

The sweet spot of a baseball bat

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The sweet spot of a baseball bat, like that of a tennis racket, can be defined either in terms of a vibration node or a centre of percussion. In order to determine how each of the sweet spots influences the “feel” of the bat, measurements were made of the impact forces transmitted to the hands. Measurements of the bat velocity, and results for a freely suspended bat, were also obtained in order to assist in the interpretation of the force waveforms. The results show that both sweet spots contribute to the formation of a sweet spot zone where the impact forces on the hands are minimised. The free bat results are also of interest since they provided particularly elegant examples of wave excitation and propagation, suitable for a student demonstration or experiment. © 1998 American Association of Physics Teachers.

I. INTRODUCTION

Given the popularity of baseball,¹ and the natural curiosity of physicists to understand collision phenomena, it is surprising that very few measurements have ever been made on the behaviour of a baseball bat when it impacts with a ball. In this paper, measurements are presented on (a) the bat velocity and (b) the impact forces transmitted to the hands, as a result of the collision between a baseball bat and ball. The primary objective was to examine the nature of the sweet spot on a baseball bat. The sweet spot is an impact point, or a narrow impact zone, where the shock of the impact, felt by the hands, is reduced to such an extent that the batter is almost unaware of the collision. At other impact points, the impact may be felt as a painful sting or jarring of the hands, particularly if the impact occurs at a point well removed from the sweet spot. It has been proposed² that two such spots exist, one corresponding to a vibration node and the other corresponding to the centre of percussion. However, it appears that no direct measurements have previously been made to determine which of the two spots is the more significant or whether the sweet spot is perhaps an impact zone encompassing both sweet spots. The two proposed sweet spots are typically only a few centimeters apart, so it is not an easy task to separately identify the two spots in terms of differences in the qualitative feel of the impact.³

The vibration modes of a baseball bat are easily determined,^{3–5} at least for a freely suspended bat (e.g., one that is suspended by a light string). The fundamental mode of a wood bat has a node located typically about 17 cm from each end and a frequency typically of about 170 Hz. The next mode has a frequency of about 530 Hz, with a node located about 5 cm from the centre of the bat toward the handle, another node about 13 cm from the barrel end, and a third node about 7 cm from the handle end. The second mode is easily excited since the impact duration is about 1.5 ms, roughly matching the half-period of the second mode. The frequency spectrum of the impulse (a half-sine waveform) peaks at zero frequency and extends beyond the frequency of the second mode. However, higher frequency modes are excited with much smaller amplitude than the first and second modes. An impact at the node of the fundamental mode will excite the second mode, but not the fundamental mode. Similarly, an impact at the node of the second mode will excite the fundamental mode, but not the second mode. An impact at any other location will excite both modes with

amplitudes that increase as the distance from the respective nodes increases. Consequently, the vibrations felt by the hand are minimised for impacts between about 13 and 17 cm from the end of the barrel, as indicated by the “sweet vibrations” zone in Fig. 1. The vibration modes of a hand-held bat are similar to those of a free bat, but the hands have a strong damping effect, with the result that all vibrations are damped within one or two cycles.

The centre of percussion (COP) is not a unique point on the barrel since it is defined in terms of a conjugate point in the handle.^{2,6,7} Each point in the handle is associated with a different COP in the barrel. When a ball strikes a freely suspended bat, the bat will translate in the direction of the incident ball and it will also rotate about the centre of mass. The combined effects of translation and rotation result in a shift of the actual axis of rotation, away from the centre of mass toward the handle. The axis passes through the conjugate point for that particular impact. If b is the distance from the impact point to the centre of mass, and x is the distance from the centre of mass to the conjugate point, then $x = I_{\text{cm}}/(Mb)$ where I_{cm} is the moment of inertia of the bat about its centre of mass and M is the mass of the bat. Consequently, if the bat is pivoted about an axis through the conjugate point, and if the ball strikes the corresponding COP, then there is no reaction force at the pivot point. It is therefore reasonable to expect that every point under the hands will have its own COP and that an impact at any given COP will generate zero reaction force at the corresponding conjugate point under the hands. Brancazio⁸ has suggested that the sweet spot zone might therefore encompass all corresponding COP points along the barrel. There is, however, an uncertainty in this argument since a hand-held bat is not freely suspended, nor is it pivoted about a single axis, and may behave differently as a result of the reaction forces exerted by the two hands. Furthermore, the axis of rotation may even shift as a function of time since the ball remains in contact with the bat for about 1.5 ms, the transmission time of the pulse from the impact point to the hands is about 1 ms, and the impact forces on the hands last for about 20 ms as a result of the induced vibrations and the induced motion of the bat.

II. EXPERIMENTAL PROCEDURE

Measurements of the force of the bat on the hands, and measurements of bat vibrations at other locations on the bat,

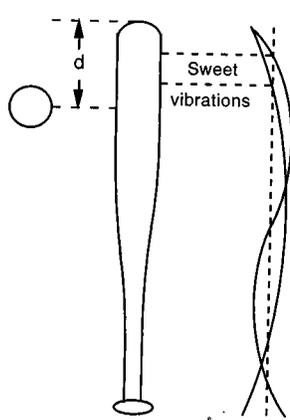


Fig. 1. The fundamental and second vibration modes of a baseball bat.

were made using two identical piezoelectric ceramic disks taped onto the outer surface of the bat handle. The disks were 9 mm in diameter, about 0.3 mm thick, and were extracted from miniature, inexpensive piezo buzzers. The disks had a mass of only 1.8 g each, and therefore had a negligible effect on the behaviour of the bat. Buzzers commonly available from electronics shops or in musical greeting cards contain piezo disks 20 mm in diameter and are a bit too large to be attached to the handle, but could be used if they are first sawn or cut to a smaller size. A flat surface was filed at various positions along the bat in order to tape the disks firmly onto the bat. The disks were electrically connected to 10-M Ω oscilloscope probes via very light connecting leads taped to the bat close to the disks to avoid any spurious response due to independent motion of the leads. The output from a piezo element is directly proportional to the applied force and was typically about 0.5 V in amplitude in the experiments described below. The polarity was chosen so that both piezo elements gave a positive output when compressed. The signals were recorded on a digital storage oscilloscope which was dc coupled to monitor both the dc and ac components of the waveforms, and which was triggered about 2 ms before the impact of the ball on the bat.

Qualitatively similar results were obtained when the piezo disks were bonded to the bat with super-glue. However, the output of bonded disks was found to be larger, by a factor of about 4 due to a significant response to bending of the disks. The bending response (common to all piezo materials) was detected by dropping a super ball directly on the piezo, giving a positive output. When a ball was dropped on the opposite side of the bat, the piezo output was negative, due to the opposite curvature of the bat. However, when the disks were simply taped to the bat, the response remained positive when the ball was dropped either directly on the piezo disk or on a point diametrically opposite the disk. Since the object of the experiment was to detect the force acting normal to the disk, and not the bending moment of the bat, all results presented in this paper were obtained by taping the disks to the bat.

Piezo disks behave electrically as a capacitor with an induced charge proportional to the applied force. The response is also determined by the rate at which the capacitor discharges through the high impedance probes. The time constant was approximately 50 ms and was determined by loading the piezo disks suddenly and observing the exponential decay of the output. Similarly, when the piezo disks were

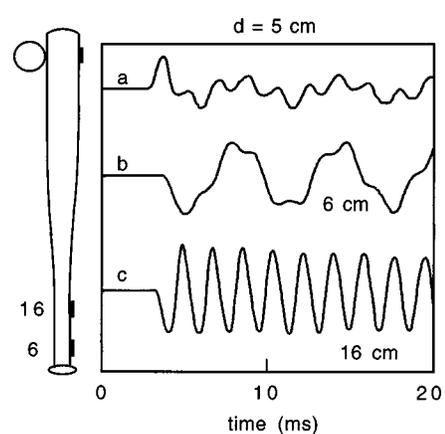


Fig. 2. Vibrations induced in a freely suspended bat when a baseball impacts at distance $d=5$ cm from the end of the barrel. The waveforms show the outputs of three piezo disks located (a) 5 cm from the end of the barrel, (b) 6 cm from the end of the handle, and (c) 16 cm from the end of the handle. All three piezos were mounted in line on the same side of the bat, directly opposite the impact point. The vertical amplitude of trace (a) is reduced by a factor of 5 compared with traces (b) and (c).

rapidly compressed by the hands, the output was typically about 2 V, decreasing exponentially to zero after about 200 ms.

The bat chosen was a Louisville Slugger model R161 wood bat of length 84 cm (33 in.) with a barrel diameter of 6.67 cm ($2\frac{5}{8}$ in.) and mass $M=0.885$ kg (31 oz). The centre of mass of the bat was located 27.6 cm from the barrel end of the bat. The bat was held horizontally by two hands firmly gripping the handle in the usual right-handed fashion, with the left hand closest to the end of the handle. The hands were electrically insulated from the piezo disks by means of the clear adhesive tape used to attach the disks to the handle. The ball was suspended by a 1 m length of string so that it could be struck accurately at predetermined positions on the bat, but it was not struck with the usual vigour normally associated with the game of baseball. In fact, it was struck at very low speed but with sufficient force so that the impulsive forces arising from the collision were clearly identifiable. One might suspect that such an experiment is too far removed from reality to be of any value, but the only nonlinear effect that is likely to contribute to differences in a high speed collision is that the coefficient of restitution varies slightly with ball speed. Quantitative results (other than the magnitude of the force) might therefore be slightly different in a high speed collision, but the essential physics was found to be independent of ball speed, as described below. Furthermore, the physics is also independent of the reference frame, as verified by the fact that identical results were obtained (within experimental error) regardless of whether the ball was swinging and the bat was stationary, the bat was swinging and the ball was stationary, or the bat was swung to strike the moving ball.

III. FREE BAT VIBRATIONS

Experimental results obtained with a hand-held bat are not easily interpreted without reference to the behaviour of a freely suspended bat. Figures 2–4 show the vibrations induced in the handle of a bat, suspended vertically by a 1 m length of string tied to the handle, when a baseball impacted near the end of the barrel (Fig. 2) or near the sweet spots

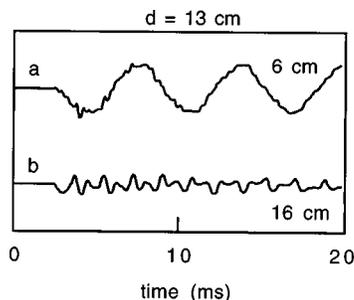


Fig. 3. Vibrations of a freely suspended bat when a baseball impacts at $d = 13$ cm. The waveforms show the outputs of two piezo disks located (a) 6 cm and (b) 16 cm from the end of the handle, as indicated schematically in Fig. 2.

(Figs. 3 and 4). Tests with other lengths of string confirmed that a 1 m length of string provided a negligible restoring force on the handle and did not interfere significantly with free rotation of the bat about its centre of mass, at least during the 20-ms period of interest following the collision.

Also shown in Fig. 2 is the response of the bat at a point on the barrel directly opposite the impact point. The fundamental mode has a frequency of 167 Hz (period 6.0 ms) with nodes 17 cm from the barrel end of the bat and 16 cm from the handle end, while the second mode has a frequency of 530 Hz (period 1.89 ms) with nodes 13 cm from the barrel end and 7 cm from the handle end. The central node of the second mode was located 47 cm from the barrel end and 37 cm from the handle end, neither in the middle of the bat nor at the centre of mass, as it would be for a uniform, cylindrical bat. All the handle waveforms in Figs. 2–4 are displayed and were recorded using the same vertical sensitivity (50 mV/div) at the oscilloscope.

These results show clearly the dominance of the first and second modes in the bat response and also show clearly the locations of the nodes for these two modes. For example, in Fig. 2(c), the first mode is absent since there is a node at the measurement point, while in Fig. 2(b), the second mode is small in amplitude since the node of this mode is close to the measurement point. Both modes are observed, in Fig. 2(a), with roughly equal amplitudes near the end of the barrel. The initial response near the impact point is approximately a half-sine waveform of duration about 1.5 ms, representing the force of the ball on the bat. The impulse propagates from the impact point to the handle in about 0.6 ms, as indicated by the time delay between waveforms (a) and (c). The funda-

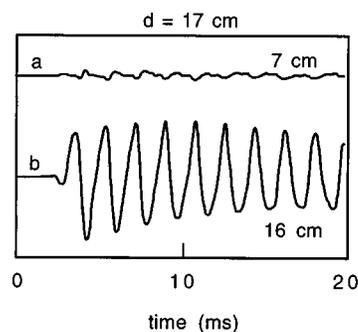


Fig. 4. Vibrations of a freely suspended bat when a baseball impacts at $d = 17$ cm. The waveforms show the outputs of two piezo disks located (a) 7 cm and (b) 16 cm from the end of the handle.

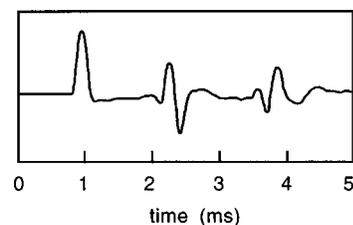


Fig. 5. Response of a piezo, 6 cm from the end of the handle, when the bat is freely suspended and a ball bearing impacts at a point diametrically opposite the piezo disk.

mental mode propagates at a lower speed since the phase velocity of a transverse wave on a beam decreases as the frequency decreases. Qualitatively, this is because a beam is stiffer at short wavelengths.

The effect of an impact at the node of the second mode, at a distance $d = 13$ cm from the end of the barrel, is shown in Fig. 3. Since the second mode is not excited and the measurement in Fig. 3(b) was made at the node of the fundamental mode, a weakly excited third mode can be observed. Figure 4 shows the response for an impact at the node of the fundamental mode. In this case, only very small amplitude vibrations are measured in the handle at the node of the second mode, but a strong second mode is observed at other positions in the handle [e.g., Fig. 4(b)].

IV. IMPULSE PROPAGATION

When a ball strikes a baseball bat held firmly by the hands, the bat transfers vibrational energy to the hands, and the hands exert an impulsive reaction force on the handle. This force is usually neglected when modelling the dynamics of the collision, since it is usually assumed that the ball will leave the bat before the impulse is transmitted to the hands.^{2,5,9} Adair¹⁰ estimated that the impulse would take about 8 ms to propagate from the impact point to the hands and back to the impact point. In fact, the measurements presented in Fig. 2 show that the impulse (or at least the high frequency components) arrives at the hands before the ball leaves the bat.

This is shown even more clearly in Fig. 5 where a 12-mm-diam ball bearing was thrown at a freely suspended bat to impact at a point 6 cm from the end of the handle, diametrically opposite a piezo bonded to the handle. The pulse width in this case was 0.2 ms, and the pulse took 1.3 ms to travel to the end of the bat and back to the handle. Dispersion of the pulse is seen after both the first and second reflections off the end of the bat, indicating that the low frequency components travel at a slightly lower speed than the high frequency components. Vibration modes are not observed in Fig. 5, at least during the first 5 ms, since the pulse width is only 0.2 ms and since the highest frequency components observed take at least 1 ms for the round trip to the end of the bat and back. The low frequency components of the pulse are relatively small in amplitude and are therefore not clearly apparent in Fig. 5. By contrast, low frequency oscillations are clearly observed in Figs. 2–4, almost from the start. When the initial pulse duration is about 1.5 ms, as in Figs. 2–4, the low frequency components are relatively large in amplitude and the leading edge of the reflected pulse catches up with the trailing end of the initial pulse after a delay of about 1.5 ms. Consequently, there is then no clearly discernible effect of pulse propagation along the bat, as is the case in Fig. 5.

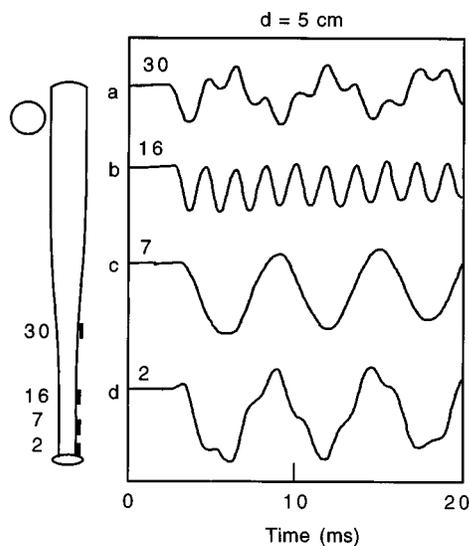


Fig. 6. Handle velocity waveforms at positions (a) 30 cm, (b) 16 cm, (c) 7 cm, and (d) 2 cm from the end of the handle, for a freely suspended bat when a baseball impacts at $d=5$ cm. All piezos were mounted in line on the same side of the bat, diametrically opposite the impact side. The vertical sensitivity of all traces is the same.

Given that the impulse propagates to the hand in a time that is an order of magnitude shorter than previously predicted, one might suspect that the results in Fig. 5 could represent an acoustic mode rather than a transverse wave. However, the speed of sound in wood is about 4000 ms^{-1} , which would give a pulse delay of only 0.21 ms from the barrel to the handle, or 0.42 ms for the round trip. This delay was confirmed by dropping a 6-mm-diam ball bearing onto an end face to excite a compressional wave that was detected by a piezo on the opposite face at the other end of the bat. The acoustic mode is also excited by the usual impact of a ball on the side of the bat, but it is not detected by a piezo mounted anywhere along the handle since the mode then propagates primarily across the barrel rather than along the bat.

V. HANDLE VELOCITY OF A FREE BAT

The results in Figs. 2–4 do not show clearly the zero frequency response of the bat resulting from translation and rotation. A much clearer picture of the zero frequency response emerges when the piezo signals are integrated using a simple RC integrator connected to the output of the $\times 10$ probe. A piezo disk attached to the bat acts as a simple accelerometer, provided the hands are not in contact with the disk. The net force acting on the disk is then $F=ma$, where m is the mass of the disk and a is the common acceleration of the bat and the disk. Consequently, the time integral of the piezo output yields a measurement of the local bat velocity resulting from the combined effects of translation, rotation, and vibration. Absolute values of the handle velocity were not calibrated since the only measurements of interest were the velocity waveforms and the relative velocities at different points along the handle.

Simultaneous measurements of the handle velocity at several different locations, for a freely suspended bat, are shown in Figs. 6 and 7, using integrators with an RC time constant of 22 ms ($R=1 \text{ M}\Omega$, $C=22 \text{ nF}$). The results are presented

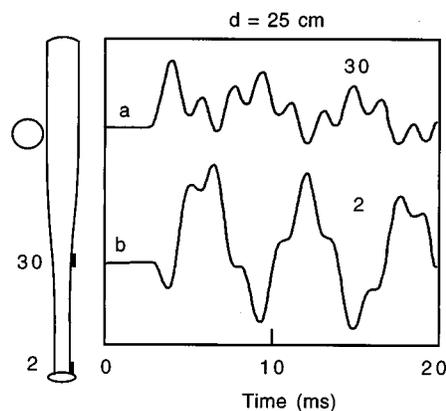


Fig. 7. Handle velocity waveforms at positions (a) 30 cm and (b) 2 cm from the end of the handle, for a freely suspended bat when a baseball impacts at $d=25$ cm. The location and vertical sensitivity of both piezos are the same as those in Fig. 6.

mainly as an aid to interpretation of the hand-held bat results, but are of considerable interest in themselves. There is a slight droop in the waveforms in Figs. 6 and 7, due to the short integration time constant, but the waveforms are easily interpreted. The ac component represents the handle velocity resulting from vibrations in the handle, and the dc component represents the combined effects of translation and rotation of the bat handle. An impact near the end of the barrel (Fig. 6) results in rotation about an axis lying between the handle and the centre of mass, with the result that the handle deflects toward the incident ball, the end of the handle having a higher dc velocity than points closer to the axis of rotation. For an impact at $d=25$ cm (Fig. 7), near the centre of mass of the bat, the axis of rotation is beyond the end of the handle, with the result that the handle deflects in the same direction as the incident ball.

Measurements such as those in Fig. 6, at the point 16 cm from the end of the handle, show that the dc component of the handle velocity at this point changes sign for impacts at either side of a point 13 cm from the end of the barrel, and is zero for an impact 13 cm from the end of the barrel [both the amplitude and the time integral of the waveform in Fig. 3(b) are essentially zero]. Consequently, the node of the fundamental mode, 16 cm from the handle end of the bat, is a conjugate point with a COP close to or coinciding with the node of the second mode at the barrel end of the bat. Similarly, it can be seen from Fig. 4(a) that the conjugate point, for an impact at $d=17$ cm, is located 7 cm from the end of the handle. The node of the second mode, at the handle end of the bat, is therefore a conjugate point with a COP coinciding with the node of the fundamental mode at the barrel end of the bat.

The main conclusions from these free bat measurements are that (a) an impulse arrives at the hand before the ball leaves the bat, (b) the velocity of the bat handle arising from vibrations in the handle is comparable with the velocity due to translation and rotation of the bat, (c) there are two node points in the handle where the vibrational velocity is essentially zero for impacts at appropriate node points in the barrel, and (d) both of the nodes in the handle happen, either by a remarkable coincidence or by design, to be conjugate points for impacts on the barrel at the node of the other mode. The shape of the bat is responsible for this coincidence. For a uniform cylindrical bat of length L , the nodes of

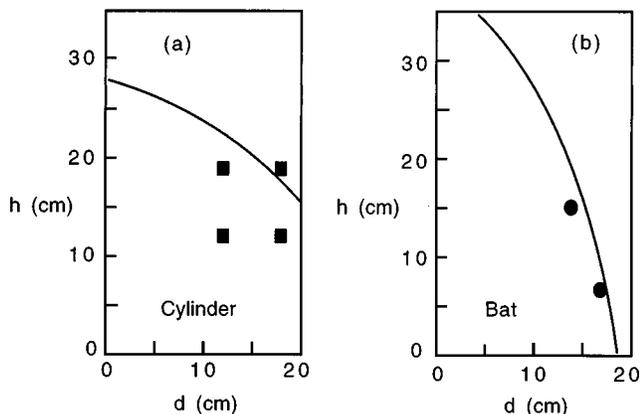


Fig. 8. Location, h , of the conjugate point as a function of the impact distance, d , for two different bats, each of mass $M=0.885$ kg and length $L=0.84$ m. h is the distance of the conjugate point from the end of the handle and d is defined in Fig. 1. The theoretical curve in (a) is for a uniform, cylindrical bat with $I_{cm}=0.052$ kg m² and the curve in (b) is for the bat used in this experiment, with $I_{cm}=0.045$ kg m². The four squares in (a) correspond to the four vibration nodes, two in the barrel and two in the handle. The two circles in (b) correspond to the two nodes in the barrel and the corresponding nodes in the handle that are almost coincident with the conjugate points in the handle.

the fundamental mode are located at a distance $0.22L$ from each end, and the nodes of the second mode are located at a distance $0.14L$ from each end.¹¹ For an impact at a node of the fundamental mode, the conjugate point is located a distance $0.20L$ from the handle end, close to the other fundamental node. For an impact at a node of the second mode, the conjugate point is located a distance $0.27L$ from the handle end, as shown in Fig. 8(a), nowhere near either of the handle nodes.

For a real bat, there are two impact points on the barrel that give rise to a corresponding point under the hands where the velocity due to rotation and translation, as well as the vibrational velocity, is essentially zero. These two points are plotted in Fig. 8(b), which shows the theoretical location of the conjugate point as a function of the impact distance from the end of the barrel when $I_{cm}=0.045$ kg m² and $M=0.885$ kg, corresponding to the measured parameters for the bat used in this study.¹² One of the two points does not lie exactly on the theoretical curve, indicating that there is not an exact coincidence of the node with a conjugate point. It would be interesting to measure and possibly optimise the locations of these points for other bat shapes.

VI. HANDLE VELOCITY OF A HAND-HELD BAT

An extensive series of measurements was made to examine the effect of the hands on the handle velocity. A sample of these results is shown in Figs. 9–11. The initial bat and ball speeds were held constant in order to provide a valid comparison of the relative handle velocities for impacts at different locations along the barrel. The handle velocity was measured simultaneously at points near but not under each hand, using integrators with a time constant of 22 ms. The small dc signal level prior to the collision was not recorded, so the waveforms in Figs. 9–11 represent the change in velocity as a result of the collision, rather than the absolute

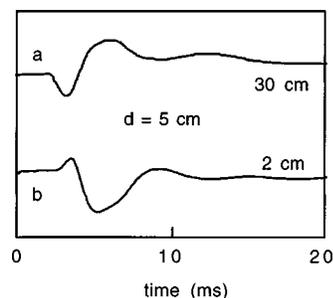


Fig. 9. Handle velocity waveforms measured simultaneously at positions (a) 30 cm from the end of the handle and (b) 2 cm from the end of the handle, for a hand-held bat when a baseball impacts at $d=5$ cm. Both piezos were mounted in line on the same side of the bat, diametrically opposite the impact side. The vertical sensitivity of all traces in Figs. 9–11 is the same.

handle velocity. Alternatively, the waveforms can be taken to represent the handle velocity following a collision with a bat that is initially at rest.

The main features of the results in Figs. 9–11 are as follows.

(1) The handle vibrations are very strongly damped by the hands, to the extent that they are barely recognisable. The damping is less severe if the grip is not as firm or if the bat is held by only one hand

(2) The handle velocity, as well as the vibration amplitude, is smallest for impacts in the sweet spot zone near the impact point $d=15$ cm. For such an impact, the axis of rotation of the bat lies under the hands, for both free and hand-held bats.

(3) For impacts outside the sweet spot zone, it is more difficult to interpret the handle motion since the rotational and vibrational components of the motion are similar in magnitude. As a result, both ends of the bat can move in the same direction while the middle of the bat moves in the opposite direction. The local, instantaneous direction of rotation at one end of the bat can therefore be opposite the local direction of rotation at the other end. For example, in Fig. 9, the handle rotates locally about an axis through the hands, since the two points on either side of the hands move with approximately equal but opposite velocities. For the first 2 ms, the handle rotates locally in the opposite direction to the rest of the bat, but the handle motion subsequently reverses to rotate locally in the same sense as the rest of the bat.

For times greater than 10 ms after the impact, Fig. 9(a) shows a constant, positive velocity and Fig. 9(b) shows a velocity close to zero. During this period, the bat rotates as a rigid body with negligible vibrational motion and with an axis of rotation near the bottom of the left hand. Prior to this period, it is not easy to distinguish the rotational from the vibrational components of the motion. However, the most likely interpretation is that the initial response of the handle is dominated by the elastic response due to vibration modes,

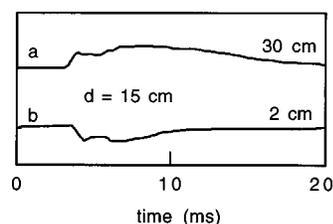


Fig. 10. As for Fig. 9, but with impact at $d=15$ cm.

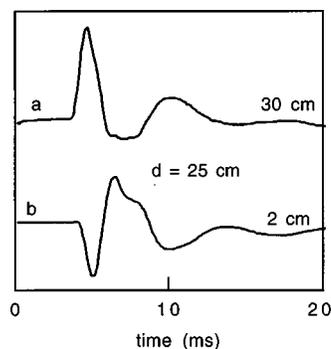


Fig. 11. As for Fig. 9, but with impact at $d=25$ cm.

and the response after 2 ms represents rotation of the bat about an axis through the hands. The initial response is similar to that shown in Fig. 6 for a freely suspended bat, but the subsequent response is different. The free bat rotates about an axis 34 cm from the end of the handle, as indicated by the results in Figs. 6 and 8(b). A larger array of piezo disks along the bat would be required to define the bat motion more precisely.

(4) For an impact at $d=25$ cm, the free bat rotates about an axis well beyond the end of the handle since the impact is close to the centre of mass of the bat (27.6 cm from the end of the barrel). No rotation is expected for an impact exactly at the centre of mass. The behaviour of a free bat, for an impact at $d=25$ cm, is shown in Fig. 7. It can be seen from Fig. 7(b) that the end of the handle actually moves toward the ball during the first 2 ms, and subsequently moves away from the ball due to rotation of the bat. The initial response is primarily due to excitation of the second vibration mode since the high frequency components of the impulse arrive at the handle before the low frequency components. The initial response shown in Fig. 11 for a hand-held bat is qualitatively similar to that shown in Fig. 7 but the initial motion of the end of the handle toward the ball is more pronounced. The negative spike in Fig. 11(b) is partly the result of excitation of the second vibration mode but its amplitude is approximately doubled (compared with the free bat response) as a result of a shift in the axis of rotation to a point under the hands. In Fig. 11, the response of the handle after the first 2 ms is determined mainly by the behaviour of the heavily damped fundamental mode. The second mode is damped within one cycle and induced motion of the bat due to translation and rotation ceases after about 10 ms.

The shift in the axis of rotation is also evident when the handle velocity is measured at a point 7 cm from the end of the handle, and the hands are moved slightly so that they do not press on the piezo at that point. In that case, the second mode does not obscure the measurement and is absent for a free bat. For a hand-held bat, and for an impact about 20 cm or more from the end of the barrel, the axis of rotation through the hands is clearly evident. This shift in the axis of rotation is also seen when the bat is held firmly by only one hand and it is also independent of the speed of the ball. The waveforms in Fig. 11 increased in amplitude by a factor of 10, but did not change shape when a ball was thrown at relatively high speed onto a stationary hand-held bat.

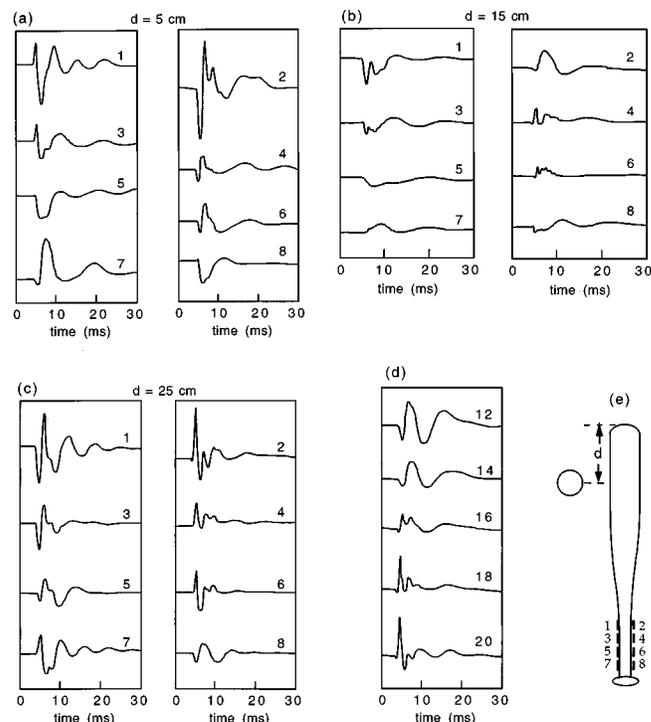


Fig. 12. Handle force waveforms for an impact at (a) $d=5$ cm, (b) $d=15$ cm, and (c) $d=25$ cm, at each of the eight positions shown in (e). Adjacent positions are 5 cm apart, with positions 1 and 2 located 21 cm from the end of the handle and positions 7 and 8 located 6 cm from the end of the handle. Positions 1, 3, 5, and 7 are in line with the impact point and positions 2, 4, 6, and 8 are on the diametrically opposite side of the bat. Window (d) shows force waveforms at position 2, for other impacts at distances $d=12$ to $d=20$ cm from the end of the barrel, as labelled. All waveforms in (a)–(d) are displayed at the same vertical sensitivity.

VII. FORCES EXERTED ON THE HAND

Measurements of the force waveforms exerted on the hands for a hand-held bat are shown in Fig. 12, for impact distances $d=5, 15,$ and 25 cm from the end of the barrel. The force waveforms were measured at eight different positions, as labelled in Fig. 12(e). These results were all obtained by allowing the ball to swing through the same angle, and by swinging the bat slowly in a horizontal plane to contact the ball just after it passed the bottom of its swing. Each of the waveforms in Fig. 12 was checked for reproducibility to within about 20% in amplitude and shape, since variations of this magnitude were commonly observed as a result of variations in grip firmness and impact location. The force waveforms were not calibrated to determine absolute magnitudes, since the object of the experiment was to determine the relative magnitudes of the forces involved.

Interpretation of the results in Fig. 12 is somewhat subjective since no instrumentation was available to monitor the pain felt by the hands in order to correlate force waveforms with pain waveforms. Nevertheless, the “feel” of the bat is quite obvious to the batter, particularly for impacts at distances between 15 and 18 cm from the end of the bat where no unpleasant sensations are felt by the hands. In terms of induced pain, a negative force corresponds to a sudden decrease in the force acting on the hands and is therefore not painful.

The main features of the results in Fig. 12 are as follows.

- (1) There is no impact point where the impact force is

negligible at all locations under the hands. For an impact in the sweet spot zone, the impact force is small at positions 5, 6, 7, and 8 (i.e., everywhere under the left hand), but a significant positive force is transmitted to the right hand, particularly at position 2 (at the base of the index finger).

(2) The force waveforms are moderately complex, but can be analysed in terms of three primary components; (a) a heavily damped fundamental vibration mode with a period of about 9 ms, (b) a heavily damped second mode with a period of about 3 ms, and (c) a narrow spike of width about 2 ms appearing at the beginning of some of the waveforms. The frequency of each vibration mode is lowered with respect to the free bat frequency, partly as a result of the heavy damping (since this is a feature of all damped oscillating systems) and partly by the additional mass of the hands and the arms.

(3) The largest forces are experienced at position 2, and the largest force component, for impacts outside the sweet spot zone, is the 2-ms-wide spike appearing at the beginning of each trace. The polarity of the spike reverses on either side of the bat, as expected, although the waveforms on diametrically opposite sides of the bat are not inverted exactly as mirror images. The polarity of the spike also reverses for impacts on either side of the sweet spot zone, in a manner that is consistent with the direction of rotation of the handle determined from the handle velocity waveforms in Figs. 9 and 11, i.e., the end of the handle moves away from the ball for an impact at $d=5$ cm, but toward the ball for an impact at $d=25$ cm. The force associated with the spike is transmitted primarily to the right hand, at positions 1–4, and is attenuated along the handle. The spike reverses polarity at the far end of the handle, indicating that the spike contains a strong component due to the second vibration mode. This is particularly evident at position 2 for the impact at $d=25$ cm, where two complete, but heavily damped, cycles of the second mode are observed. Attenuation of the spike along the handle is consistent with the fact that the second mode has a node near locations 7 and 8.

(4) Since the hand force is largest at position 2, further details are shown of the force at this position in Fig. 12(d) for other impact points. The sweet spot zone was judged qualitatively, from the feel of the bat, to extend from $d=15$ cm to $d=18$ cm. Over this range of impacts, the feel was essentially the same. This is consistent with the fact that the force waveforms in this zone are generally smaller in amplitude than at other impact points.

(5) The forces transmitted to the left hand are generally smaller than those on the right hand, but a significant positive force is apparent at position 7 for impacts near the end of the bat. This result (i.e., pain in the left hand) is consistent with qualitative experiments performed by Noble and Walker³ and Brancazio.⁸ The force at position 7 can be attributed to the fundamental mode which is seen clearly at this position in Fig. 12(c). The polarity of the fundamental mode reverses between positions 5 and 7 (or 6 and 8), indicating that the handle node for this mode has shifted from its free bat location to a point about 10 cm from the end of the handle. A similar shift in node location, toward the end of the handle, has also been observed by the author for the case of a hand-held tennis racket.

The response at position 7 depends more strongly than at other positions on the way the bat is held. The force increases significantly if the grip is more relaxed. In other

words, if the grip is not firm, the bat rotates more in the hands and the end of the handle strikes position 7 with more vigour.

VIII. CONCLUSIONS

Because of the short transmission time of an impulse along a baseball bat, the hands exert a significant force on the bat while the ball is still in contact with the bat. This effect is probably not very significant in determining the motion of the ball (for reasons such as those discussed in Ref. 11), but it has a significant effect on the behaviour of the bat both during and after the impact.

The sweet spot of a wood baseball bat was found to represent an impact zone of width about 3 cm where the force and the impulse (i.e., the time integral of the force) transmitted to the hands are both minimised. The existence of this zone can be attributed to a combination of the two factors previously identified as being responsible for two separate sweet spots. The dominant vibrations induced in a hand-held bat are heavily damped versions of the fundamental and second modes of a free bat. The vibration amplitude of both modes is sufficiently large to cause pain for any impact outside the sweet spot zone. The amplitudes of these modes are minimised for impacts at the respective barrel nodes, but an impact at either one of these modes will excite the other mode. An impact at the fundamental node is about optimum since the amplitudes of both the fundamental and second modes remain small. For impacts outside the sweet spot zone, the second mode amplitude is largest under the right hand (for a right-handed batter) and smallest near the node under the left hand. The node of the fundamental mode is also located under the left hand for a hand-held bat.

The other main factor contributing to the sweet spot zone concerns rotation of the bat in the hands. The rotation is not as simple as one might expect from an analysis of a freely suspended, rigid bat, since the handle is relatively thin and flexible. The initial direction of rotation of the handle is determined largely by the behaviour of the two vibration modes and is also effected by a shift in the axis of rotation to a point under the hands after the ball leaves the bat. Consequently, the force acting on the hands is not always as expected from pure rotation of a rigid bat. For example, an impact near the tip of a rigid, free bat causes the handle to move suddenly toward the ball, in which case a large positive force should be felt on the fingers of the right hand. In fact, the force is largest on the opposite side of the bat, at the base of the index finger. Similarly, one would expect, for an impact near the centre of mass of the bat, that the force at the base of the little finger on the left hand should be negligible compared with the force on the other side of the bat. In fact, the opposite situation is observed since the bat rotates about an axis through the hands. Nevertheless, the forces transmitted to the hands as a result of bat rotation are minimised for impacts in the zone where the centres of percussion of a free bat correspond to conjugate points under the hands. This also contributes significantly to the formation of the sweet spot zone.

The measurements reported in this paper could easily be repeated in an undergraduate laboratory, at low cost, or extended to investigate an aluminum bat or other hand-held equipment such as a tennis racket or golf club. The experiment illustrates some elementary properties of wave excitation and propagation as well as some interesting features of classical collision and rotation phenomena, and should appeal especially to students and staff interested in bat and ball

sports. The freely suspended bat experiments would be particularly suitable for this purpose since the waveforms are especially clear and informative.

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¹About 60 Australians play in US Major League, and 500,000 Australian children play closely related games such as tee ball or softball at school.

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¹¹R. C. Cross, "The dead spot of a tennis racket," *Am. J. Phys.* **65**, 754–764 (1997).

¹²For the uniform bat, $I_{\text{cm}} = ML^2/12 = 0.052 \text{ kg m}^2$. It may surprise some readers that I_{cm} decreases when mass is shifted toward one end of the bat. The reason is that the bat cm also shifts toward the heavy end, with the result that the shifted mass is closer to, rather than further from, the cm.