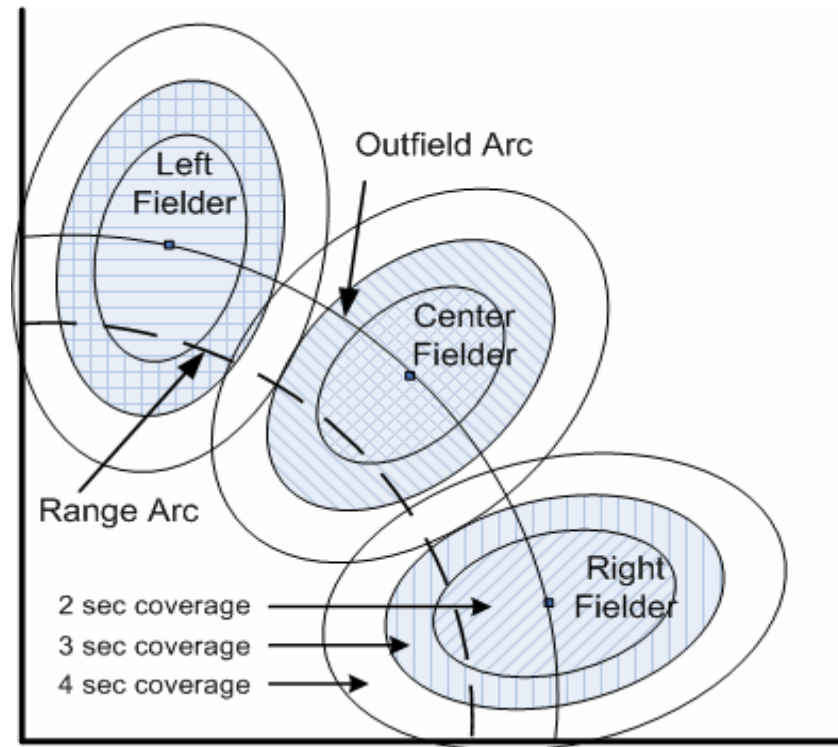


Fig. 2. Range arc, outfield arc, and defensive coverage of the outfielders.

The model assumes the speed of outfielders and infielders is 23.0 ft/sec. Outfielders' reaction loses are 8 ft (0.35 sec) forward, 12 ft (0.52 sec) side, and 15 ft (0.65 sec) backward. Infielders' loses are 12 ft side and 15 ft backward. These values were selected as "reasonable" and are not based on empirical data.

AN EXAMPLE OF VARYING OFFSETS

For an example of collision evaluation, the model is solved at offset increments of 0.25 inches upward from zero offset. Pitch backspin is -1800 rpm and speed is 85 mph. Contact occurs at the bat's point of maximum horizontal sweetness and the speed of the bat's contact point is 60 mph (average for the San Francisco Giants measured by Bahill and Karnavas, 1989). These speeds produce a CoR of 0.54. Coefficient of friction, μ , is estimated to be 0.5 (Sawicki, et al, 2003). The angles γ and ψ of ball and bat are both ten degrees. Other test values are $r_{bat} = 1.375$ in, $r_{ball} = 1.452$ in, $m_{bat} = 32.0$ oz (effective bat mass = 20.0 oz), $m_{ball} = 5.125$ oz, and ρ at standard sea level conditions. Ranges and hang times were found using a Pascal aerodynamics program. Spreadsheets developed by Alan Nathan were used to find launch values.

RESULTS AND DISCUSSION

Test results are given in Table 1. As hang time increases, probability of success decreases rapidly. Pop-ups are produced by offsets greater than 1.50 inches. These are assigned a success probability of zero. Note the model assumes no outfield barriers. In most major league stadiums, flies hit farther than 330 ft have a chance of clearing a wall (average distance = 329.5 ft down lines, 406.3 ft in center field). Thus, the model underestimates success for any range with a chance to be a home run.

The example shows how the model might be used to analyze collision parameters (e.g., offsets, bat velocity) or bat properties (e.g., bat radius). Relating initial conditions to sweetness provides a valuable criterion for these analyses.

Table 1. Launch values and probability of batter's success for non-negative offsets.

Offset (in)	Launch Velocity (mph)	Launch Angle (deg)	Backspin Rate (rpm)	Range (ft)	Hang Time (sec)	Prob. of Success
>1.50	<81.60	>57.59	>4923.67	<129.36	<6.395	0.00
1.50	81.60	57.59	4923.67	129.36	6.395	0.00
1.25	85.03	48.17	3991.48	236.71	6.652	0.00
1.00	87.76	39.42	3059.29	306.29	6.143	0.00
0.75	89.88	31.12	2127.10	321.27	5.004	0.00
0.50	91.41	23.12	1194.90	285.62	3.585	0.09
0.25	92.39	15.31	262.71	213.05	2.204	1.00
0.00	92.83	7.59	-669.48	122.30	1.070	0.63

REFERENCES

- Adair, R. K. (2002) *The Physics of Baseball*, 3rd edn. HarperCollins, New York.
- Bahill, A. T. and Baldwin, D. G. (2004) The vertical illusions of batters. *Baseball Research Journal*, **32**:26-30.
- Bahill, A. T. and Baldwin, D. G. (2003) The rising fastball and other perceptual illusions of batters. In: *Biomedical Engineering Principles in Sports*. (Ed. G. Hung). Kluwer Academic, New York.
- Bahill, A. T. and Karnavas, W. J. (1989) Determining ideal baseball bat weights using muscle force-velocity relationships. *Biological Cybernetics*. **62**:89-97.
- Goldsmith, W. (1960) *Impact: The Theory and Physical Behavior of Colliding Solids*. Edward Arnold Publ. Ltd., London.
- Sawicki, G. S., Hubbard, M. and Stronge, W. J. (2003) How to hit home runs: Optimum baseball bat swing parameters for maximum range trajectories. *Am. J. Phys.* **71**:1152-1162.
- Watts, R. G. and Bahill, A. T. (2000) *Keep Your Eye on the Ball: Curve Balls, Knuckleballs and Fallacies of Baseball*. W. H. Freeman, New York.
- Watts, R. G. and Baroni, S. (1989) Baseball-bat collisions and the resulting trajectories of spinning balls. *Am. J. Phys.* **57**:40-45.