Statcast and the Baseball Trajectory Calculator

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Statcast and the Baseball Trajectory Calculator

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Baseball's flirtation with technology began in 2005 when PITCHf/x® by Sportvision[1] started to be installed in major league ballparks. Every stadium had the system operational by 2007. Since then, the trajectories of over six million pitches have been measured to within about half an inch using three 60-Hz video cameras to track the position of the ball.

In the 2015 season that modest flirtation exploded into a total technology love affair with the debut of Statcast.[2] Instead of just tracking each pitch, Statcast follows everything on the field all the time: the pitched and batted baseball as well as all the players. This new system uses Doppler radar supplied by Trackman[3] to follow the ball and video tracking by ChyronHego[4] to monitor the players.

The dual technologies are needed because Doppler radar is ideal for tracking the ball with typical speeds between 30 and 120 mph. This is roughly the same speed range used by law enforcement radar to measure the speed of your car. Video doesn't work to track the ball because it is so small that the variations in the background of the camera image create a signal-to-noise issue.

Radar is a poor choice to measure the motion of players because their speeds rarely exceed 20 mph, resulting in too small a Doppler shift to measure above the noise. Since players move against a constant green turf background, they can easily be followed within a video.

While Major League Baseball made the decision early on to allow free access[5] to PITCHf/x data, they unfortunately have been less forthcoming with Statcast information, which is more challenging to share because of the huge data sets. We know of only two publicly available sources: MLB's Statcast Leaderboard[6] and Baseball Savant.[7] The former has a limited selection of information but is easy to use. The latter contains a far more complete data set, but can be more challenging to navigate.

We intend to introduce physics teachers (and hopefully, in turn, their students) to the Statcast data set and a powerful spreadsheet called the "Trajectory Calculator." The potential investigations of the national pastime that these tools allow are far too broad to enumerate here. So, we'll just settle for the basic application described below.

A look at some Statcast data

Alex Rodriguez (A-Rod) of the New York Yankees is, by all measures, one of the greatest players of all time. In his 20+ years he has amassed over 3000 hits and is fourth all-time on the home run list. Like so many celebrities, his personal lapses became public. He was suspended for the entire 2014 season for the use of performance-enhancing drugs. He returned in 2015 with a vengeance, clubbing 33 long balls.

MLB's Statcast Leaderboard shows his longest home run[8] from 2015 was whacked in the second inning on April 17 at Tropicana Field in Tampa Bay, FL.[9] The Statcast data show:

Distance[10]	470.5 ft
Launch Speed	107.3 mph
Launch Angle	26.1°
Height[11]	96.0 ft

Your students might immediately want to plug the launch speed and launch angle into the range equation,

$$R = \frac{v_0^2}{g}\sin 2\theta, \tag{1}$$

where v_0 is the launch speed, g is the acceleration due to gravity, and θ is the vertical launch angle. Assuming they have the skills to navigate the challenges of English[12] units, they will immediately see a serious discrepancy. Their result is 608.3 ft, far in excess of the 470.5 ft claimed by Statcast.

The forces on a homer in flight

Newton's laws tell us the ball will do what it does because of the forces that act on it. Figure 1 shows these forces as red arrows. The blue arrow represents the velocity of the ball, while the circular green arrow indicates the backspin on the ball.

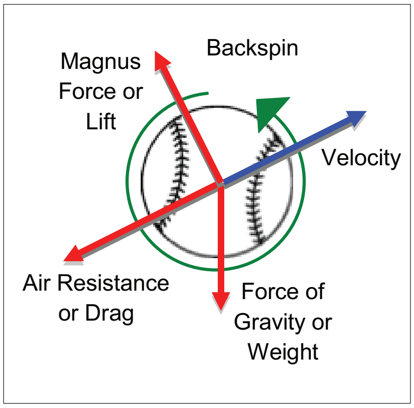

Fig. 1. The forces (red) on a ball in flight with backspin are lift, drag, and weight. The backspin (green) and velocity (blue) are also shown.

There are two things that exert forces on a long fly ball: Earth pulls it down and the air pushes it around. The force that the air exerts on the ball is complex and subtle. That's part of the reason it is usually considered as two distinct forces, the drag (or air resistance) and the Magnus force (or lift). If gravity were the only force, the range equation would have given the correct distance.

Air resistance or drag always acts opposite the velocity of the ball and is primarily responsible for the discrepancy your students discovered from the range equation. Drag, as the

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name implies, slows the ball down. You have experienced air drag every time you stuck your hand out the window of a moving vehicle. The force the air exerts on your hand can be quite strong at highway speeds that, incidentally, are around the average speed of a homer in flight.

The magnitude of the drag force is given by

$$F_{\mathrm{D}} = \tfrac{1}{2} C_{\mathrm{D}} A \rho v^2, \tag{2}$$

where C_D is the drag coefficient, A is the cross-sectional area of the ball, ρ is the air density, and v is the speed of the ball relative to the air.[14]

The other force exerted by the air is the Magnus force or lift. It is always perpendicular to the velocity and is in the direction of the spinning motion of the front of the ball as it moves through the air. Due to the spin, the front of the ball in Fig. 1 is moving mostly up the page and slightly to the left, matching the direction of the Magnus force.

It is of interest to note that if you use the kinematic equations to calculate the maximum height during the flight of the ball, you will get a value around 75 ft. This is well below the 96 ft measured by Statcast. So, the lift provided by the Magnus force is substantial.

In analogy to the drag, the Magnus force can be written as

$$F_{\mathrm{M}} = \tfrac{1}{2} C_{\mathrm{L}} A \rho v^2, \tag{3}$$

where C_L is the lift coefficient.[14] You might notice that the Magnus force doesn't appear to depend upon the spin rate. Of course it does and that dependence is hidden in the lift coefficient. In the simplest model of a baseball in flight, the lift coefficient is proportional to the ratio of spin rate to the velocity. As a result, the Magnus force is proportional to both spin rate and velocity, each to the first power.

The backspin on a ball on a typical home run trajectory (25 to 35° launch angle) will result in a Magnus force that is more vertical than horizontal. This upward component partially counteracts gravity, so that the ball rises to a greater height, allowing it to stay in the air longer and travel farther.

In summary, the drag will tend to slow the speed and lower the distance of a potential homer. On the other hand, backspin on such a ball will result in an upward Magnus force helping the ball stay up longer and go further.

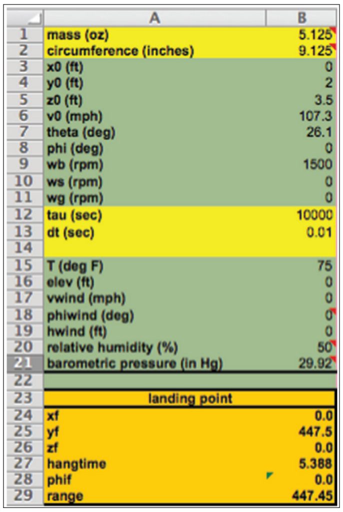
The Trajectory Calculator

Calculating the trajectory of a homer given the launch speed and launch angle becomes a complex chore of applying Newton's second law using the three forces in Fig. 1 to find the acceleration. From the acceleration the speed and position can be extracted. Much of the complication comes from the fact that the drag and Magnus forces are velocity dependent.

Wouldn't it be nice if someone had gone to all the trouble to build an Excel spreadsheet to do all this work for you? Well, someone did! The Trajectory Calculator is a powerful tool for examining the flight of a baseball, whether it is a pitched or batted ball. Go download[15] a copy!

Earlier you may have realized the trajectory not only depends upon the launch speed and launch angle, but also on the spin of the ball. While the Statcast database contains the spin, that information is not presented on the Statcast Leaderboard. So, as an example of one way to use the Trajectory Calculator, let's find the backspin on A-Rod's homer.

Start by opening the "Batted Ball" tab, part of which is shown in Fig. 2. The names of the necessary input parameters are in column A while their values go in column B. We'll go through each parameter for the sake of clarity.



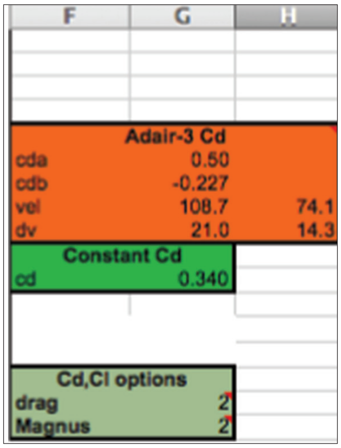
	A	B
1	mass (oz)	5.125
2	circumference (inches)	9.125
3	x0 (ft)	0
4	y0 (ft)	2
5	z0 (ft)	3.5
6	v0 (mph)	107.3
7	theta (deg)	26.1
8	phi (deg)	0
9	wb (rpm)	1500
10	ws (rpm)	0
11	wg (rpm)	0
12	tau (sec)	10000
13	dt (sec)	0.01
14		
15	T (deg F)	75
16	elev (ft)	0
17	vwind (mph)	0
18	phiwind (deg)	0
19	hwind (ft)	0
20	relative humidity (%)	50
21	barometric pressure (in Hg)	29.92
22		
23	landing point	
24	xf	0.0
25	yf	447.5
26	zf	0.0
27	hangtime	5.388
28	phif	0.0
29	range	447.45

Fig. 2. The input and output parameters for the trajectory of a batted ball.

The mass and circumference of the ball are already entered when you downloaded the spreadsheet.

The coordinate system has its origin at the back point of home plate. The z-axis is vertical, the y-axis points directly to centerfield, and the x-axis points to the catcher's right. Let's assume the initial position of the batted ball is right down the middle of the plate, 3.5 ft above the ground and 2 ft in front of the back of home plate. After all, most balls hit out of the park are pretty "fat" pitches.

The launch speed and vertical launch angle for A-Rod's homer are given by Statcast. Phi is the azimuthal angle, with zero being directly toward centerfield. Let's just leave it at zero for now. The entry labeled "wb" is the backspin, which is one of the parameters that we would like to find. For now let's just leave the downloaded value of 1500 rpm. The sidespin is labeled "ws" and "wg" is the "gyro spin" or the spin component in the direction of the velocity of the ball. Let's assume these two are zero for simplicity.



F	G	H
Adair-3 Cd		
cda	0.50	
cdb	-0.227	
vel	108.7	74.1
dv	21.0	14.3
Constant Cd		
cd	0.340	
Cd,Cl options		
drag	2	
Magnus	2	

Fig. 3. The lift and drag models and coefficients.

Tau is the time constant for spin decay, which is set to be a big number, implying that the spin stays constant throughout the flight of the ball. dt is the time step for the calculations. The value that comes with the spreadsheet usually works quite well. The homer was hit in an indoor ballpark. So, we'll leave the weather-related parameters as they come with the download.

Figure 3 shows the section of the spreadsheet related to the drag and lift coefficients. There are differing models for each that can be chosen using the bottom light green box of Fig. 3. For simplicity, we'll stick with Magnus option 2 and a constant drag coefficient, also option 2.

As mentioned earlier, the drag is primarily responsible for the discrepancy in the range discovered by your students. In addition, the drag will have a larger effect on the distance than the backspin. Therefore, keeping the backspin (cell B9) at the value of 1500 rpm, which is in the right "ballpark" for typical long fly balls, let's adjust the value of C_d (cell G11) until the range in cell B29 matches the Statcast distance of 470.5 ft. We got a C_d value 0.316 to three significant figures.

Examining the graph of height above the ground vs. the horizontal distance traveled near A33 in the spreadsheet, you'll see the maximum height is about 92.2 ft. This value disagrees with the Statcast measurement of 96.0 ft. The discrepancy is associated with the assumed backspin being a little too small. Increasing the backspin will give a greater height but also a somewhat greater distance. Therefore, you will need to both increase the backspin and increase the drag coefficient.

The point is, you can adjust *both* the backspin and the drag coefficient until you match the Statcast height and distance. In this way, you'll establish the values of both parameters. Without going overboard on significant figures, we got a C_d of 0.326 with a backspin of 1860 rpm.

We have only scratched the surface with this one example. There are a virtually unlimited number of questions and ideas your students can explore once they master the Trajectory Calculator. Here are some extensions to this exercise:

1. ***Study the effect*** of changing temperature or elevation.
2. ***Examine the changes*** in distance due to head or tail winds.
3. ***Determine the slope*** of distance vs. exit speed with everything else fixed.
4. ***Find the sidespin*** needed to shorten the distance by 20 ft.
5. ***Find the launch angle*** that gives the longest distance.

Additional applications of the Trajectory Calculator can be found at the Physics of Baseball[16] website or The Hardball Times.[17] Now, play ball!

References

1. Their baseball website has many videos describing the system at http://www.sportvision.com/baseball/pitchfx%C2%AE .
2. Here is a sample of the data that Statcast can bring to a television broadcast: http://m.mlb.com/video/v31459495/?partnerId=as_mlb_20140311_19868264 .
3. Their baseball website is at https://trackmanbaseball.com/. Additional info can be found at http://baseball.physics.illinois.edu/trackman.html .
4. Info on their player tracking system can be found at http://chyronhego.com/sports-data/player-tracking .
5. The use of PITCHf/x data is described in: David Kagan, "The anatomy of a pitch: Doing physics with PITCHf/x data," *Phys. Teach.* **47**, 412–416 (2009).
6. http://m.mlb.com/statcast/leaderboard .
7. https://baseballsavant.mlb.com/ .
8. Here's a video of this homer: http://m.mlb.com/video/v78076783/?game_pk=413799 .
9. There is a thorough discussion of both home runs hit by A-Rod in this game at http://www.hardballtimes.com/a-tale-of-two-dingers/.
10. There is confusion on several Statcast pages at mlb.com as to what distance is actually given. The distance on the page we cite is from the back of home plate to the spot on the ground where the ball would strike if it were to get there unobstructed. That measured trajectory up to the point where the ball strikes an impediment is used to extrapolate to get this value.
11. This is the maximum height of the ball during its flight.
12. The authors make no apology for the use of English units throughout this paper. After all, they are the traditional units of the national pastime!
13. See David Kagan and Alan M. Nathan, "Simplified models for the drag coefficient of a pitched baseball," *Phys. Teach.* **52**, 278–280 (2014).
14. Alan M. Nathan, "The effect of spin on the flight of a baseball," *Am. J. Phys.* **76**, 119–124 (Feb. 2008).
15. The Trajectory Calculator is currently undergoing an update. The version referred to in this paper can be found at http://baseball.physics.illinois.edu/trajectory-calculator-old.html .
16. http://baseball.physics.illinois.edu/statcast.html.
17. http://www.hardballtimes.com/?s=%22trajectory+calculator%22 .

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