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Modeling Pitch Trajectories in Fastpitch Softball

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Abstract We use the fourth-order Runge-Kutta method to numerically integrate the equations of motion for a fastpitch softball pitch and in this way create a model from which we can compute and display the trajectories of drop balls, rise balls and curve balls. By requiring these pitches to pass through the strike zone, and by making reasonable assumptions about the initial speed, launch angle and height of a pitch, we predict an upper limit on the lift coefficient that agrees with experimental data. We also predict the launch angles necessary to put various pitches in the strike zone and a value of the drag coefficient that agrees with experimental data. Finally, we compare pitches that look similar to a batter starting her swing, yet diverge before reaching home plate, using the analysis of a batter's swing given by Adair to predict when a pitch is likely to be missed (or fouled) by a batter.

Keywords Softball · pitching · differential equations · Runge-Kutta

1 Introduction

The game of fastpitch softball has been played since the late 1800's and is currently a very popular women's sport in American high schools and colleges. Although the trajectories of many different spherical sports balls have been

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investigated [1], those of a fastpitch softball have only recently begun to be studied [2,3].

In this paper we present a computer model based on Newton's equations that simulates the motion of fastpitch softball pitches. By assuming reasonable values for the angle, speed and height off the ground of a softball as it leaves the pitcher's hand, we compute and display the trajectories of rise balls, drop balls and curve balls. We then use the model to predict an upper bound on the lift coefficient of a fastpitch softball and show that this bound agrees with experimental data. We also predict the launch angles necessary for various pitches to pass through the strike zone, and a value of the drag coefficient which is consistent with experimental data. Using the time analysis of a batter's swing presented by Adair [4], we then discuss when a given pitch has a good chance of fooling a batter into starting her swing before she can accurately assess the trajectory of the ball and thus cause her to miss (or foul) it. Finally, we discuss how the agreement of our model with experimental data gives us confidence that it can be used to explore such questions as what happens when a fastpitch softball has its axis of rotation at an arbitrary angle, and what happens when forces other than gravity, drag and the Magnus force act on the ball.

2 Assumptions and initial conditions

The coordinate system used in our analysis has the x axis along a horizontal line from the pitcher to home plate, the y axis perpendicular to the ground, and the z axis perpendicular to the x and y axes according to the right hand rule. The origin of the coordinate system is on the ground directly beneath the point where the ball leaves the pitcher's hand. The pitch is thrown from just above the origin at an initial height y_0, so the coordinates of the launch point are $x_0 = 0, y_0, z_0 = 0$.

We use spherical coordinates to describe the launch angles of a pitch, but with the mathematician's choice of the labels θ and φ. In other words, the angle φ is the angle the velocity vector makes with the z-axis, and θ is the angle made with the x-axis by the projection of the velocity vector in the $x - y$ (vertical) plane. Note that a pitch which remains in the vertical plane (and does not curve) has a constant angle of $\varphi = 90°$. If the pitch is thrown perfectly horizontally (parallel to the ground) it has $\theta = 0°$. A nice illustration of this coordinate system (shown with the y-axis in the horizontal rather than the vertical direction) is given online by Arnold [8].

We model the motion of the fastpitch softball pitch by assuming that once the ball leaves the pitcher's hand it is acted upon by three forces: gravity, air resistance and the Magnus force. The magnitude of the force of gravity is

$$F_{gravity} = mg \tag{1}$$

where m is the mass of the softball and g is the gravitational acceleration, whose magnitude[1] is 32 ft/s^2 (9.8 m/s^2) and whose direction is along the negative y axis.

The magnitude of the force of air resistance is expressed in the standard form [9]

$$F_{drag} = \frac{1}{2} C_D \rho A v^2, \tag{2}$$

where C_D is the drag coefficient, A is the cross-sectional area of the ball, v is the speed of the ball, and ρ is the density of air which, to an accuracy of two significant figures, is 1.2 kg/m^3 for temperatures within the range of 60° F to 90° F (16°C to 32°C) [10]. The drag force acts in the direction opposite to that of the velocity.

In order to simplify the computer code we defined the constant

$$C \equiv \frac{1}{2} C_D \rho A \tag{3}$$

so the drag force could be written as

$$F_{drag} = C v^2. \tag{4}$$

As we discuss below, after putting in the appropriate value for C_D we find that $C = 1.55 \times 10^{-3}$ kg/m.

The standard expression for the magnitude of the Magnus force is [9]

$$F_{Magnus} = \frac{1}{2} C_L \rho A v^2, \tag{5}$$

where C_L is the lift coefficient. The Magnus force is created by the spin of the ball, and depending upon the direction of the spin axis, it leads to pitches that rise, curve or drop as they travel from the pitcher to home plate. The Magnus force acts in a direction perpendicular to both the angular and translational velocity of the ball. More precisely, if $\boldsymbol{\omega}$ is the angular velocity vector then the Magnus force is in the direction of $\boldsymbol{\omega} \times \mathbf{v}$.

Putting these forces into Newton's second law we find

$$m\dot{v}_x = -C v_x \sqrt{v_x^2 + v_y^2 + v_z^2}, \tag{6}$$

$$m\dot{v}_y = -mg - C v_y \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{7}$$

$$+\frac{1}{2} C_L \rho A \left(v_x^2 + v_y^2 + v_z^2\right) \sin\alpha, \tag{8}$$

$$m\dot{v}_z = -C v_z \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{9}$$

$$+\frac{1}{2} C_L \rho A \left(v_x^2 + v_y^2 + v_z^2\right) \cos\alpha. \tag{10}$$

[1] Because the values of some of our initial conditions are only reasonable approximations, we only use two significant figures throughout the paper.

In these equations, the superscript dot signifies a time derivative. The x component of the velocity is denoted by v_x and similarly for the y and z components. The direction of the Magnus force vector is specified by the angle α, which lies in the y-z plane with $\alpha = 0$ when the Magnus force vector points along the positive z axis.

The initial speed of the pitch is v_i, and the initial values for v_x, v_y, and v_z are given in the spherical coordinates described at the beginning of this section:

$$\begin{aligned} v_{x,i} &= v_i \sin\varphi \cos\theta \\ v_{y,i} &= v_i \sin\varphi \sin\theta \\ v_{z,i} &= v_i \cos\varphi. \end{aligned}$$

The fastpitch softball is an optic yellow sphere with at least 88 raised red thread stitches. Although the dimensions of softballs vary slightly, we assume [11] that each softball has a circumference of 12 inches (0.305 m), a radius 1.9 inches (4.83 cm) and a weight of 6.5 ounces (which corresponds to a mass of 0.184 kilograms).

Some of the initial conditions will be the same for all of the pitches we consider. The front of the softball pitching rubber (the side closest to the batter) is 43 feet (13.1 m) from the back of home plate [11] and, unlike in baseball, where there is a pitcher's mound, the rubber from which fastpitch softball pitchers throw is at the same level as the batter. Since a pitcher is permitted to take one stride towards the batter during her delivery, if we assume her stride is 3.0 feet (0.913 m) then every pitch travels 40 feet (12.2 m) in the direction of the x-axis from its release point to home plate.

The ball is released from a point just above the knee of the pitcher. As she steps forward to deliver a pitch her knee drops so we assume the release point is 1.5 feet (0.46 m) off the ground. This value of y_0 is close to those recorded by Nathan [3] (1.8 ft, $\sigma = 0.2$ ft) from four fastpitch softball pitchers. Finally, we assume the initial speed of the softball is 65 mph (29 m/s), which is consistent with values measured by Nathan [3] ($v_i = 65\,\text{mph}$, $\sigma = 5$ mph) in over 3500 pitches, and with values reported by others.[2]

Next we want to determine a numerical value for the drag coefficient C_D. If we assume the fastpitch softball is a scaled up baseball then both the baseball and the softball will have the same C_D when they have the same Reynolds number

$$Re = \frac{\rho D v}{\mu}, \tag{11}$$

where ρ is the density of air (whose numerical value is given just above Eq.(3)), D is the diameter of the ball, v is the speed of the ball relative to the air, and

[2] The Lisa Mize Fastpitch Academy reports the range of speeds for college fastpitch softball pitches as 59 to 70 miles per hour [12].

$\mu = 1.85 \cdot 10^{-5}$ N·s/m^2 is the dynamic viscosity of air [9]. To an accuracy of two significant figures the diameter of a regulation baseball is 2.9 inches (7.4 cm), so a softball moving at 65 mph should have the same properties as a baseball moving at

$$v = \frac{(3.8)(65\,\text{mph})}{(2.9)} = (1.31)(65\,\text{mph}) = 85\,\text{mph}. \tag{12}$$

Since the drag coefficient for a baseball moving with a speed of 85 mph is about 0.33,[3] we used this value of C_D in our equations. We note that this number falls within the range of drag coefficients for fastpitch softballs found experimentally by Nathan ($C_D = 0.31, \sigma = 0.04$, private communication), which supports our assumption that a fastpitch softball can be considered as a scaled up baseball.

The dimensions of the strike zone vary from batter to batter. In this paper we assume the strike zone starts 1.5 feet (0.46 m) above the ground (at the height of the top of the batter's knees) and ends 3.75 feet (1.1 m) above the ground (at the height of the batter's forward armpit). We take the width of the strike zone as the width of home plate (17 inches) plus the diameter of a softball (3.8 in) since the rules specify that a pitch is a strike if any part of the ball crosses over the width of home plate.[4] Thus, we take the width of the strike zone to be 20.8 in (0.53 m). The strike zone provides the boundary for pitches in our analysis; that is, we require all of our pitches to pass through the strike zone.

The last parameter we need to discuss is the lift coefficient C_L. For baseballs, the degree to which C_L is affected by the orientation of the stitches with respect to the spin axis of the ball is not completely understood.[5] Some analyses ignore the effect of seam orientation on the lift coefficient [13, 14, 15] while others indicate that there are at least a few cases in which the orientation of the seams can have a substantial effect on C_L [6].

We decided to let our model determine the range of allowed values for C_L for fastpitch softballs. Our approach was to numerically integrate the equations of motion (6) - (10) with a C++ program that implements the fourth-order Runge-Kutta method. Given that many of the input parameters are only accurate to two significant digits, we used a step size of 0.02 s because smaller step sizes produced identical values (to two significant digits) for the outputted quantities of interest.

The program is set up so that we first input the fixed parameters discussed above, then choose the launch angles θ and φ, and then let the program determine the values of C_L for which the ball passes through the strike zone. More specifically, after entering the fixed parameters discussed above, we chose θ and φ for the pitch in which we were interested and then allowed C_L to vary

[3] See the graph on page 8 of Reference [4].

[4] Since our program is tracking the center of mass of the softball, a strike occurs when its position is one radius from either side of home plate.

[5] See pp. 14-15 of Reference [4].

Table 1 Conditions for drop ball pitches

θ (degrees)	Range of C_L
5.0	0.00 – 0.05
5.5	0.00 – 0.11
6.0	0.00 – 0.16
6.5	0.00 – 0.22
7.0	0.00 – 0.27
7.5	0.00 – 0.33
8.0	0.04 – 0.38
8.5	0.10 – 0.44
9.0	0.16 – 0.50
9.5	0.21 – 0.56
10.0	0.27 – 0.62

from a minimum of 0.00 to a maximum of 1.00 in order to find the values of C_L which put the ball in the strike zone for those particular values of θ and φ.

3 Results

3.1 The drop ball

In the case of a drop ball, the Magnus force points in the negative y direction ($\alpha = -90°$). Since the only other force acting in the y direction is gravity, the softball will only pass through the strike zone if there is a positive launch angle θ.

Table 1 shows the conditions necessary for a drop ball to pass through the strike zone for a range of launch angles. For example, if the drop ball leaves the pitcher's hand at an angle of $\theta = 6.0°$ then the lift coefficient must be between 0.00 and 0.16 in order for the ball to cross home plate in the strike zone. The reason there is a range of acceptable values for the lift coefficient is because the ball will be a strike if it passes anywhere between the top and bottom of the strike zone. Note that here, and in what follows, $C_L = 0.00$ indicates a pitch for which the Magnus force is zero. Technically, a pitch for which the Magnus force is zero is not a drop ball (or a curve ball or a rise ball), but we include this value of C_L in the tables because it represents the lower bound of the allowed values of the lift coefficient.

Figure 1 shows the trajectory of a drop ball with a launch angle of $\theta = 6.0°$ and a lift coefficient $C_L = 0.15$. Note that the scale on the y axis is smaller than the scale on the x-axis, so the vertical part of the trajectory displayed in the figure is somewhat exaggerated. In Figure 1, and in all subsequent figures, $x = 0$ ft is the x-coordinate of the point at which the pitch is released from the pitcher's hand and $x = 40$ ft is the x-coordinate of the far side of home plate.

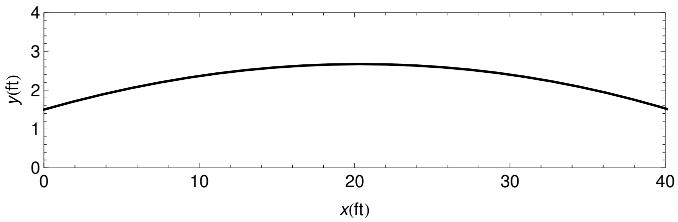

Fig. 1 The trajectory of a 65 mph (29 m/s) drop ball with an initial angle $\theta = 6.0°$ and a lift coefficient $C_L = 0.15$.

Table 2 Conditions for rise ball pitches

θ (degrees)	Range of C_L
0.0	0.51 – 0.85
1.0	0.40 – 0.73
2.0	0.29 – 0.62
3.0	0.18 – 0.51
4.0	0.06 – 0.40
5.0	0.00 – 0.29
6.0	0.00 – 0.18
7.0	0.00 – 0.07

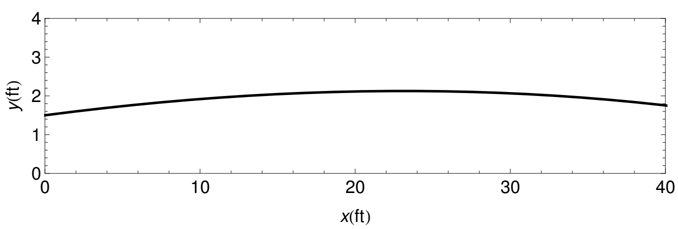

Fig. 2 The trajectory of a 65 mph (29 m/s) "rise ball" with an initial angle $\theta = 3.0°$ and a lift coefficient $C_L = 0.20$. Note, however, that this pitch does not rise throughout its complete trajectory.

3.2 The rise ball

In the case of a pure rise ball, the Magnus force points in the positive y direction ($\alpha = 90°$). Table 2 shows the range of values of C_L for which the ball will pass through the strike zone for various values of the launch angle θ. When $\theta \geq 8°$ there is no (positive) value of C_L for which the pitch will be a strike.

For $\theta = 3.0°$ and $C_L = 0.20$, the pitch stays in the strike zone but, as Figure 2 shows, the ball does not continue to rise throughout its complete trajectory. Figure 3 shows a rise ball pitch with $\theta = 6.0°$ and $C_L = 0.18$ that does rise during its whole trip to home plate.

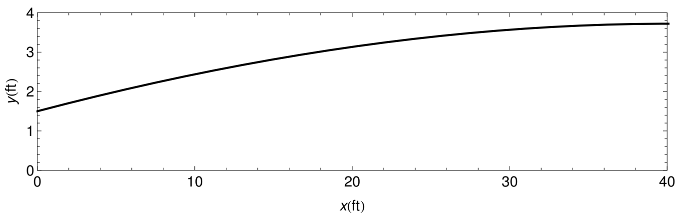

Fig. 3 The trajectory of a 65 mph (29 m/s) rise ball with an initial angle $\theta = 6.0°$ and a lift coefficient $C_L = 0.18$. For these values of θ and C_L the ball rises throughout the whole trajectory.

Table 3 Conditions for curve ball pitches

φ (degrees)	Range of C_L
90.5	0.00 – 0.21
90.0	0.00 – 0.27
89.5	0.06 – 0.32
89.0	0.12 – 0.38
88.5	0.17 – 0.44
88.0	0.23 – 0.49
87.5	0.29 – 0.55

3.3 The curve ball

In the case of a pure curve ball, the Magnus force points in the negative z direction ($\alpha = 180°$) and the softball curves to the pitcher's left. (By calling this pitch a "curve ball" we are implicitly assuming the pitcher is right-handed. If the pitcher were left-handed this same pitch would be called a "screw ball.")

Table 3 shows the conditions necessary for a curve ball to pass through the strike zone. In each case we have assumed a value of θ which keeps the ball within the strike zone's vertical dimensions.

Pitches with values of φ less than 90° begin with a component of velocity in the positive z direction, opposite to the direction in which the ball will curve, which means they first travel slightly to the pitcher's right before they curve to the left. The trajectory of a curve ball with launch angles $\theta = 4.5°$ and $\varphi = 90°$, and a lift coefficient $C_L = 0.15$, is shown in Figure 4. We stopped calculating trajectories when $\varphi = 87.5°$ because, as we discuss in Section 5, fastpitch softball pitchers rarely produce a pitch with a value of C_L higher than about 0.30.

3.4 Other pitches

We can also compute and graph trajectories of pitches with the Magnus force vector pointing in any direction α in the y–z plane and any valid launch angles θ and φ, which we can think of as "rising screw balls," "falling curve balls," etc. Indeed, one way our model can be used is to display the trajectories of

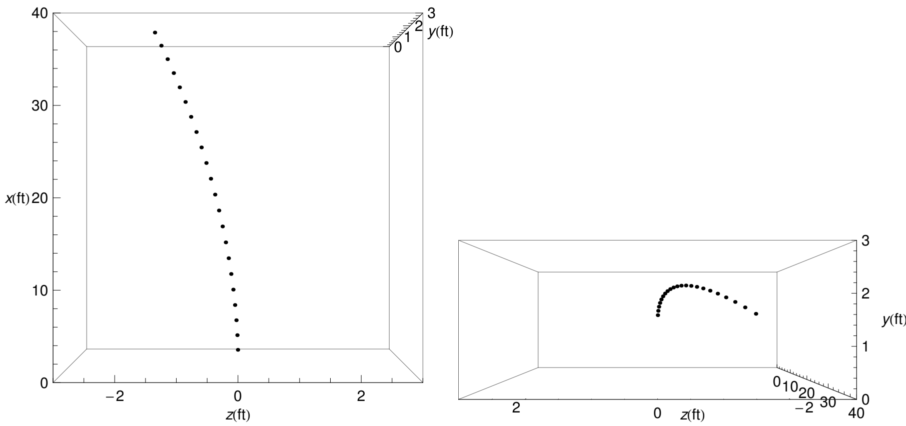

Fig. 4 The trajectory of a 65 mph (29 m/s) curve ball with $\theta = 4.5°$, $\varphi = 90°$ and a lift coefficient $C_L = 0.15$. (a) A view of the trajectory from above. (b) The batter's view of the trajectory. Note that in both cases the scale on the x axis is larger than the scales on the y and z axes.

these pitches for various values of C_L to estimate how much spin would be necessary to keep them in the strike zone.

4 Striking out the batter

We can use our results in conjunction with the time analysis of a typical batter's swing given by Adair [4] to get a better understanding of when a batter is most likely to miss (or foul) a pitch that passes through the strike zone. Our approach is to find out where the ball is when the batter must initiate her swing. If one type of pitch (say a rise ball) cannot be distinguished from another type of pitch (say a drop ball) before this time then the batter is likely to miss (or foul) the ball as it passes over home plate. Similarly, if a curve ball hasn't begun to curve when the batter must begin her swing then she is also likely to miss (or foul) it.

The first step in our approach is to determine the total time the ball is in the air as it travels from the pitcher to home plate. Our numerical integration of Newton's equations shows that, for a softball whose initial speed is 65 mph and whose drag coefficient is 0.33, this time is 0.45 s. If we assume no drag force, so the softball is traveling at a constant speed of 65 mph from its release point to home plate, the time of flight is 0.42 s.

Note that the time for a baseball traveling at a constant speed of 85 mph to reach home plate, which is 56 ft away from the point where the pitch is released, is also 0.45 s. Similarly, if the baseball is traveling at 90 mph, it takes 0.42 seconds to reach home plate. Thus, a softball player must judge and react to a pitch in a similar short time interval as a baseball player, and Adair's analysis of a baseball batter's swing is also valid for the swing of a fastpitch softball player.

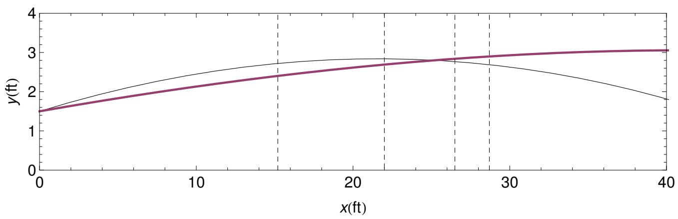

Fig. 5 The trajectory of a 65 mph (29 m/s) rise ball with an launch angle $\theta = 4.0°$ and a lift coefficient $C_L = 0.30$ together with the trajectory of a drop ball with an launch angle of $\theta = 7.0°$ and a lift coefficient $C_L = 0.25$. The first dotted line, at $x = 15.2$ ft (4.6 m), shows the location of the ball at the last possible time by which the batter can start the Looking phase. The dotted lines at $x = 22.0$ ft (6.7 m), $x = 26.5$ ft (8.1 m) and $x = 28.7$ ft (8.7 m) show the respective last locations of the ball at which the batter can start the Thinking, Action and Batting phases. Given the scale on the y-axis, the locations of the two pitches are almost indistinguishable to the batter during the time just before she initiates her swing, but when the pitches cross home plate they are over a foot apart.

Adair's analysis of what happens during the batter's swing separates the batter's complete response into four parts: Looking, Thinking, Action, and Batting.[6] The Looking portion of the swing takes about 75 milliseconds and is the time it takes the batter to cognize that the ball has left the pitcher's hand. The next part of the batter's process is the Thinking portion. During this part of the swing, which takes about 50 milliseconds, the brain estimates the ball's trajectory. The next part of the batter's process is the Action portion, which takes about 25 milliseconds. This is the time period during which the brain tells the muscles to begin the swing. Finally, there is the Batting portion of the process, which takes approximately 150 milliseconds, during which the batter sets the bat in motion until it hits the ball in the middle of home plate. Adair says an experienced player can make minor adjustments in the motion of the bat for about the first $50 - 100$ milliseconds of the Batting portion of the swing, but these adjustments most likely won't result in a solid hit if the trajectory is not what the batter expected at the end of the Action portion of the swing.

In Figure 5 we show the locations of a rise ball and a drop ball during each of the four parts of the batter's process. To do this, we first determined the time at which the ball would be directly over home plate and called this the end of the Batting portion of the swing. We then worked backwards from this time to determine where the ball is 150 milliseconds earlier, at the beginning of the the Batting portion, and continued in this way to determine where the ball is at the beginning of the Action, Thinking and Looking phases.

Figure 5 shows how hard it is for a batter to assess the trajectory of a pitch in time to get a solid hit. As the figure shows, the batter must commit to her swing at the end of the Action portion of her process, when the drop and

[6] See pp. 38–46 of Reference [4]. A beautiful pictorial representation of Adair's time-analysis of a batter's swing is shown in the article "Hitting a Baseball: The Hardest Thing To Do In Sports," by Dan Peterson `http://www.axonpotential.com/hitting-a-baseball-the-hardest-thing-to-do-in-sports`.

rise ball trajectories are almost indistinguishable. When the balls cross home plate, however, the two trajectories are over one foot apart(!), so if the batter has made the wrong choice at the beginning of her swing she most likely will miss or foul the ball.

Figures like Figure 5 can be created to compare the trajectories of any two pitches, and thus can be used to get an idea of how likely it is that a batter will have difficulty distinguishing one pitch from another by the time she has to commit to a specific swing. They also can be used to show the trajectory of a single pitch, such as a curve ball, to see where the ball is when a batter must initiate her swing. As such, if the initial conditions for a specific pitcher (her launch speed, launch height, etc.) are used in the program, figures like Figure 5 could be useful in predicting whether her pitching will be effective against an opposing team.

5 Lift coefficients for a fastpitch softball

Although equations (2) and (5), in which the drag and lift coefficients are defined, have the same form, the coefficients themselves have different functional dependencies. For example, whereas the drag coefficient depends upon properties intrinsic to the ball and the air through which it travels, heuristic arguments [4, 9] suggest that the lift coefficient should depend upon the spin ω of the ball and its linear speed v through the air, both of which can vary from pitch to pitch. Consequently, although the drag coefficient should be essentially the same for all pitches thrown at roughly the same speeds, we expect the lift coefficient to vary, and more generally, to lie within a bounded range determined by the maximum and minimum values of the spin and initial speed given to the ball.

Nathan [9] investigated lift coefficients for baseballs, first extracting values of C_L from data and then examining their functional dependence on the Reynolds number and a quantity called the "spin factor," which is defined as $S = \omega R/v$. He found that for 75 mph $< v <$ 100 mph and $0.15 < S < 0.25$, which are the ranges most relevant to baseball, C_L is independent of Reynolds number but does depend upon S. Nathan says his data is in "excellent agreement" with the parameterization of Sawicki *et.al.* [13]

$$C_L = 0.09 + 0.6S \tag{13}$$

when $S > 0.1$, and

$$C_L = 1.5S \tag{14}$$

when $S \leq 0.1$.

In order to investigate how C_L and S are related for fastpitch softballs we need to know typical values for a softball's angular speed ω. RevFire© makes equipment which they claim measures ω for fastpitch softballs to within $\pm$ 0.25 revolutions per second [16, 17, 18]. Their measurements show that the average value of ω for drop balls is 20 revolutions per second (rps), for curve balls and screw balls is 21 rps, and for rise balls is 22 rps. More generally, they found that for most pitches 17 rps $< \omega <$ 32 rps. Using these values of ω we find that the average value of S is about 0.22 for all four types of softball pitches when they are moving with a speed of 65 mph, and that S is within the range $0.18 < S < 0.34$. Note that this range of values has significant overlap with the corresponding range (mentioned above Eq. (13)) for baseballs.

In Section II we calculated a theoretical value of the drag coefficient based on the assumption that a fastpitch softball moving at 65 mph can be treated as a scaled-up 85 mph baseball, and we found that the calculated value was in excellent agreement with experimental data. Because of this, and because the range in which the spin factor S falls for a fastpitch softball has significant overlap with the range of S for a baseball, we will assume the allowed values of C_L for a fastpitch softball can also be computed from Eqs. (13) and (14). Making this assumption, we find that the average value of C_L for drop balls, rise balls, screw balls and curve balls moving at 65 mph should be around 0.22, and that C_L should fall within the range $0.20 < C_L < 0.29$. Nathan (private communication) reports that a preliminary analysis of data taken from over 3500 fastpitch softball pitches of *all kinds*, thrown by four pitchers, found a similar upper bound for C_L (about 0.30) although a somewhat lower mean $(0.13, \sigma = 0.06)$. Of course the mean is determined by how many of each type of pitch was thrown, which was not recorded.

The fact that we expect C_L to be bounded above by 0.30, and that this expectation is in agreement with experimental data, allows us to use the results presented in Tables I, II and III to draw several interesting conclusions about launch angles. First, according to Table I, drop balls which pass through the strike zone can't have launch angles greater than about ten degrees. This prediction is consistent with the data presented by Nathan [9], which showed that θ has an average value of $7.4°$ with a root mean square value of $2.3°$. Note that Nathan's launch angle data were taken for all types of pitches, not just drop balls. Second, according to Table II, rise balls must have a nonzero launch angle in order to pass through the strike zone and this angle must be greater than two degrees. This prediction is consistent with the data presented by Nathan [9], which showed that there were no launch angles θ less than two degrees. Third, according to Table III, a pitch curving to the pitcher's left must be launched with a horizontal angle less than $2.5°$ to the right of the line between the pitcher and home plate if it is to have a chance of passing through the strike zone. At present there are no data with which to compare this prediction.

6 Conclusion

In this paper we have presented a model based on Newton's Laws from which the trajectories of various pitches in fastpitch softball can be calculated and displayed. We used this model to graph the paths followed by drop balls, rise balls and curve balls for different choices of launch angles and lift coefficients, and to determine which combinations of these parameters result in pitches that pass through the strike zone. We then used the model, along with an analysis presented by Adair, to predict when a pitch is likely to be missed or fouled by a batter. Finally, we considered lift coefficients C_L for fastpitch softballs. We predicted that C_L should be bounded from above by 0.30, a result confirmed by recent experimental data. We then used this upper bound to place limits on the launch angles for which various pitches will stay within the strike zone, and showed these limits also agree with experimental data.

Although the model is quite straightforward, the fact that its predictions agree with experimental data gives us confidence that it can be used to successfully answer more questions about fastpitch softball pitches then the ones discussed here. For example, in the interest of brevity, we only considered pure drop balls, pure rise balls and pure curve balls. However, the model can also be used to find the trajectories of curve balls that are rising, drop balls that are curving, etc. simply by changing the magnitude and direction of the angular velocity vector in the code. Also, it would be easy to analyze pitches with speeds other than 65 mph (which we took as the average speed of a fastpitch softball pitch), such as the maximum and minimum speeds achieved by pitchers or the speeds of balls thrown by a particular pitcher, to see how other variables, such as the lift coefficient, must change in order for various types of pitches to pass through the strike zone. Finally, the model as presented here includes the usual forces of gravity, drag and the Magnus forces, but pitches have been observed which are hypothesized to have been acted upon by forces dependent on the orientation of the softball seams.[7] Once explicit expressions are proposed for these forces, they can be tested by putting them into this model to see if the resulting trajectories agree with those observed.

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[7] In some cases there are additional "side or cross forces" on a pitched baseball. These forces seem to occur in special cases when the spin axis passes through the smooth part of the baseball (the cowhide) rather than the rough part (the seams). They are discussed by Wu and Gervais [5], Cross [6], and most recently by Borg and Morissey [7]. For an interesting example of such a pitch, see `http://www.nytimes.com/2012/08/05/sports/baseball/pitch-by-yankees-freddy-garcia-tests-physics-experts.html?_r=0`.

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