What Is The Hawkeye Spin Data Teaching Us?
II: Lift and Side Forces

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I. INTRODUCTION

This article is the second in a series in which I discuss the new things we are learning from the Hawkeye pitch-tracking system. In the first article,[1] hereafter referred to as I, the formalism was developed for using the constant acceleration parametrization of the trajectory (9P) along with the Cartesian spin components, ω_x, ω_y, and ω_z, to separate the transverse acceleration (and therefore the movement) into Magnus and non-Magnus components. To achieve this, one first finds the transverse acceleration, which is the component of the acceleration (with gravity removed) in the direction perpendicular to the velocity and which determines the movement. One expresses the transverse acceleration as the vector sum of Magnus and non-Magnus components. The essential idea is that if the Magnus component is known, then the non-Magnus component can be determined. The drawback of this formalism is that the separation can only be done if the magnitude of the Magnus component is known.

In this installment, the formalism is developed further based on an idea from Glenn Healey.[2] Recall that the transverse acceleration is given by[1]

$$\vec{a}_T = \vec{a} - \vec{g} + [(\vec{a} - \vec{g}) \cdot \langle\hat{v}\rangle] \langle\hat{v}\rangle \,. \tag{1}$$

Whereas the previous formalism separated the transverse acceleration (T) into Magnus (M) and non-Magnus (N) components,

$$\vec{a}_T = \vec{a}_M + \vec{a}_N \,, \tag{2}$$

the new formalism separates T into "lift" (L) and "side" (S) components,

$$\vec{a}_T = \vec{a}_L + \vec{a}_S \,, \tag{3}$$

with $\vec{a}_L$ and $\vec{a}_S$ mutually perpendicular (i.e., $\vec{a}_L \cdot \vec{a}_S$=0) . The lift component is in same direction as the Magnus force, i.e., in the $(\hat{\omega} \times \hat{v})$ direction, whereas the side component is in the $\pm\hat{v} \times (\hat{\omega} \times \hat{v})$ direction. As we shall see in the next section, this method of separation has the advantage that it can be done exactly, using the spin components but without previous knowledge of the lift coefficient. But that advantage comes at a cost, which will also be discussed.

II. SEPARATING INTO LIFT AND SIDE COMPONENTS

The relationship among the three vectors in Eq. 3 is shown in Fig. 1, in the plane perpendicular to the mean velocity vector. Since the magnitude and direction of the transverse acceleration (T) is known, together with the direction of the lift vector (L), the entire right triangle is completely determined:

$$\vec{a}_L = |\vec{a}_T| \cos\theta_{TL} \left[\frac{\hat{\omega} \times \hat{v}}{|\hat{\omega} \times \hat{v}|} \right] = |\vec{a}_T| \cos\theta_{TL} \left[\frac{\hat{\omega} \times \hat{v}}{|\sin\theta|} \right], \tag{4}$$

and

$$\vec{a}_S = |\vec{a}_T| \sin\theta_{TL} \left[\frac{\pm\hat{v} \times (\hat{\omega} \times \hat{v})}{|\hat{v} \times (\hat{\omega} \times \hat{v})\,|} \right] = |\vec{a}_T| \sin\theta_{TL} \left[\frac{\pm\hat{v} \times (\hat{\omega} \times \hat{v})}{|\sin\theta|} \right], \tag{5}$$

where $\cos\theta_{TL} = \hat{a}_T \cdot \hat{a}_L$, the terms in brackets are the direction unit vectors, $|\sin\theta|$ is the spin efficiency, and θ is the angle between the spin and velocity vectors. In the limit of unit spin efficiency ($\hat{\omega} \cdot \hat{v}$=0 and $|\sin\theta|$=1), the direction of $\vec{a}_S$ reduces to $\pm\hat{\omega}$. The sign in Eq. 5 is the same as the sign of $\hat{y} \cdot (\hat{a}_T \times \hat{a}_L)$.

It is useful to express the spin vector in spherical coordinates as follows:

$$\omega_x = \omega \sin\Theta \cos\Phi \quad \omega_y = \omega \cos\Theta \quad \omega_z = \omega \sin\Theta \sin\Phi\,, \tag{6}$$

where the polar angle Θ is the angle of the spin with respect to the y axis and the azimuthal angle Φ is the angle of the projection of the spin in the xz plane with respect to the x axis. While the total spin rate ω and the azimuthal angle Φ are publicly available, the polar angle Θ is not. For many pitches, ω_y is small so that Θ is close to 90○.

As before, the transverse acceleration determines the movement:

$$M_{T,x} = \frac{1}{2} a_{T,x} t^2 \quad M_{T,z} = \frac{1}{2} a_{T,z} t^2 \quad M_T = \sqrt{M_{T,x}^2 + M_{T,z}^2} \quad \phi_T = \arctan\left(\frac{M_{T,z}}{M_{T,x}}\right), \tag{7}$$

where t is the flight time from release to the front of home plate, M_T is the total movement, and ϕ_T is the direction of the movement in the xz plane. Similar equations hold separately for the movement due to the lift and side forces; in Eq. 7, simply replace T by either L or S. As a result, Eqs. 4–7 are completely sufficient for determining the lift and side contributions to the x and z movements.

However for a pitched baseball, it is always true that the ball travels primarily in the -y direction, so that $|v_y| \gg |v_x|, |v_z|$. Under these circumstances, the equations can be manipulated to find the following very simple expressions for the lift and side movements:

$$M_{L,x} \approx M_T \cos(\phi_T - \phi_L) \cos\phi_L \quad M_{L,z} \approx M_T \cos(\phi_T - \phi_L) \sin\phi_L \,, \tag{8}$$

and

$$M_{S,x} \approx \pm M_T \sin(\phi_T - \phi_L) \sin\phi_L \quad M_{S,z} \approx \mp M_T \sin(\phi_T - \phi_L) \cos\phi_L \,, \tag{9}$$

where ϕ_L is the direction of the Magnus force in the xz plane:

$$\phi_L = \arctan\left(\frac{\omega_x v_y - \omega_y v_x}{\omega_y v_z - \omega_z v_y}\right) . \tag{10}$$

In Eq. 9, the upper/lower sign applies if $\hat{y} \cdot (\hat{a}_T \times \hat{a}_L)$ is postive/negative. It is very simple to confirm that

$$M_{T,x} = M_{L,x} + M_{S,x} \quad M_{T,z} = M_{L,z} + M_{S,z} \,, \tag{11}$$

as must be the case. Using a broad range of simulated pitches representative of those thrown by MLB pitchers, I have confirmed that the movements calculated from Eqs. 8–9 differ from the exact movement by no more than ± 0.1 inches.

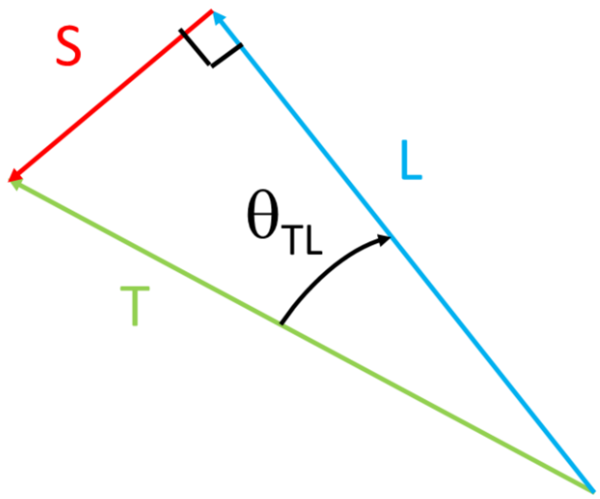

FIG. 1: Separation of the transverse acceleration T into mutually perpendicular lift (L) and side (S) components in the plane perpendicular to the mean velocity vector, where θ_{TL} is the angle between the transverse and lift vectors.

III. APPROXIMATIONS

The only term in Eqs. 8–9 that depends on knowledge of the spin components is the Magnus movement direction ϕ_L, through Eq. 10. An appealing approximation is to neglect the terms involving v_x and v_z in Eq. 10, resulting in

$$\phi_L \approx -\arctan\left(\omega_x/\omega_z\right), \tag{12}$$

which is identical to Φ in Eq. 6 and publicly available, allowing calculations of the lift and side contributions to the movement without full knowledge of the spin components. In effect, this approximation is equivalent to setting Θ=90^circ. In fact, this very approximation was used in the analysis of Smith, *et al.*[3] to demonstrate that the direction of the actual movement (ϕ_T) does not always coincide with the direction of movement inferred from the Magnus force (ϕ_L). Simulations similar to those discussed above show that for spin efficiencies exceeding ∼0.7, this approximation can lead to discrepancies in movement of up to $\sim \pm 1$ inch. For smaller spin efficiencies, the discrepancies can be much larger. The bottom line is that, while the approximation is useful and appealing, it should be used with caution.

An alternative procedure for obtaining the separation of the movement into lift and side components using only publicly available data is that used by Healey.[2] As a reminder, those data include the 9P parametrization of the trajectory, the total spin ω, and the azimuthal spin angle Φ. The argument involves two key assumptions:

- The lift component is entirely due to Magnus
- The magnitude of the Magnus component is known exactly from ω and the spin efficiency[4]

In effect, for a given pitch there is a unique spin polar angle Θ such that the movement due to the lift is equal to that expected from Magnus. Having obtained that angle, the spin vector is completely determined and the separation can be done exactly. It is an innovative technique. It will be interesting to see how well other quantities that are derived from this process (e.g., the spin efficiency) compare with those obtained directly from the spin vector.

IV. LIFT, SIDE, AND MAGNUS COEFFICIENTS

Although not necessary, it is sometimes convenient to characterize the magnitude of the transverse, lift, and side forces by transverse, lift and, side coefficients, respectively:

$$C_T = \frac{|\vec{a}_T|}{K\langle v^2\rangle} \quad C_L = C_T \cos\theta_{TL} \quad C_S = C_T \sin\theta_{TL}\,, \tag{13}$$

where

$$C_T = \sqrt{C_L^2 + C_S^2}\,. \tag{14}$$

The factor K is given by

$$K = \frac{1}{2}\frac{\rho A}{m}\,, \tag{15}$$

where m and A are the mass and cross sectional area of the ball, respectively, and ρ is the density of the air. Both C_L and C_S can be determined from the preceding formulas. However, it is important to keep in mind that C_L need not be identical to the corresponding coefficient for the Magnus force, the force that depends on the spin. To avoid confusion, I will refer to the Magnus lift coefficient as C_{LM}, which is the quantity that is related to the spin factor $S = R\omega/v$ and measured in the laboratory experiments shown in Fig. 1 of I. The difference $C_L - C_{LM} \equiv C_{LN}$ is a measure of the contribution of non-Magnus forces to the lift. So while it is true that all of C_S is due to non-Magnus forces, some of C_L may also be due to non-Magnus forces. Without independent knowledge of C_{LM}, we simply do not know.

V. AN EXAMPLE USING PSEUDO-DATA

An example of how to apply this formalism to real analysis is given in Fig. 2. The example utilizes the publicly available data for Alex Cobb's sinker, the mean parameters of which are given in Table I. The calculations utilize the approximation in Eq. 12. With those assumptions, the preceding formulas are used to calculate the total, lift, and side contributions to the movement. The pitch-by-pitch results are shown in the figure. Note that in this example, the net effect of the side force is to increase the total movement by $\sim 6\%$ and to shift the direction by $\sim 18°$ toward more arm-side and less upward. The side force is perpendicular to the lift force.

There is a curious feature in the data that deserves some explanation. It appears that the side contribution to the movement (red points) lies in a very narrow angular band compared to that of the lift and total movement. That is simply an illusion due to the fact that the magnitude of the side movement is small. In fact, one can see from the plot that the spread of red points gets larger as the magnitude of the side lift gets larger. Indeed, the data show that the standard deviation of the side movement angle is essentially identical to that of the lift movement angle.

TABLE I: Mean parameters of Alex Cobb's sinker.

ω (rpm)	S	ϕ_T (deg)	ϕ_L (deg)	ϕ_S (deg)	MT (inch)	ML (inch)	MS (inch)
2071	0.195	141.6	123.8	213.7	20.8	19.7	6.3

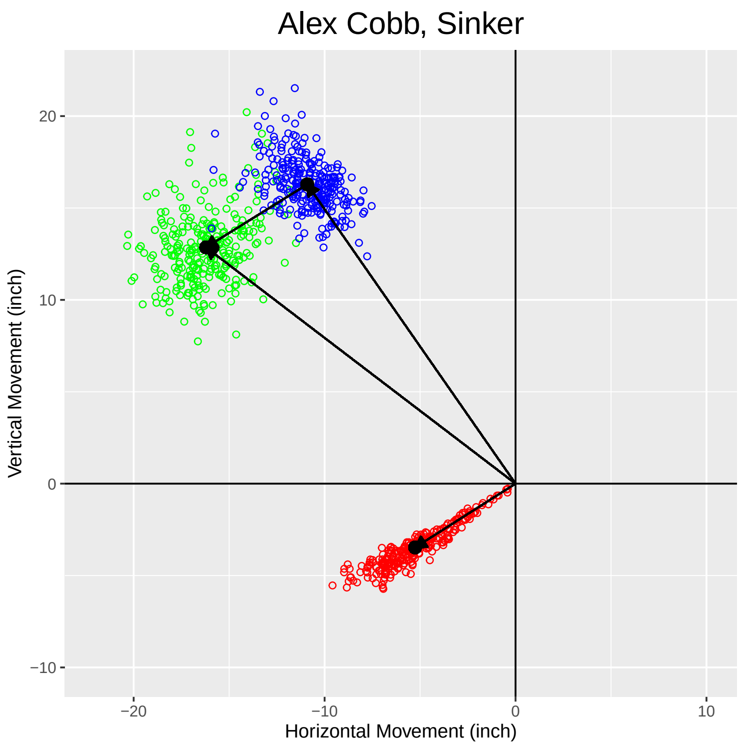

FIG. 2: Movement in the xz plane, shown from the catcher perspective, for Alex Cobb's sinker. The green, blue, and red clusters are the total, lift, and side movements, respectively, while the black dot is the mean value of each cluster. The arrows are lines connecting the mean values and show the vector nature of their mutual relationship.

VI. SUMMARY AND OPEN QUESTIONS

The revised formalism discussed here allows us to do a unique separation of the pitch movement into components parallel to (lift) and perpendicular to (side) the direction of the Magnus force, independent of the size of C_{LM}. While approximation methods exist, to do a proper separation requires knowledge of the full 3D spin components.

One important open question is the separation of the lift component into Magnus and non-Magnus components. Better information on the relationship between the Magnus lift coefficient and spin (both magnitude and direction) will allow us to do that separation. It is hoped that such information will soon be forthcoming from careful analysis of Statcast data.

Another open question is the relationship between the non-Magnus movement, whether side or lift, and seam orientation. In principle, the same Hawkeye cameras that measure spin rate and spin axis should be able to provide information on seam orientation. Our ability to address this question must await the release of such data.

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[1] Alan M. Nathan, What Is The Hawkeye Spin Data Teaching Us? I: Determining Magnus and non-Magnus Movement, January 5, 2020 (unpublished).

[2] G. Healey and L. Wang. Analyzing the Side Force on a BaseballUsing Hawk-Eye Measurements, January 13, 2020 (unpublished).

[3] Barton Smith, Alan Nathan, and Harry Pavlidis, Not Just About Magnus Anymore, Baseball Prospectus, November 5, 2020.

[4] G. Healey and L. Wang. Combining Radar, Weather, and Optical Measurements to Model the Dependence of Baseball Lift on Spin and Surface Roughness, August 18, 2020 (unpublished).