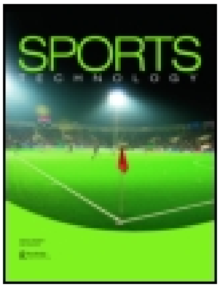

Sports Technology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rtec20

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Available online: 17 May 2012

Sports Technology, 2012, iFirst article, 1–6

Routledge
Taylor & Francis Group

Reducing the effect of the ball on bat performance measurements

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(*Received 3 March 2011; accepted 28 April 2011*)

Abstract

A simple physical model is presented that relates the ball–bat coefficient of restitution e to the ball coefficient of restitution e_0 and dynamic stiffness k_0. The model is used to develop a technique to normalize e to values of e_0 and k_0 for a "standard ball." The efficacy of this normalization technique is demonstrated by comparison with experimental data. It is shown to be vastly superior to a widely used technique that is based on the physically unjustified assumption that the ratio e/e_0, commonly referred to as the Bat Performance Factor or BPF, is independent of both e_0 and k_0. A residual, but much reduced, dependence of the normalized e on k_0 is observed and is shown through finite element simulations to be due to a dependence of the bat stiffness on the ball stiffness.

Keywords: *baseball, softball, performance, coefficient of restitution*

Introduction

In recent years, work has been done to measure and regulate the performance of non-wood baseball and softball bats. The measurement technique involves projecting a ball from a high-speed cannon onto a stationary bat and measuring the speed of the ball both before and after the collision. From these measurements, a value can be derived for the ball–bat coefficient of restitution (COR) e, which is a measure of energy dissipation in the ball–bat system. If e is to be a meaningful metric of bat performance, it is necessary to control the properties of the balls used to measure it. One such ball property is e_0, the COR of the ball when colliding with a massive rigid object, which determines the fraction of compressional energy stored in the ball that is returned as kinetic energy. A second ball property is k_0, the effective spring constant or "dynamic stiffness" of the ball when colliding with a massive rigid object. For a given bat, the ball stiffness controls how the initial energy is partitioned between compressional energy stored in the ball and that stored in the bat. The larger the ball stiffness, the less compressional energy is stored in the ball, leading to less overall energy dissipation and larger e. This phenomenon is popularly known as the "trampoline effect" (Cross, 2011; Nathan, Russell, & Smith, 2004).

Based on these general ideas, a highly-simplified theoretical model is constructed in Section 2 that describes the dependence of e on e_0 and k_0. This model is used to develop a technique to normalize e to a "standard ball" with COR e_{0S} and stiffness k_{0S}. In Section 3, an experiment is described to test this normalization technique, with the results and discussion presented in Sections 4 and 5, respectively. The technique is applied to a practical problem in Section 6. A summary and conclusions are given in Section 7.

Theoretical considerations

Two-spring model for the ball–bat collision

The starting point is a two-spring model for the ball–bat collision, Figure 1, which was previously developed by Cross as a model for the trampoline effect in the interaction of tennis balls with the racket strings (Cross, 2000). In this model, the ball and bat are each represented as masses on linear lossy springs, with force constants k_0 and k_1, respectively. We hereafter refer to k_0 as the "dynamic stiffness" of the ball. The two springs mutually compress each other, converting the initial center-of-mass (CM)

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ISSN 1934-6182 print/ISSN 1934-6190 online © 2012 Taylor & Francis
http://dx.doi.org/10.1080/19346182.2012.663533

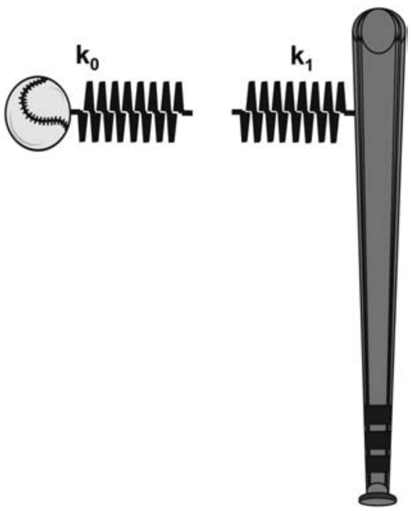

Figure 1. Simplified physical model for the ball-bat collision.

kinetic energy entirely into compressional potential energy. The fundamental equation for the energy dissipated in the collision is:

$$1 - e^2 = (1 - e_0^2)f_0 + (1 - e_1^2)f_1 \quad (1)$$

where f_0 and f_1 are the fraction of the initial CM energy stored in the ball and bat, respectively; the quantities $(1 - e_0^2)$ and $(1 - e_1^2)$ are the fraction of stored energy that is dissipated in the ball or bat; and $(1 - e^2)$ is the fraction of total CM energy that is dissipated in the collision. For linear springs, $f_0 = k_1/(k_1 + k_0)$ and $f_1 = k_0/(k_1 + k_0)$. Defining $r \equiv k_1/k_0$, which is the ratio of energy stored in the ball to that stored in the bat, Eq. 1 can be rearranged to obtain:

$$e^2 = \frac{re_0^2 + e_1^2}{1 + r} \quad (2)$$

Assuming no losses in the bat (i.e., $e_1 = 1$), a reasonable assumption for impacts near the sweet spot of the bat (Nathan, 2000), then Eq. (2) can be rewritten to obtain the result of Cross (2000):

$$e^2 = \frac{re_0^2 + 1}{1 + r} \quad (3)$$

Eq. 3 is the basis for our normalization procedure.

A plot of e vs. r is shown in Figure 2(a) for several different values of e_0. The limiting cases have simple physical interpretations. For $r \gg 1$, essentially all of the CM energy is stored in the ball, none in the bat, and e approaches e_0, the value for the ball alone, independent of r. This regime is typical of wood bats and low-performing hollow bats. In the opposite regime $r \ll 1$, very little energy is stored in the ball, so that e approaches 1 (or more generally e_1) independent of e_0. In the intermediate range, e is larger than e_0, as some of the energy that might have been stored and mostly dissipated in the ball is instead stored in the bat. For modern hollow metal or composite bats, r is generally in the range 2–15, a range in which e depends on both ball properties, e_0 and k_0.

Normalizing to a standard ball

Suppose a ball of known COR e_0 and dynamic stiffness k_0 is used to measure the ball–bat COR for a particular bat, obtaining e. Given that information, a technique is sought to predict the ball–bat COR e_S when the same bat is tested with a "standard" or normalizing ball with COR e_{0S} and dynamic stiffness k_{0S}. In the context of the two-spring model, an exact procedure can be obtained via Eq. 3. After some algebraic manipulation, our proposed normalization prescription is obtained:

$$\text{two} - \text{spring model: } e_S = \sqrt{\frac{r_S e_{0S}^2 + 1}{1 + r_S}} \quad (4)$$

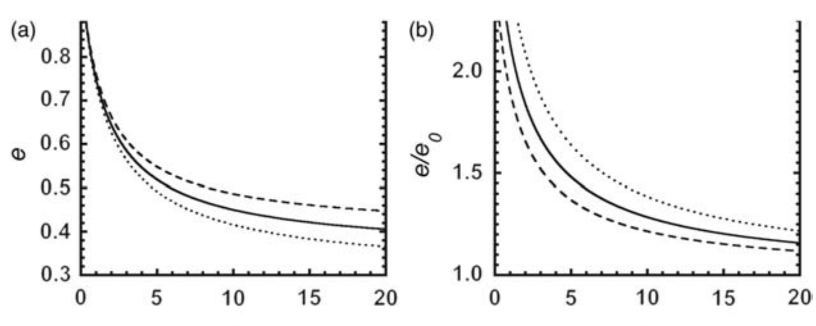

Figure 2. (a) Plot of e vs. r (Eq. 3) for three values of e_0. (b) Plot of the ratio e/e_0, commonly called the BPF, vs. r for three values of e_0, demonstrating that the BPF is not independent of either ball COR or dynamic stiffness. The dashed, solid, and dotted curves correspond to $e_0 = 0.40$, 0.35, and 0.30, respectively. For the bat and balls studied experimentally, $2.6 < r < 5.1$.

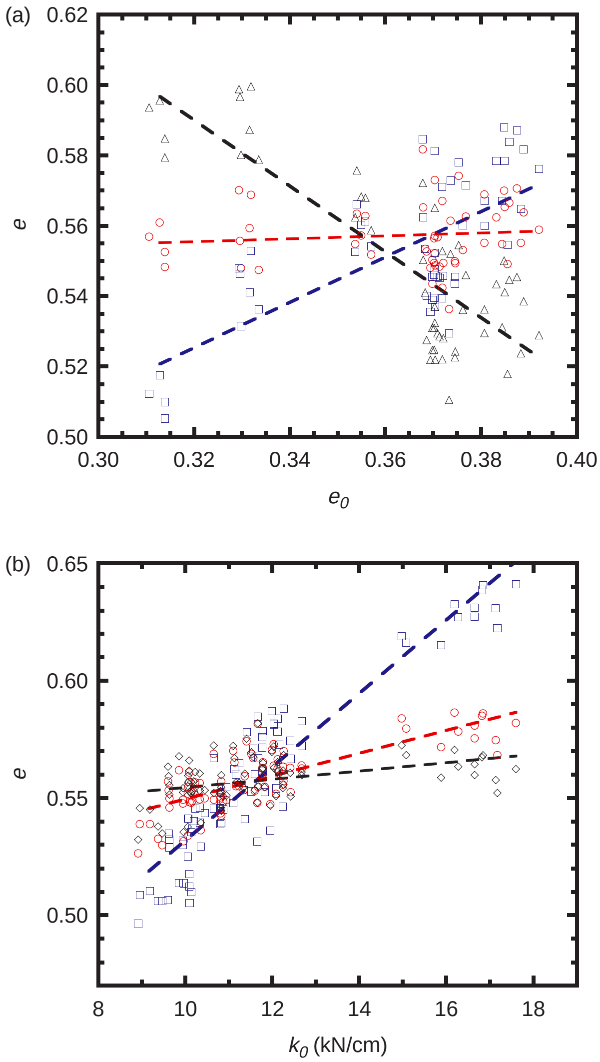

Figure 3. Results for the ball-bat COR e plotted as a function of (a) ball COR e_0 or (b) ball stiffness k_0. The blue open squares are the unnormalized values. The normalized values are shown as the red open circles (Eqs. 4–5), open black diamonds (Eqs. 7–8), or open black triangles (Eq. 6). The data in (a) are those with k_0 in the range 10.0-12.2 kN/cm. The dashed lines are linear fits to the data.

where r_S is the ratio of bat stiffness to the stiffness of the standard ball, and is given by:

$$r_S \equiv \frac{k_1}{k_{0S}} = \frac{k_0}{k_{0S}} \frac{1-e^2}{e^2-e_0^2} \tag{5}$$

A different normalization procedure (Brandt, 1997) is widely used and is based on the assumption that the ratio e/e_0, commonly known as the Bat Performance Factor or BPF, is a property of the bat alone and independent of both e_0 and k_0. The BPF normalization is given by the formula:

$$e_{S,BPF} = e\left(\frac{e_{0S}}{e_0}\right) \tag{6}$$

However, the BPF assumption is not in general consistent with the two-spring model. Indeed, a careful inspection of Eq. 4 or Figure 2(b) shows that e/e_0 is independent of e_0 and k_0 only in the limit $r \gg 1$, i.e., only for wood or low-performing hollow bats. Moreover, the BPF technique does not correct for differences in the ball dynamic stiffness.

Experiment

The bat and ball testing facility at the Sports Science Laboratory at Washington State University (Smith, 2008; Smith & Cruz, 2008) was used to study the dependence of e on the ball properties e_0 and k_0, with the specific goal of testing the normalization procedure of Eqs. 4–5. The measurements consisted of firing a softball from an air cannon at 49.2 ± 0.4 m/s (110 ± 1 mph) onto a stationary bat that is mounted horizontally and is free to pivot about a point on the handle 15 cm from the knob. The speeds of the incoming and rebounding ball are measured, from which the ball–bat COR is derived using standard formulas (Nathan, 2003). From past experience with such measurements (Smith, 2008), the ball–bat COR is determined with a root-mean-square precision of approximately 0.005. The measurements utilized 78 different standard softballs, whose COR and dynamic stiffness were determined in supplemental experiments (ASTM, 2010; Smith, Nathan, & Duris, 2010) and ranged from 0.31–0.39 and 8.9–17.5 kN/cm (5100–10,000 lb/inch), respectively.

The bat studied was a high-performing non-wood softball bat, the Louisville Slugger Catalyst of length 0.86 m (34 inch) and weight 0.75 kg (26.5 oz). As we will discuss shortly, the r values for this bat and the balls used were in the range 2.6–5.1. From Figure 2, we see that in this regime the ball–bat COR is a much stronger function of k_0 than of e_0, whereas the BPF is a strong function of both k_0 and e_0. This bat should therefore be particularly useful both for testing our procedure for normalizing to dynamic stiffness and for distinguishing the two techniques for normalizing to COR. The impact location was fixed at 16.5 cm (6.5 inch) from the barrel tip. The standard ball used to obtain normalized COR values had $e_{0S} = 0.36$ and $k_{0S} = 11.7$ kN/cm (6700 lb/inch).

Results

The results of the measurements are presented in Figure 3, where the plotted values represent individual impacts. Figure 3(a) shows the ball–bat COR values plotted versus e_0, along with linear fits. Given the strong dependence of e on k_0, the results in Figure 3(a) are shown only over the limited range of dynamic stiffness 10.0-12.2 kN/cm. The two-spring

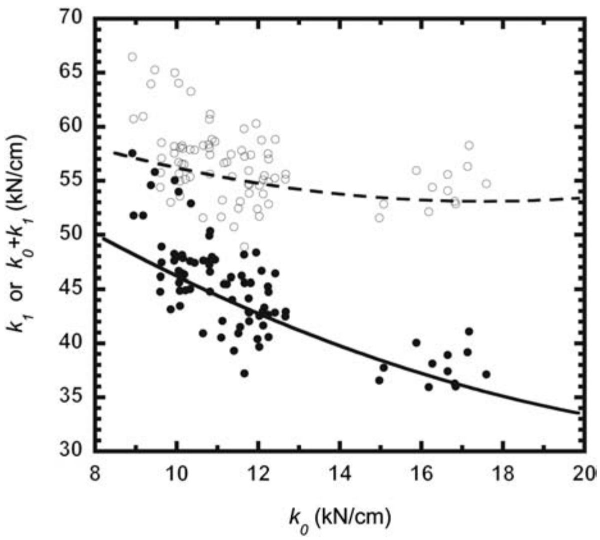

Figure 4. Bat stiffness k_1 (closed points) as a function of ball stiffness k_0, as derived from the data and Eq. 5; the solid curve is from the corresponding finite element simulation. The open points are the sum $k_0 + k_1$; the dashed curve is from the corresponding finite element simulation. Both the data and the simulation show that the bat stiffness decreases with increasing k_0 but the sum of bat and ball stiffness is approximately constant.

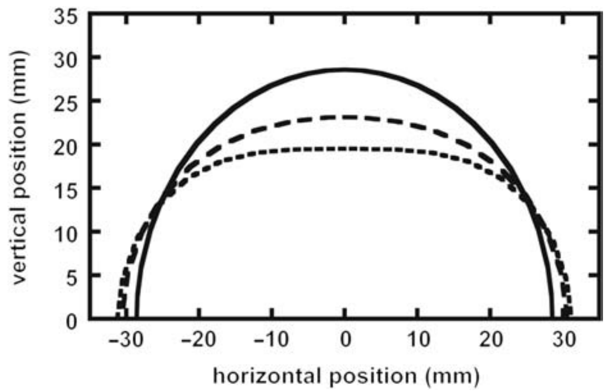

Figure 5. Comparison of the bat cross-sectional profile for simulated impacts with a ball of high stiffness (dotted curve) and low stiffness (dashed curve). The solid curve is the bat undeformed profile.

normalization prescription, Eqs. 4–5, removes essentially all the dependence on e_0, reducing the slope of the linear fit by a factor of fifteen. On the other hand, the BPF normalization technique, Eq. 6, overcorrects for e_0, resulting in a slope larger in magnitude and opposite in sign compared with the uncorrected data. Figure 3(b) shows all 78 COR values plotted versus k_0 and demonstrates that e has an approximately linear dependence on k_0 with a slope of 0.0157 cm/kN. The slope is reduced by a factor of three by the two-spring normalization. Ideally, the normalized slope would be zero, so there is some additional dependence of e on k_0 that is not accounted for by the two-spring model. Using Eq. 5, the bat stiffness is estimated to be approximately 45 kN/m (26,000 lb/inch), so that r falls in the range 2.6–5.1. That the BPF technique works so poorly can be easily understood from Figure 2(b), given the range of r. Indeed, the experimental BPF values are far from constant, ranging from 1.4–1.8 for the 78 impacts.

Discussion

While the two-spring normalization technique substantially reduces the dependence of e_S on k_0 and e_0, a residual dependence of e_S on k_0 was nevertheless observed. A possible key to understanding the origin of this residual dependence may be found in Figure 4, which shows that the bat stiffness k_1 (Eq. 5) decreases with increasing k_0, while in the two-spring model k_1 is assumed to be independent of k_0. To study the limitations of the two-spring model, the bat–ball impact was modeled numerically using finite elements. The model was constrained to simulate the bat performance test described in Section 3. The bat was modeled using a linear elastic material, where the modulus was adjusted to achieve the desired performance level (i.e., k_1). The ball was modeled using a linear viscoelastic material, with the viscoelastic properties adjusted to reproduce approximately the values of e_0 and k_0 used in the experiment (Smith & Duris, 2009). Simulated impacts between these balls and bats of differing stiffness were consistent with the predictions of the two-spring model (Figure 2). Namely, when r is small, e is more sensitive to k_0 than to e_0; when r is large, e is more sensitive to e_0 than to k_0. Details of the finite element analysis are given in the thesis of Faber (2010).

From the simulated impact, the bat stiffness could be found from the ratio of the peak contact force to the corresponding deformation of the bat profile. These values are plotted as a curve in Figure 4 and show a similar decrease in bat stiffness with increasing k_0 to the experimental data. Upon closer inspection, the simulations reveal the origin of this behavior, as illustrated in Figure 5, which shows the bat deformed shape with a high-stiffness ball and low-stiffness ball. The simulations show that the high-stiffness ball is better able to maintain its spherical shape upon impact with the bat, while the bat exhibits noticeable local deformation in the form of a flat region at the center of the contact region. The net result is a smaller effective contact area and a correspondingly larger bat deformation for a given impact force, resulting in a smaller k_1. Qualitatively, the effect of nonconstant k_1 on the normalization procedure is understood as follows. As k_1 decreases with increasing k_0, the bat-to-ball stiffness ratio r decreases more rapidly with increasing k_0 than it otherwise would, so that the technique described in

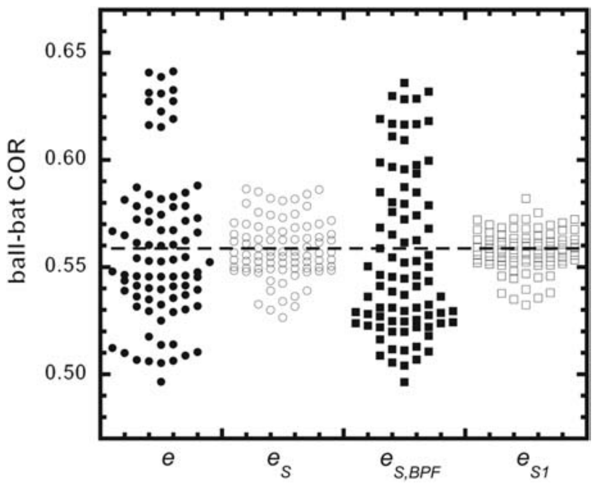

Figure 6. Scatter plot of all the data showing the distribution of ball–bat COR un-normalized values and various normalized values. Closely spaced points are displaced horizontally for clarity.

Section 2.2 will under-correct for ball stiffness, exactly as the data show. The simulations further show that this effect is larger for high-performance than for low-performance bats.

An interesting feature of Figure 4 is that both the data and the simulations show that the sum of ball and bat stiffness, $k_0 + k_1$, is nearly independent of k_0 over the region 10–18 kN/cm. This result can be used to improve the effectiveness of the normalization method by providing a prescription for finding the stiffness of the bat when impacted by the standard ball:

$$e_{S1} = \sqrt{\frac{r_{S1} e_{0S}^2 + 1}{1 + r_{S1}}} \tag{7}$$

with the modified stiffness ratio:

$$r_{S1} = r_S + \frac{k_0 - k_{0S}}{k_{0S}} \tag{8}$$

where r_S is given by Eq. 5. Figure 3(b) shows the values of e_{S1} obtained in this manner. The slope of e_{S1} versus k_0 is reduced by a factor of nine relative to the uncorrected values and a factor of three relative to e_S.

It is perhaps surprising that both the data and simulation show a ball stiffness independent of the bat stiffness; i.e. the different bat profiles described in Figure 5 might also affect the ball stiffness. A possible explanation comes from the simulations themselves. During impact, bat barrels with high stiffness will tend to better hold their round cross-section, resulting in higher deformation in the ball, both in magnitude and rate. In contrast to the bat's linear-elastic response, the ball's time-dependent nature causes increased material stiffness as the strain rate increases. Thus, a barrel with high stiffness impacting a ball may result in lower ball stiffness (due primarily to increased ball deformation from the bat retaining its cylindrical shape) or higher ball stiffness (due primarily to increased deformation rate). These competing effects seem to offset each other, resulting in no net change in ball stiffness within the accuracy of the numerical model (Faber, 2010).

All of the results are presented as scatter plots in Figure 6. This plot demonstrates clearly the spread of uncorrected values (rms = 0.036), along with the improvement obtained by normalizing using e_S (rms = 0.014), the further improvement using e_{S1} (rms = 0.010), and the poor results using the BPF technique (rms = 0.038).

A practical application

While the experiment investigated a high-performance bat, a practical application of the normalization technique applies to low-performance bats, such as those mandated by the NCAA (2009). Low performance means bats with very little trampoline effect, which in the two-spring model means that $r \gg 1$. Under such conditions, e should be insensitive to the precise value of r so that the following approximate normalizing expression can be derived:

$$r \gg 1 \text{approximation: } e_S \approx \sqrt{e^2 + e_{0S}^2 - e_0^2} \tag{9}$$

As a numerical example, consider a ball with e_0 in the range 0.45–0.49, with a stiffness ratio $r_S = 25.8$ and a normalizing COR $e_{0S} = 0.47$. Over the range of e_0, the un-normalized COR varies from 0.481 to 0.518. Using the exact expression Eq. 4, e normalized to a constant value of 0.500 independent of e_0. Using the approximation expression Eq. 9, the normalized value ranges from 0.502 to 0.498, a factor of ten reduction relative to the un-normalized values. Thus, the approximation works very well.

Summary and conclusion

We have presented a model of the ball–bat collision that explicitly demonstrates the dependence of the ball–bat COR e on the COR e_0 and dynamic stiffness k_0 of the ball. We have used this model to develop a technique for normalizing e to properties of a standard ball. We have tested the model with a high-performance softball bat. We have shown that the dependence of e on e_0 is removed by the normalization. We have also shown experimentally that the ratio e/e_0, known as the BPF, depends on both e_0 and k_0, as predicted by the two-spring model. Therefore, it is not surprising that the BPF method fails to normalize performance for the bat tested. The data show that there is a strong nearly linear dependence of e on k_0 and have shown that the normalization technique, while not perfect, reduces that dependence by about a factor of three. We have shown through finite element simulations that the residual dependence of e_S on k_0 is due to a dependence of k_1 on k_0. The simulations lead to an improved normalization technique that reduces the dependence on k_0 by factor of nine. Finally we have derived an approximate normalization expression that is valid for low-performing bats.

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