Simplified Models for the Drag Coefficient of a Pitched Baseball

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The classic experiment to measure the drag coefficient involves dropping coffee filters.[1] Wouldn't it be more fun to try something different? In fact, an experiment on the drag force is conducted nearly 4000 times a day during the baseball season and you have free access to this PITCHf/x data![2]

A one-dimension model

Let's begin by making the simplest approximation that during the short time of the pitch the ball moves along a horizontal path. So, we can ignore gravity for now. The only force on the ball is the drag, as shown in Fig. 1. Using Newton's second law,

$$F_D = ma_y, \tag{1}$$

where m is the mass of the ball and a_y is its acceleration. We are using the standard PITCHf/x coordinates where the direction from the catcher toward the pitcher is the y-axis, the vertical axis is z, and the x-axis is to the catcher's right. The drag force is written,

$$F_D = \tfrac{1}{2} C_D A \rho v_y^2, \tag{2}$$

where ρ is the density of the air, v_y is the speed of the ball, A is the cross-sectional area, and C_D is the drag coefficient[3] we want to measure. By equating Eq. (1) and Eq. (2), you can see that the acceleration is proportional to the square of the speed. Since the acceleration is opposite to the velocity, the ball will slow down at a non-constant rate. Substituting into the second law and solving for the drag coefficient,

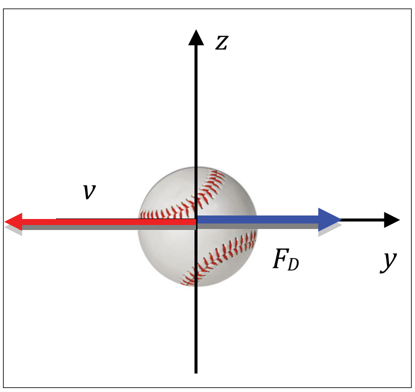

Fig. 1. The horizontal force on a ball in one-dimensional flight is just the drag F_D.

$$C_D = \delta \frac{a_y}{v_y^2}, \tag{3}$$

where we have defined the quantity

$$\delta \equiv \frac{2m}{\rho A}. \tag{4}$$

We will call this quantity the "drag length" because, when divided by the dimensionless drag coefficient, it is the distance an object would have to travel before the velocity falls by a factor of e.

The drag length

Let's call a timeout to discuss some interesting and relevant features of the drag length. The rules of the game[4] state, "The ball shall be a sphere formed by yarn wound around a small core of cork, rubber or similar material, covered with two strips of white horsehide or cowhide, tightly stitched together. It shall weigh not less than five nor more than 5-1/4 ounces avoirdupois and measure not less than nine nor more than 9-1/4 inches in circumference." The median values[5] of the weight and circumference are $W = 0.320$ lb, corresponding to $m = 0.145$ kg, and $c = 0.760$ ft (23.2 cm).

It looks like we'll also need to convert the drag length into English units by multiplying top and bottom by the acceleration due to gravity, g.

$$\delta \equiv \frac{2mg}{\rho g A}. \tag{5}$$

Since the weight is just mg, we can make that substitution. The density in English units is in force per unit volume as opposed to the mass per unit volume of SI units. So, we'll need to substitute $\rho_E = \rho g$. Finally, in terms of the weight and circumference the drag length becomes

$$\delta = \frac{8\pi W}{\rho_E c^2}. \tag{6}$$

Now it gets a bit tricky. The density of air varies[6] quite a bit with temperature,[7] humidity,[8] air pressure,[9] and altitude.[10] To get a sense, let's assume a standard atmosphere. The warmest day during the baseball season in 2013 at Coors Field in Denver was 95 °F and 35% humidity at 5184 ft above sea level. The resulting air density ρ_E is about 0.058 lb/ft^3. Compare that with the coldest day at AT&T Park in San Francisco at 48 °F and 35% humidity at 63 ft above sea level, giving a density of 0.077 lb/ft^3. The depth to which you take your students into these issues depends upon your personal taste and their interest level.

We will use the numerical values from a certain magical night in San Diego, July 13, 2013. The game time temperature was 70 °F and it remained constant throughout the evening. Petco Park is 13 ft above sea level. The average humidity that day was 69% and the air pressure was 29.94 in. The resulting value for the air density was $\rho_E = 0.0747$ lb/ft^3. Plugging the values into Eq. (5) gives $\delta = 186$ ft.

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Calculating the drag coefficient

Since 2007, Major League Baseball (MLB) has measured the trajectory of every pitch to within about one inch. While the PITCHf/x system tracks the trajectory, it only reports the three components of the initial position vector at 50 ft from home plate, the three components of the initial velocity vector at 50 ft, and the three components of an average acceleration vector over the entire flight. These nine parameters are found by least squares fitting so they reconstruct the entire trajectory with minimum error. The data are available online[2] from the PITCHf/x server. They are also available in an easy-to-read format for selected pitches,[11] including the pitch to be discussed here—the last pitch of Tim Lincecum's no-hitter July 13, 2013. The initial position, initial velocity, and acceleration vectors are:

$$\begin{aligned}\mathbf{r}_0 &= (-1.566\text{ ft})\hat{i} + (50.00\text{ ft})\hat{j} + (5.780\text{ft})\hat{k}\\ \mathbf{v}_0 &= (2.631\tfrac{\text{ft}}{\text{s}})\hat{i} + (-122.644\tfrac{\text{ft}}{\text{s}})\hat{j} + (-3.435\tfrac{\text{ft}}{\text{s}})\hat{k}\\ \mathbf{a} &= (-6.387\tfrac{\text{ft}}{\text{s}^2})\hat{i} + (25.067\tfrac{\text{ft}}{\text{s}^2})\hat{j} + (-21.81\tfrac{\text{ft}}{\text{s}^2})\hat{k}.\end{aligned} \tag{7}$$

Looking back at Eq. (3), we need the y-components of the acceleration and velocity. The simplest thing to do to get a first approximation is use the y-component of the initial velocity. The result is $C_D = 0.310$.

For some students, this is a good place to stop. However, we might be able to do a little better. Since the acceleration from PITCHf/x is an average (in the least squares sense), a better approximation is likely to be obtained using the average velocity instead of the initial velocity. Using the data above to estimate the average y-component of the velocity (under the assumption of constant acceleration from y = 50.00 ft to the front of home plate at y = 1.417 ft) gives $\langle v_y \rangle$ = –117.46 ft/s.

Some students would be surprised to learn that the ball slows appreciably during the flight of the pitch. In this case, the speed drops from about 84 mph to 77 mph. This confusion might be due to the fact that the quoted pitch speed is a single value. It is always the higher initial speed. Since we are trying to find the drag coefficient, it must be slowing as it moves. Anyway, using the average speed gives a drag coefficient of $C_D = 0.338$, which is nearly 10% higher than the first approximation.

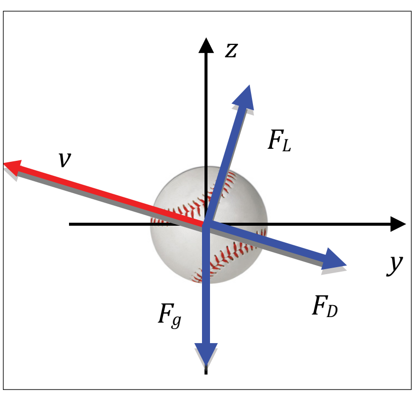

Fig. 2. The forces on a ball in flight are the drag, F_g, the lift, F_L, and the weight, F_g.

A three-dimension model

If your students are up to it, a still better approximation can be found by considering a full three-dimensional treatment. All the forces on a pitched baseball[12] are shown in Fig. 2. The drag force, F_D, is opposite the velocity, while the lift force due to the spin and the Magnus effect, F_L, is perpendicular to the velocity. There is also gravity, F_g. Using Newton's second law,

$$\mathbf{F}_L + \mathbf{F}_D + \mathbf{F}_g = m\mathbf{a}, \tag{8}$$

where m is the mass of the ball and $\mathbf{a}$ is its acceleration. Taking the dot product with the velocity vector, $\mathbf{v}$, gives

$$\mathbf{F}_L \cdot \mathbf{v} + \mathbf{F}_D \cdot \mathbf{v} + \mathbf{F}_g \cdot \mathbf{v} = m\mathbf{a} \cdot \mathbf{v}. \tag{9}$$

Since the lift is always perpendicular to the velocity, that term is zero. The drag is always anti-parallel so the dot product results in a minus sign. Writing the weight in terms of the gravitational acceleration gives

$$-F_D v - mgv_z = m\mathbf{a} \cdot \mathbf{v}, \tag{10}$$

where v_z is the z-component of the velocity vector. Solving for the drag,

$$F_D = -\frac{m}{v}\left(gv_z + \mathbf{a} \cdot \mathbf{v}\right). \tag{11}$$

Using the expression for the drag force from Eq. (2), as well as the definition of δ from Eq. (4), we can write the drag coefficient as

$$C_D = -\delta \frac{gv_z + \mathbf{a} \cdot \mathbf{v}}{v^3}. \tag{12}$$

Calculating the remaining components of the average velocity yields $\langle v_x \rangle$ = 1.320 ft/s and $\langle v_z \rangle$ = –7.945 ft/s. Continuing with the computation,

$$\begin{aligned}\langle v \rangle &= \sqrt{\langle v_x \rangle^2 + \langle v_y \rangle^2 + \langle v_z \rangle^2} = 117.74\tfrac{\text{ft}}{\text{s}},\\ g\langle v_z \rangle &= -225.64\tfrac{\text{ft}^2}{\text{s}^3}, \quad \text{and}\\ \mathbf{a} \cdot \langle \mathbf{v} \rangle &= -2779.4\tfrac{\text{ft}^2}{\text{s}^3},\end{aligned} \tag{13}$$

finally yielding the result[13] $C_D = 0.346$, about 3% above the last value from the one-dimension model.

Commentary

While there is a 10% difference between using the initial velocity compared with the average velocity, there is only a 3% difference between the one-dimensional and the three-dimensional models. This is not surprising given that the y-component of the velocity dominates the three-dimensional calculation. The question as to whether the values are correct is a bit more challenging.

The drag coefficient for a smooth sphere is known to be about 0.5 in this velocity range. It is also known that increasing the roughness of the surface actually reduces the drag. For example, for golf balls the drag coefficient can be as low as 0.25. Given that the surface of a ball is not exactly smooth and it has seams, our result seems reasonable.

Our values agree with Adair.[14] Variations in the drag coefficient for nearly 8000 pitches thrown at the same venue over a range of weather conditions including wind give values from 0.28 to 0.58.[15] So, we can certainly say that our values are consistent with what is known about the drag coefficients of pitched baseballs.

Now you can use your coffee filters for more productive activities—like brewing some go-juice!

References

1. For example, http://www.vernier.com/files/sample_labs/PWV-13-COMP-air_resistance.pdf.
2. For instructions to get to and use the PITCHf/x data set, see D. T. Kagan, "The anatomy of a pitch: Doing physics with PITCHf/x data," *Phys. Teach.* **47** (7), 412 (Oct. 2009).
3. The drag coefficient is known to be a function of the speed. Since most professionally pitched baseballs are thrown within the narrow range of 70 to 100 mph, we'll neglect this complication here. See http://www.grc.nasa.gov/WWW/k-12/airplane/balldrag.html.
4. *Official Baseball Rules* (2013 ed.), Rule 1.09.
5. No apologies! The traditions of the national pastime give it an exemption from the constraints of the SI system. If the units bother you, then the fact that the PITCHf/x system has a serious significant figure problem will send you into orbit.
6. An online calculator that assumes standard pressure can be found at http://www.denysschen.com/catalogue/density.aspx. A formula that omits elevation can be found at "Density of Moist Air," in *CRC Handbook of Chemistry and Physics,* 63rd ed., edited by David R. Lide (CRC Press, Boca Raton, FL, 2005). There is a spreadsheet available at http://baseball.physics.illinois.edu/trajectory-calculator.html that will calculate the density of air and use it to predict the trajectory of a pitched or hit ball.
7. Game time temperature can be found in the box score at http://MLB.com.
8. The local humidity can be found at http://www.wunderground.com/history/ as can other historical weather data.
9. Historical pressure data can be found at http://trexwww.ucc.nau.edu/cgi-bin/daily_average.pl?user_param=64101.
10. The ball park elevations can be found at http://baseballjudgments.tripod.com/id62.html.
11. See http://phys.csuchico.edu/baseball/POBActivities/pitchfx/.
12. For a complete and thorough discussion of the forces on a baseball in flight, see A. M. Nathan, "The effect of spin on the flight of a baseball," *Am. J. Phys.* **76**, 119–124 (2008).
13. Comparing calculated versus measured values for the drag coefficient can be used as a test of the PITCHf/x system. See http://baseball.physics.illinois.edu/LiftDrag-1.pdf.
14. R. K. Adair, *The Physics of Baseball,* 3rd ed. (Harper & Row, Publishers Inc., New York, 2002), p. 8, Fig. 2.1.
15. See the graph in the link of Ref. 13.

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