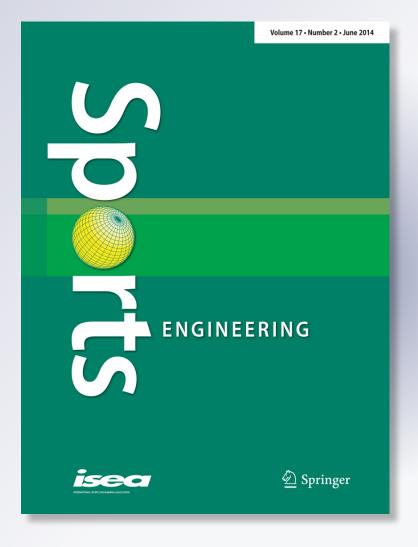
Field and laboratory measurements of softball player swing speed and bat performance

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INVITED PAPER

Field and laboratory measurements of softball player swing speed and bat performance

Lloyd Smith · Jeff Kensrud

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Abstract The swing speed of the bat is one of the most important factors affecting the hit-ball speed. Most field studies tend to focus on measuring ball speed, which is easier to measure and quantify than bat speed. For this reason, relatively little data exist describing bat motion in field conditions. The following describes a relatively large swing speed field study involving bats of the same model with nearly constant weight and varying inertia. The study was conducted using right-handed batters on a regulation outdoor field with a live pitcher. Swing speed was measured by tracking markers on the bat with two high-speed video cameras so that the bat markers could be traced in three-dimensional space. The ball motion was tracked using the same high-speed video cameras and a threedimensional Doppler radar system. Bat swing speed was observed to be proportional to the batter skill level and the normalised swing speed increased with decreasing bat inertia. The bat centre of rotation during impact was close to the knob of the bat. The bats were tested under controlled laboratory conditions using a standardised performance test. The field and laboratory results showed good agreement including the hit-ball speed and the subtle effect of bat inertia on the maximum performance location. The vibrational response of the bats was considered using modal analysis. The maximum performance location was correlated with the node of the first vibrational mode.

Keywords Softball · Swingspeed · Bat performance · Sweetspot · Modal analysis

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1 Introduction

Amateur baseball and softball regulating bodies have allowed hollow bat designs for over 40 years. In contrast to solid wood bats, the performance of hollow bats has increased over time as materials have improved and designs have evolved. To maintain a competitive offensive balance in play, regulating associations have sought to limit hollow bat performance. Methodologies to measure bat performance in a laboratory setting are well established and routinely conducted to a high degree of accuracy [1]. To characterise the hitting performance of a bat in play from laboratory measures, it is necessary to quantify the factors affecting bat swing speed. For example, if two bats have the same performance at the same swing speed, the bat that can be swung faster will perform better in the field. The relationship between the hit-ball speed, v_h , and bat swing speed, v_s , is

$$v_{\rm h} = e_{\rm a} v_{\rm p} + (1 + e_{\rm a}) v_{\rm s} \tag{1}$$

where e_a is a property of the bat, known as the collision efficiency, and v_p is the ball pitch speed [2]. While Eq. (1) is relatively simple and agrees well with field testing [3], its components depend on many factors, some of which are not well understood. The collision efficiency depends on the bat's coefficient of restitution, e, the bat's mass moment of inertia, I, and the impact location, q (measured from the bat's pivot point). Thus, it is possible to have a bat with high e produce a lower v_h than a bat with low e. The bat's coefficient of restitution can be found from field measurements, but is more commonly measured in a laboratory setting [4]. The coefficient of restitution of a bat is not constant, but depends on the properties of the ball (which in turn are sensitive to temperature and humidity) the impact speed [5] and the impact location. The bat swing



Table 1 Wass and Deriviniance properties of the field study be	Table 1 Mass and perfor	mance properties	of the	field	study	bats
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Inertia (kg m ²)	Length (mm)	Weight (kg)	Balance point (mm)	COP (mm)	e	e_{a}
0.1263	863	0.797	443	691	0.493	0.087
0.1494	863	0.826	475	712	0.497	0.132
0.1666	864	0.824	514	712	0.492	0.135
0.1857	861	0.824	545	726	0.495	0.173
0.2030	863	0.825	573	737	0.486	0.188

speed (a focus of this work) depends on the ability of the batter, the impact location on the bat, and the bat's instantaneous centre of rotation during impact.

Many players believe that hollow bats outperform wood bats in the field because hollow bats are lighter and can be swung faster. Unfortunately, it is difficult to predict how swing speed is affected by the inertial properties of the bat. Moreover, few systematic studies of bat speed may be found in the literature, reflecting the fact that bat speed is difficult to measure and can depend on conditions not easily controlled or quantified. Despite these difficulties, previous studies [6-9] have shown a qualitative dependence of bat speed on inertial properties. Unfortunately, because of the difficulty in doing such measurements and due to the selection of bats used in these studies, the dependencies are usually not quantified nor are the effects of mass and inertia separately determined. The limited availability of quantitative results provided the motivation for the current study, the results of which will improve laboratory bat performance measures.

2 Field study protocol

The field study considered the effects that bat inertia has on the swing speed of a softball bat. Five aluminium bats of the same shell, each 0.86 m (34 in.) in length, were used. The bats had nearly the same weight, but varied in inertia in five uniform increments, where inertia was referenced to a point 0.15 m (6 in.) from the knob end of the bat [10] as described in Table 1. Inertia was changed by moving weight between the proximal and distal ends of the bat.

The study was conducted on an open field during daylight hours in Oklahoma City, Oklahoma, USA. To accommodate the video camera orientation, only right-handed batters were used. The study consisted of 29 batters with an average age of 33.3 years (8.0), an average height of 1.8 m (0.07) and an average weight of 103 kg (17). Batter skill was ranked in seven levels and varied from expert (1) to recreational (7). All batters swung each of the five bats in random order. To reduce fatigue effects, the

3 Speed measurements

The batter's swing speed was measured using two high-speed video cameras ($1,200 \times 800$ pixels at 1,000 fps) as depicted in Fig. 1. The cameras were approximately 45° from each other, 4.6 m above the ground and 4.6 m from home plate. Two 1.2 m \times 1.2 m panels, each with an array

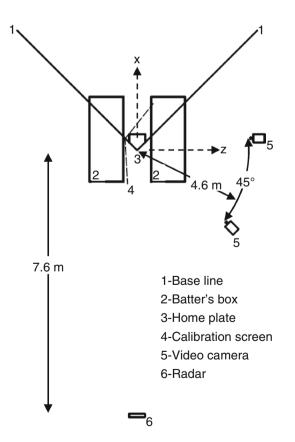
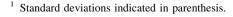


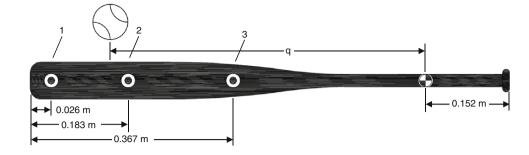
Fig. 1 Schematic of field study equipment layout





batters worked in pairs, alternating after swinging each bat ten times. Each batter was allowed a few practice swings before hitting with a new bat. All pitches were delivered using a live pitcher (not a machine) following the regulation arc height, where an average pitch speed of 11 m/s (0.6) was achieved. All bat–ball impacts used regulation 52/300 ASA slow-pitch softballs.

Fig. 2 Schematic of bat with tracking dots (numbered 1, 2, 3), laboratory test pivot location, and ball impact location



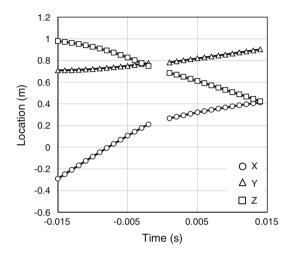


Fig. 3 Coordinates of marker one on a bat for a representative swing (*points*), where the location during impact (t = 0) was not included in the curve fits (*lines*). The coordinates are relative to the right-handed coordinate system defined in Fig. 1

of 36 equally spaced markers, were used to calibrate the camera locations. The sound of the bat–ball impact was used to trigger the cameras. Video was saved from each camera for each swing using 15 frames prior to impact and 15 frames after impact. White tracking markers were placed at three locations on each bat as shown in Fig. 2.

The ball and bat markers were tracked in three-dimensional space using commercial software (ProAnalyst 3D Professional). The cameras were calibrated using the 1.2 m square calibration panels, from which a mean calibration error of 6 mm was reported. The coordinates for the ball and each marker on the bat were fit to second-order polynomial equations, as shown in Fig. 3 for marker one of a representative swing. The *x* coordinate increased with time as the bat primarily moved towards the pitcher. The slope of the *x* coordinate decreased after impact, due to the momentum transfer to the ball and associated slower bat speed. The *y* coordinate (elevation) was nearly constant, showing the swing was relatively level. The *z* coordinate started positive and decreased, characteristic of a hit towards left field.

Because impact causes a motion discontinuity, motion prior to impact was fit separately from motion after impact,

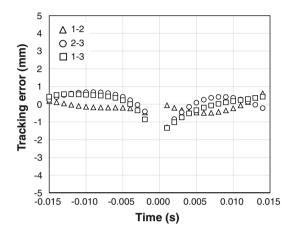


Fig. 4 Measured marker distance minus the marker distance from the video analysis for the swing presented in Fig. 3

resulting in eight sets (x, y, z) of polynomial equations for each swing. The accuracy of the video tracking was checked by evaluating the distance between the bat markers, as shown in Fig. 4 for the swing in Fig. 3. Here, the measured marker spacing was subtracted from the marker distance as determined from the video for three cases: 1-2, 2-3, and 1-3. In this example, the error for 1-2 was less than the other cases, suggesting that the tracking of marker 3 had more error than markers 1 and 2. Due to the large dataset and time required for manual tracking, only the swings where error in the distance between markers exceeded 6 mm were corrected through manual tracking.

Because of the varied light conditions that inevitably occur in an outdoor field study, some swings could not be accurately tracked and were not used. A threshold was set using the standard deviation of the difference between the video and measured bat marker spacing for each swing. Swings where the standard deviation was more than 6 mm were not used. The ball was tracked using its spherical shape, so that marker spacing could not be used to evaluate its tracking accuracy. Since the fitted ball trajectories excluded impact, they should be smooth and follow a second-order polynomial. Ball trajectories where the root mean square deviation between the video locations and polynomial fit was greater than 2.5 mm were not used. Bat



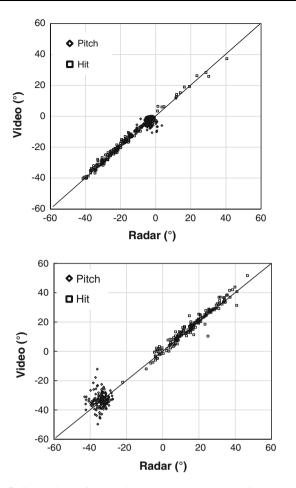


Fig. 5 Comparison of the horizontal (top) and vertical (bottom) ball trajectory angles between the video and radar

and ball speeds were obtained by differentiating the polynomial equations with respect to time. In all, data from 1487 swings were used.

In addition to the video tracking, a Doppler radar system (Trackman, DN), as indicated in Fig. 1, was used to track the ball motion. The unit was placed 7.6 m behind home plate and was able to record ball speed and trajectory. Approximately, 300 hits were captured using both the video and radar systems. A comparison between the video and radar system ball angles in the horizontal and vertical planes is presented in Fig. 5. In nearly all instances, the two independent measures of ball trajectory agreed to within 5°. The magnitude of the pitched and hit-ball speeds is compared between the video and radar systems in Fig. 6. The agreement is again favourable, and on average to within 2 m/s.

4 Swing speed

To find the bat's instantaneous centre of rotation, the velocity vector of each marker was found at t = 0 from the

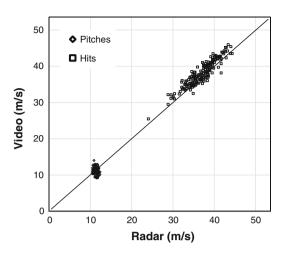


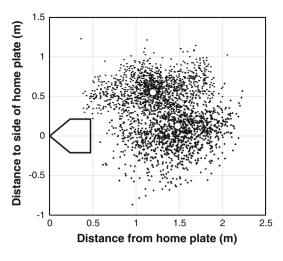
Fig. 6 Comparison of the magnitude of the ball speed between the video and radar tracking systems

curve fits prior to impact with the ball. The magnitude and direction of the redundant marker velocity vectors were extrapolated to the common instantaneous centre of rotation. The location of the knob, impact location, and instantaneous bat centre of rotation at t = 0 are presented in Fig. 7.2 The impact location occurs, on average, 1.4 m in front of the plate, while the batter typically stands 0.2 m in front of the plate. The batter's stride was apparently responsible for this motion towards the pitching mound, which may be the result of low pitched balls (in slow-pitch softball, the batter has sufficient time to adjust to the trajectory of the ball after it is pitched). The average impact location was 0.25 m further from home plate than the average knob location. This is consistent with the left field ball placement, typical of right-handed batters. The instantaneous centre of rotation occurred just off of the knob and was close to the batter's wrist (43 mm axially from the knob and 43 mm towards the batter). The errant points showing centres of rotation between 0.2 and 1 m are due to unusual batter motion, not tracking noise, and were included in the average.

The rotational bat swing speed was found by dividing the linear speed of marker 1 by the distance to the instantaneous centre of rotation. The average swing speed is shown for each batter skill level in Fig. 8. As observed in the figure, higher swing speeds were generally observed for batters with higher skill level. Some variation in the correlation between swing speed and player skill level was observed. As swing speed is only one aspect of a batter's ability, the discrepancy could be related to other factors contributing to player ability and the accuracy of the ranking system itself. The range in average batter swing



² By convention, distance from home plate is measured from its apex, as expressed in Fig. 7.



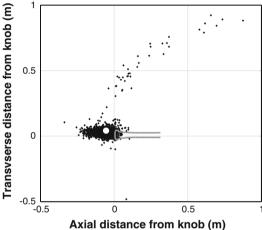


Fig. 7 Locations at impact. *Top* knob location (–) and impact location (+) relative to home plate (*black lines*). *Bottom* instantaneous centre of rotation relative to the knob (*grey lines*). *White solid circles* are averages

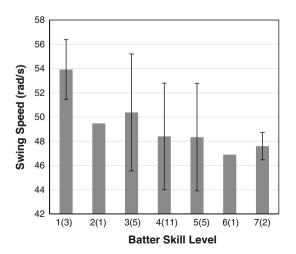


Fig. 8 Batter swing speed as a function of batter skill level. Group size indicated in *parenthesis*; *error bars* are one standard deviation for each skill level

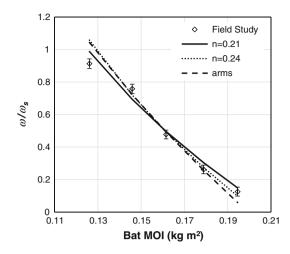


Fig. 9 Swing speed power as a function of bat inertia

speeds is relatively large, and far exceeds the range in performance of differing bat models. Since swing speed is the largest factor contributing to the hit-ball speed (Eq. 1), the contribution of player ability should not be neglected when considering bat performance.

Laboratory tests of bat performance can accurately determine energy dissipation from impact, but rely on empirical measures of batter swing speed to correlate with field hit-ball speeds. Of particular interest is the effect of the bat's inertial properties on swing speed, ω . Swing speed has been described using [11, 12]

$$\frac{\omega}{\omega_{\rm s}} = \left(\frac{I_{\rm s}}{I}\right)^n \tag{2}$$

where I and I_s are inertias of the test and standard bats, respectively, ω_s is the batter's average swing speed and the power n describes the dependence of swing speed on bat inertia. The normalised rotational swing speed, ω/ω_s , is thus unity when $I = I_s$. Clearly Eq. (2) is a simplification of the inertial effect contributing to batter swing speed. Since Eq. (2) does not include the batter's own inertia, it incorrectly predicts an infinite swing speed as bat inertia approaches zero. It is nevertheless a convenient expression to describe swing speed over a limited range in bat inertia. Powers of n = 0, $\frac{1}{4}$, and $\frac{1}{2}$, for instance, describe constant swing speed, constant power, and constant energy, respectively.

To determine the dependence of bat inertia on swing speed, each swing speed was normalised using the average from the respective batter, as described in Eq. (2). The normalised swing speeds were averaged for each bat as shown in Fig. 9. The lightest bat had considerably less inertia than most batters prefer. It is perhaps for this reason that the swing speed for this bat departs from the trend apparent from the remaining bats. When Eq. (2) is fit to the average swing speed for each bat, n = 0.21, while when the swing speed of the lightest bat is removed, n = 0.24.



If we assume that a player delivers constant energy when swinging bats of different inertia, the normalised swing speed may be expressed as

$$\frac{\omega}{\omega_{\rm s}} = \left(\frac{I_{\rm s} + I_{\rm p}}{I + I_{\rm p}}\right)^{1/2} \tag{3}$$

where I_p is the inertia of the portion of the batter contributing to the swing speed and may be found from [3, 13]

$$I_{\mathbf{p}} = I\left(\frac{1}{2n} - 1\right) \tag{4}$$

Thus, for 0.21 < n < 0.24, $I \approx I_p$, showing that the portion of the batter contributing to swing speed is roughly equal to the inertia of the bat. Equation (4) is included in Fig. 9 and agrees closely to Eq. (2) for n = 0.24.

5 Bat performance

When collecting data for measuring swing speed, nearly every swing can be used, as contact with the ball is immaterial. In comparison, field measures of hit-ball speed have considerable variation given the high sensitivity of the ball impact location on bat performance and the difficulty that batters experience in achieving optimal impact with the ball. It is common practice, therefore, to take a fraction of the highest performing hits, since those hits are most likely to have optimal impact conditions and describe the maximum bat performance. The following results were taken from the top 10 % of the hits for each bat, comprising approximately 30 hits per bat.

Figure 10 contains the average hit-ball speed of each bat used in the field study as a function of bat inertia. While the data show a general trend of increasing performance with bat inertia, the effect is small, and thus influenced by scatter inherent with human subjects.

The field study bats were also tested under controlled laboratory conditions. The laboratory test is explained in detail elsewhere [4], and is described here briefly. The test involved impacting an initially stationary bat with a ball projected at 49 m/s. The bat was free to pivot after impact. The ball speed was the sum of the pitch and bat speeds so that the laboratory test described play conditions. The ball was projected to achieve normal rebounds to obtain the maximum bat performance. The bat was impacted in 12 mm increments along its length until the maximum performance was obtained. Ball speed after and before impact was measured, the ratio of which is the collision efficiency [2]. The maximum hit-ball speed for each bat was found from the laboratory test using Eqs. (1) and (2),

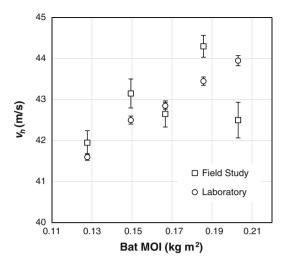


Fig. 10 Hit-ball speed as a function of bat inertia for impact under field and laboratory conditions (*error bars* represent one standard deviation of the mean for each case)

where $w_s = 53.4 \text{ rad/s}$, $I_s = 0.165 \text{ kg m}^2$, n = 0.24, and $v_p = 11.1 \text{ m/s}$.

In Fig. 10, the average field study performance exceeded the laboratory performance in some cases. This is expected since v_h is a strong function of v_s , and the field study v_s will have variation associated with human subjects. Both the field study and laboratory results show that the hit-ball speed increases with bat inertia as has been observed elsewhere [11, 12].

Bats of a similar design that differ only in inertia will have the same coefficient of restitution. The coefficient of restitution, e, and the collision efficiency, e_a, are related through

$$e_{\mathbf{a}} = \frac{e - r}{1 + r} \tag{5}$$

where, for a pivoted bat

$$r = \frac{mq^2}{I} \tag{6}$$

m is the ball mass and q is the distance from the impact to the pivot location [2]. Some batters prefer using low inertia bats, concluding that the increased swing speed will result in higher hit-ball speed. Decreased inertia also results in a lower collision efficiency, however (Eq. 5). Thus, it is not obvious if decreasing inertia will increase or decrease the hit-ball speed. Equation (1) may be used to consider the competing effects of inertia on the hit-ball speed. Figure 10 shows that the hit-ball speed increases with bat inertia. Thus, as bat inertia increases, the associated increase in the collision efficiency more than compensates for the decrease in swing speed to achieve higher hit-ball speed. Many batters may still prefer a light weight bat, however, since a



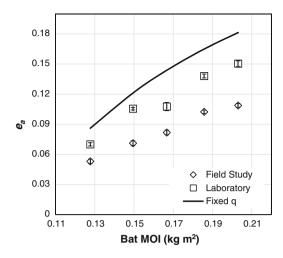


Fig. 11 Maximum collision efficiency as a function of bat inertia. Field study values are the average of the top 10 % for each bat, while the laboratory values are the average of three values at the maximum e_a location. *Error bars* represent one standard deviation of the mean

light bat improves the batter's ability to achieve contact with the ball.

The results of the field study may be used to obtain the collision efficiency of each impact through Eq. (1). The peak collision efficiency (average of the top 10 %) is presented in Fig. 11 as a function of bat inertia. Scatter in the results is due to the quality of the bat-ball impacts, where collinear impacts near the bat's maximum performance location produce higher collision efficiencies. Note that the field study results have less scatter in Fig. 10 than Fig. 11. This is expected, since the collision efficiency is not sensitive to variations in swing speed. The results of laboratory bat tests [4] are included in Fig. 11. The laboratory tests involve only collinear impacts, and thus describe the upper limit of performance observed in the field study. The dependence of the collision efficiency on inertia shows remarkable agreement between the field and laboratory results.

Equation (5) describes the effect of inertia on e_a for bats with the same e. This is shown in Fig. 11 under "Fixed q," where e and q were taken from the lightest bat. Equation (5) significantly overestimates the effect of inertia on e_a . The discrepancy between Eq. (5) and the experimental data is due to a dependence of the peak performance location on I, which is explored in the next section.

6 Sweet spot

Batters often refer to the sweet spot of a bat as the location that produces the least sensation to their hands and the highest hit-ball speed. While bat speed increases linearly with the distance from the pivot, the collision efficiency

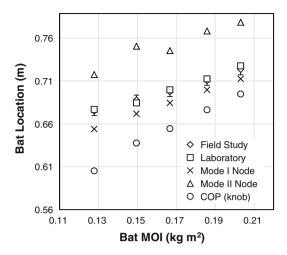


Fig. 12 Bat location, with respect to the knob, as a function of bat inertia. Field study values taken from the average impact location of the top 10 % hit-ball speeds for each bat. Laboratory values were taken from the location producing the highest hit-ball speed (impacted at 12 mm increments). *Error bars* represent one standard deviation of the mean

usually peaks 175 mm (7 in.) from the distal end of the bat. The peak hit-ball speed occurs just outside of the location of the maximum collision efficiency, where the linear effect of increased bat speed exceeds the non-linear decrease in collision efficiency. The average impact location (from the top 10 %, based on coefficient of restitution) for each bat is presented in Fig. 12. The peak e location from the laboratory tests is also included in Fig. 12 and agrees with the 50 mm change in impact location, over the five bats, found from the field study. Batters are apparently aware of this fact, where the average impact location for all hits of each bat showed a similar dependence on I as the top 10 % of hits.

Two explanations for the dependence of the sweet spot location on bat inertia have been proposed. The first concerns the bat's centre of percussion, which is a function of the bat's inertia and the pivot location. The bat centre of percussion is often found using ASTM F2398, which reports a location close to the bat's peak performance location and shows a similar dependence on inertia as *e* does. ASTM F2398 uses a pivot location that is 150 mm (6 in.) from the knob, however. When the centre of percussion is found for a pivot location at the knob, the centre of percussion lies 20–70 mm (0.7–2.7 in.) inside of the sweet spot, as observed here and elsewhere [2]. It is unlikely, therefore, that the COP is responsible for the dependence of the location of maximum *e* on *I*.

Another explanation for the dependence of the bat's sweet spot on bat inertia concerns the bat's vibrational response. When a ball is well hit (i.e. high hit-ball speed), contact with the ball is almost imperceptible by the batter. When a ball is poorly hit, however, (i.e. away from the



sweet spot) the batter's hand will sting from vibrations in the bat. It has been suggested that node locations of the fundamental vibrational modes correspond with the bat's sweet spot location [14]. To test this theory, the vibrational frequencies and mode shapes of each of the bats used in the field study were measured using modal analysis. This was accomplished by attaching an accelerometer (PCB Piezotronics, Model # 352C22, Depew, NY, USA) to the bat at 0.38 m (15 in.) from the knob and striking the bat with an instrumented impact hammer (PCB Piezotronics, Model # 350B23, Depew, NY, USA) at 12 mm increments along its length. The bat was given a free-free support by supporting it horizontally on compliant foam pads at the proximal and distal ends. The mode shapes were obtained from the frequency response function of each test, as is routinely done in modal analysis.

The node locations for the first and second modes are included in Fig. 12. The mode I nodes are inside of the field study and laboratory sweet spot locations within 8 and 18 mm, respectively. One would expect the sweet spot to occur just outside of the minimum vibration location, where the increased bat speed would exceed the effect of vibrational energy dissipation. The mode II nodes are 30-60 mm *outside* of the sweet spot locations. The sweet spot should occur near the location of minimum vibration of the bat, which is influenced by multiple vibrational modes. The first mode apparently has a dominant effect, given its proximity to the sweet spot (in comparison to the mode II locations), and the decreased amplitude of the mode II vibrations (vibrational amplitude decreases with increasing frequency, and the mode II frequencies were nearly 4 times higher than the mode I frequencies). The results suggest that free vibrational response may be a good indicator of forced vibrational behaviour during the batball impact. While this relationship has been predicted by others [5], the current results represent the first experimental verification [14].

7 Summary

The foregoing has considered field measurements of softball player swing speed and bat performance. The average bat-ball impact was observed to occur nearly 1.4 m in front of the apex of home plate. The average bat instantaneous centre of rotation at impact was near the knob of the bat. Batter swing speed decreased with increasing bat inertia, while the collision efficiency increased with bat inertia. The field measurements of bat performance were in good agreement with standardised laboratory bat performance tests and showed that the hit-ball speed increases with bat inertia. The maximum bat performance location moved away from the pivot as inertia increased, corresponding to the node location of the first vibrational mode. The contribution of player ability on hit-ball speed was shown to be large, and should not be neglected when considering bat performance.

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