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A Comparative Study of Baseball Bat Performance

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Abstract The results of a comparative study of five aluminum and one wood baseball bats are presented. The study includes an analysis of field data, high-speed laboratory testing, and modal analysis. It is found that field performance is strongly correlated with the ball-bat coefficient of restitution (BBCOR) and only weakly correlated with other parameters of the bat, suggesting that the BBCOR is the primary feature of a bat that determines its field performance. It is further found that the instantaneous rotation axis of the bat at the moment of impact is very close to the knob of the bat and that the rotational velocity varies inversely with the moment of inertia of the bat about the knob. A swing speed formula is derived from the field data and the limits of its validity are discussed. The field and laboratory measurements of the collision efficiency are in generally good agreement, as expected on theoretical grounds. Finally the BBCOR is strongly correlated with the frequency of the lowest hoop mode of the hollow bats, as predicted by models of the trampoline effect.

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1 Introduction

In recent years, the issue of baseball and softball bat performance has attracted considerable interest among players, the regulating associations, and even scientists. The interest arises primarily as a result of new technologies which have produced bats that seemingly perform significantly better than the traditional wood bat. The evidence for better performance is partly anecdotal and partly statistical (e.g., greater number of runs scored per game when non-wood bats are used). However, the most compelling evidence comes from both carefully controlled laboratory experiments [1] and well-executed field studies [2,3], both of which have conclusively demonstrated the improved effectiveness of non-wood bats. Along with these recent experimental developments has come improved theoretical understanding of how bats work [4,5] and how laboratory experiments can be used to predict field performance [6–8].

The focus of the present paper is a reexamination of the batting cage data of Crisco, et al. [2,3]. In that experiment, a collection of skilled college-level batters used a variety of different bats, some wood and some aluminum, to swing at pitches projected from a pitching machine with speeds in the range 20-29 m/s (45-65 mph). High-speed motion-analysis techniques were used to track the trajectories of the bat and of the the pitched and batted ball in the vicinity of the ball-bat impact region, so that the pre-collision and post-collision ball and bat speeds and the impact point along the axis of the bat could be accurately measured, among other things. It was demonstrated conclusively that the average batted ball speed for a selection of aluminum bats was greater than that for a particular wood bat. Some of the aluminum bats performed similarly to the wood bat but some outperformed the wood bat by a statistically significant amount. Moreover, maximum batted ball speed was shown to be highly correlated with bat speed. In a plot of batted ball speed versus bat speed at the point of contact (see Fig. 6 of Crisco, et al. [3]), a linear relationship was observed between maximum batted ball speed and bat speed, with a slope which was nearly the same for each bat. However, for a given bat speed, the maximum batted ball speeds were different for each bat. These maximum values were recognized to be due to an inherent property of the bat and were postulated to be related to the ball-bat coefficient of restitution (BBCOR) but might also be due to additional factors [3]. The larger values observed for some of the aluminum bats relative to the wood bat were taken as indirect evidence for a trampoline effect. The trampoline effect is due to the elastic deformation of the barrel wall upon contact with the ball, resulting in less deformation of the ball, less overall energy loss, and a higher BBCOR [5]. Very little correlation was observed between batted ball speed and pitch speed. The role played by the inertial properties of the bat, principally its moment of inertia (MOI) about the knob, was also discussed. An inverse relationship between MOI and swing speed was qualitatively demonstrated but no quantitative relationships were developed. Consequently it was not possible to draw any conclusions about the importance of MOI for batted ball speed.

At the time of their publication Crisco, et al. did not have access to a theoretical formalism for further interpretation of the data. Now that such a formalism has been developed [7], it is of interest to reanalyze the batting cage data within the framework of that formalism. Such an analysis is the focus of the present paper. One goal is to identify the particular characteristics of the bats that determine their batting cage performance, specifically the relative roles played by the BBCOR and the MOI. One of the principal tenets of the theoretical formalism is that laboratory measurements of certain performance metrics can be used to predict field performance. Therefore a second goal is to test experimentally this very important feature. This test necessitated additional laboratory measurements on the batting cage bats. A third goal is to quantify the dependence of a batter's swing speed on the inertial properties of a bat, principally the MOI, by developing a universal formula consistent with the experimental data. A final goal is to examine the relationship between the BBCOR and the barrel flexibility through laboratory measurements of the frequency of hoop modes in these bats [8]. The experimental methods are described briefly in Sec. 2, while the bulk of the paper is the presentation of the analysis in Sec. 3. A summary of our findings is given in Sec. 4.

2 Experimental Methods

The primary data analyzed are the batting cage data from the extensive study by Crisco, et al., which has been described in detail in previous publications [2,3] and briefly in the preceding section. A total of seven bat models were used: two wood bats with similar properties that were averaged together (W) and five aluminum bats (M1-M5), with inertial properties listed in Table 1. The following quantities used in the present analysis were determined from the bat and ball trajectories: the velocity vectors of the pitched and batted ball just before and after impact; the impact location along the axis of the bat; and the rotational velocity and axis of rotation of the bat at impact.

The identical bats that were used in the batting cage study were subsequently tested at the Sports Science Laboratory at Washington State University. Since a description of the facility and apparatus has been described extensively in a recent publication [1], only a brief description is presented here. The measurements consisted of firing a baseball from a high-speed cannon at an initial velocity $v_i \approx 60.8$ m/s (136 mph) onto a stationary bat and measuring the rebound velocity of the ball v_f. The initial velocity was chosen to approximate closely the relative ball-bat speed from the batting cage study. The bat is horizontal and supported by clamping it at the handle to a structure that is free to pivot about a vertical axis located 15 cm (6 in.) from the knob. The ball passes through several planes of light screens, which allows v_i and v_f to be measured. The measurements followed the ASTM F2219 protocol [9]. A full scan over the impact location range z=10-20 cm (4-8 in.) was completed for each of the aluminum bats, where z is measured relative to the barrel tip. Unfortunately, for the wood bat only points in the range 10-15 cm (4-6 in.) were obtained prior to the bat breaking.

The collection of bats from the batting cage study were also tested in the acoustics laboratory at Kettering University to determine the frequency of the lowest order hoop mode through experimental modal analysis. Each bat was supported by rubber bands at the handle and barrel ends. A small accelerometer was attached to the bat, approximately 12.7 cm from the tip of the barrel. A small hammer with a force transducer in the tip was used to tap the bat at 2.5-cm intervals along the length its length. A frequency response function consisting of the ratio of acceleration divided by force was recorded with a two-channel FFT analyzer for each impact location. The STAR Modal software program [10] was used to fit all of the frequency response functions and extract frequencies, mode shapes, and damping parameters for each of the resulting vibra-

Table 1 Physical characteristics of the wood and aluminum bats used in the batting cage study. The first column labels the bat type, which is W for wood bat and M1-M5 for the aluminum bats. The center of mass CM is measured from the knob end of the bat; the moments of inertia I_{knob} is with respect to the knob. The moment of inertia can be computed about other points using the parallel axis theorem. The last column is the number of impacts N satisfying the data quality cuts. The lengths are accurate to 0.15 mm, the masses to 0.15 g, the CM locations to 0.7 mm, and the moments of inertia to 0.4 g-m^2.

bat	length m (in)	mass kg (oz)	CM m (in)	I_{knob} kg-m^2 (oz-in^2)	N
W	0.86 (33.9)	0.88 (31.1)	0.58 (22.9)	0.346 (18,895)	51
M1	0.84 (33.1)	0.86 (30.2)	0.53 (20.8)	0.293 (16,000)	33
M2	0.84 (33.0)	0.83 (29.3)	0.52 (20.6)	0.282 (15,395)	40
M3	0.84 (33.2)	0.81 (28.5)	0.53 (20.8)	0.277 (15,141)	11
M4	0.84 (33.1)	0.79 (28.0)	0.54 (21.4)	0.278 (15,190)	17
M5	0.86 (33.9)	0.85 (29.9)	0.54 (21.3)	0.303 (16,571)	35

tional modes. The frequencies of the lowest order hoop mode are shown in Table 2. There is no frequency for the wood bat since, being solid, it does not exhibit a hoop mode.

3 Analysis and Results

3.1 Theoretical Formalism

In this section the formalism [7] that will be used in the analysis of the batting cage data is summarized. The starting point is the fundamental equation that relates batted ball speed, denoted herein by v_f, to the pitched ball speed v_i and the bat speed v_{bat}:

$$v_f \;=\; e_A v_i \;+\; (1+e_A)v_{bat}\,, \tag{1}$$

where e_A is the so-called collision efficiency, a joint property of the ball and bat. In turn, e_A is related to the BBCOR, denoted by e, via the expression

$$e_A \;=\; \frac{e-r}{1+r}\,, \tag{2}$$

where r is the bat recoil factor that depends on the inertial properties of the bat. For a free bat,

$$r \;=\; m_{ball}\left(\frac{1}{M} \;+\; \frac{(z-z_{cm})^2}{I_{cm}}\right)\,, \tag{3}$$

where m_{ball} and M are the ball and bat masses, respectively, I_{CM} is the moment of inertia of the bat about the center of mass, and $(z-z_{CM})$ is the distance of the impact point z from the center of mass.

For the batting cage data, the pitch speed v_i, the bat speed v_{bat} at the moment and location of impact, and the batted ball speed v_f, were used to extract the collision efficiency e_A by inverting Eq. 1

$$e_A = \frac{v_f - v_{bat}}{v_i + v_{bat}}\,, \tag{4}$$

where all speeds are taken as positive. Then Eq. 2 was used to extract the BBCOR, assuming the recoil factor Eq. 3 appropriate for a free bat. The justification for that procedure has been discussed at length by Nathan [4,7]. For the laboratory data, the collision efficiency is also calculated from Eq. 4, with v_{bat}=0.

One of the principal uses of this formalism is in the regulation of bat performance. Measurements of e_A are done in the laboratory using the same technique described in Sec. 2; these measurements are used along with Eq.1 and a prescription for pitch and bat speed to predict batted ball speed in the field. An essential element of this technique is that the values of e_A measured in the laboratory are identical to those in the field [7], a theoretical prediction that will be tested experimentally in Sec. 3.3. Using the batting cage data to derive a prescription for bat speed is presented in Sec. 3.4.

3.2 Bat Performance Analysis

The goal of this analysis is to use the formalism of Sec. 3.1 to identify properties of the bats that lead to differences in v_f in the batting cage experiment. In doing so, it is important to realize that the formalism only applies to head-on collisions between ball and bat, so it is necessary to select events for analysis that best approximate this condition. Fig. 1 is a composite plot of all bats and all impacts of the normalized BBCOR values versus impact location z, with a normalization procedure described below. It is evident from this plot that there is a narrow band of points, shown as the closed points, that varies smoothly with z about a parabola, shown in the figure as a solid curve. The closed points constitute the head-on impacts, with most of the other (open) points falling below the smooth curve, exactly as expected for more oblique impacts where the collision does not occur squarely on the axis of the bat. The closed points are those that survive the data quality cuts, which are now described.

First a vertical mismatch cut was made to assure a head-on collision. The mismatch is the distance of closest approach of the bat and ball trajectories, which was specified to be less than 2.5 cm (1 in.), where 0 corresponds to a head-on collision. Second a cut was made to assure that the inbound and outbound ball speeds were of the highest accuracy. These speeds were calculated using both a finite difference and a linear fit. The difference in these values was required to be less than 0.09 m/s (0.2 mph) for the inbound speed and 0.22 m/s (0.5 mph) for the outbound speed. Finally, several angular cuts were made to select only line-drive hits. First, the elevation angle above the horizontal of the ball after impact was limited within ±16° of horizontal. Second, the outbound angle of the ball relative to the field was limited to be within 20° on either side of the pitcher-catcher line. Third, the difference between the inbound angle and outbound angle of the ball was limited to less than 20°. As a result of these "data quality cuts," the total number of impacts was reduced from 503 to 187, distributed among the bats as shown in Table 1, with normalized BBCOR values shown as the closed points in Fig. 1.

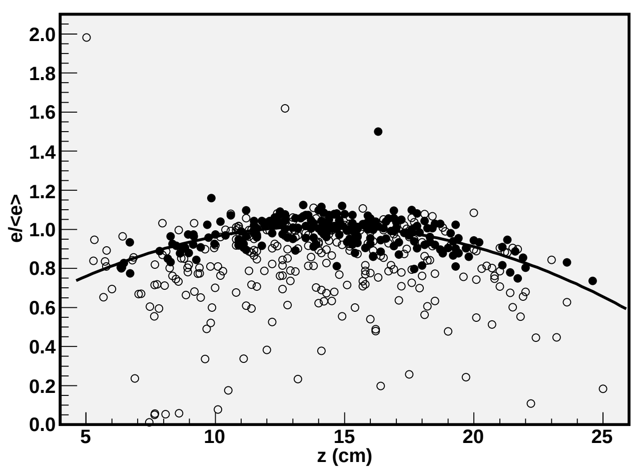

Fig. 1 A composite plot for all bats and all impacts of the normalized BBCOR values versus impact location, with a normalization procedure described in the text. The closed points are those surviving the data quality cuts, also described in the text, with the curve a parabolic fit to guide the eye. The obvious outlier at z=16 cm was not included in the analysis. The open points are those not surviving the cuts.

A box plot showing the distribution of v_f for these bats is shown in Fig. 2, and the mean values are given in Table 2. These mean values are robust under different prescriptions for obtaining the means (e.g., Gaussian fit, simple average, average with outliers removed, etc.). Moreover, they represent averages over batters with different skill levels [2]. The results indicate a definite ordering of the bats, with M2 being the highest performing at over 45 m/s (101 mph); M3 and M4 at 43.3-43.9 m/s (97-98 mph); M1 and M5 at 42.3-43.0 m/s (95-96 mph); and the wood bat W being the lowest performing at 40.9 m/s (91 mph). Also shown in Fig. 2 are box plots showing the distribution of e and e_A for each bat, with mean values given in Table 2. The mean values of e ($\langle e \rangle$) were used to obtain the normalized BBCOR values shown in Fig. 1. The correlation between v_f and either e or e_A is shown in Fig. 3. These plots show that the ordering of bats based on e is *identical* to the ordering based on v_f, suggesting that e plays an important role in distinguishing the performance of one bat from another. Note, however, that the ordering based on e_A is not the same as the ordering based on v_f.

Given this formalism, it is now possible to explain some of the qualitative observations of Crisco, et al. One observation is the strong dependence of v_f on bat speed and weak dependence on pitch speed, both of which immediately follow from Eq. 1 along with the empirical result that $e_A \ll 1$ for each of the bats. Another is that the slope of v_f versus v_{bat} is nearly the same for each bat; from Eq. 1 the slope is $1+e_A$, which varies by only a few percent among the different bats, in agreement with the observation. The inherent property that determines maximum batted ball speed for a given bat speed is e_A, which in turn depends both on the BBCOR (just as speculated) and on the inertial properties of the bat through the recoil factor.

To emphasize further the important role played by the BBCOR, the quantity v_f^* given by the expression

$$v_f^* = \left[\frac{0.5 - r}{1 + r}\right] v_{ball} + \left[\frac{1.5}{1 + r}\right] v_{bat} \,. \qquad (5)$$

is calculated for each impact. This expression is derived by combining Eqs. 1 and 2, with e set to the constant value of 0.5. It represents the expected value of v_f for each impact if all bats had the same fixed value of $e = 0.5$, so that any difference among the bats must come from factors *other than* differences in e. The results are shown as a box plot in Fig. 2, with mean values given in Table 2. The spread in mean values of v_f^* (0.80 m/s or 1.8 mph), is considerably smaller than the spread in mean values of v_f (4.5 m/s or 10.1 mph). This is displayed graphically in Fig. 3, which shows the correlation between the mean values of each quantity. From this analysis, it is concluded that e is the primary factor that distinguishes performance among the

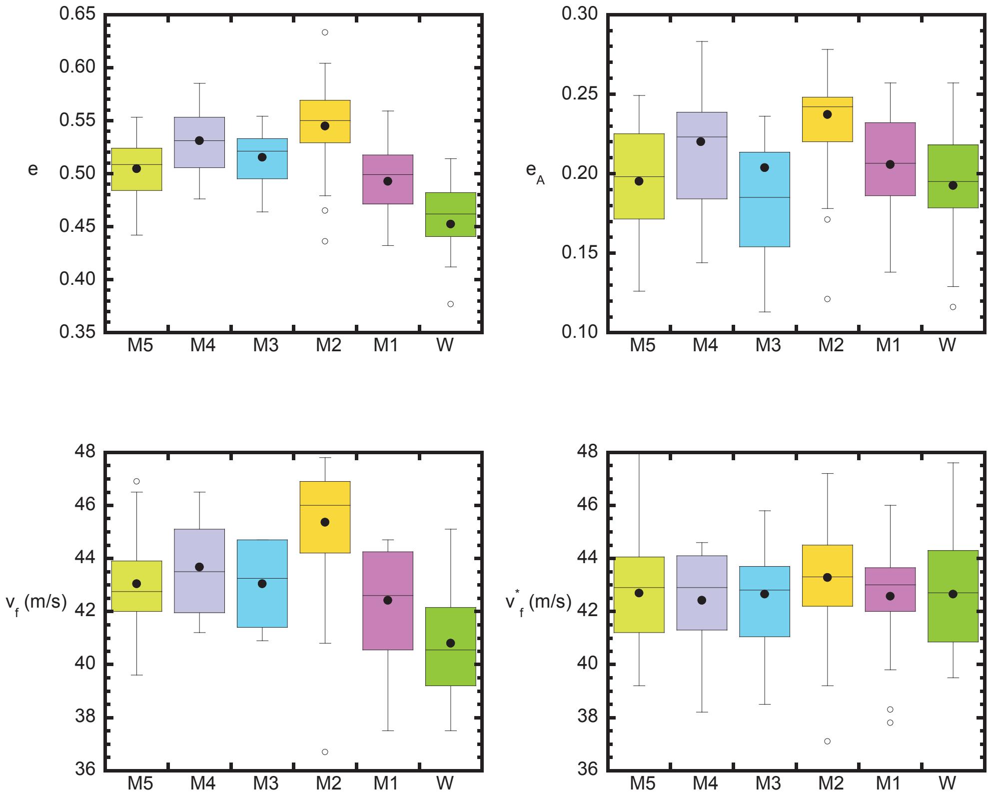

Fig. 2 Box plot of the BBCOR (upper left), collision efficiency (upper right), and ball exit velocities v_f (lower left) for hits in the impact range 8.9 to 19.1 cm (3.5 to 7.5 in.) from the barrel end of the bat. For each bat, the closed point represents the mean value, the horizontal line is the median value, and the shaded region is bounded by the upper quartile (U) and lower quartile (L), with the interquartile distance D equal to U-L. The flags are the bounds of points within the range (L-1.5D)-(U+1.5D), while the open points are events lying outside those bounds. The lower right panel is a box plot of the adjusted ball exit velocity v_f^*, which is calculated from Eq. 5. Differences in v_f^* among the bats are due entirely to differences in inertial factors rather than to differences in e.

Table 2 Average quantities for each bat inferred from the batting cage data. The quantities tabulated are the mean angular velocity about the knob, the mean ball exit speed v_f, the mean value of the v_f^*, the mean collision efficiency e_A and the mean BBCOR e. The uncertainties on the least-significant digit, given in parentheses, are the standard deviation of the mean. The quantity v_f^* is calculated based on Eq. 5. The last column is the frequency of the lowest hoop mode.

bat	ω_{knob} rad/s	v_f m/s [mph]	v_f^* m/s [mph]	e_A	e	$\mathrm{f_{hoop}}$ Hz
W	43.1(2)	40.9(3) [91.4(7)]	42.7(2) [95.6(5)]	0.193(4)	0.452(5)	—-
M1	44.8(4)	42.3(4) [94.6(8)]	42.6(2) [95.4(5)]	0.208(6)	0.494(6)	2334
M2	45.7(5)	45.4(4) [101.5(9)]	43.2(4) [96.7(8)]	0.233(4)	0.545(6)	1720
M3	46.1(4)	43.3(7) [96.8(15)]	42.6(2) [95.4(5)]	0.204(9)	0.515(11)	1908
M4	46.4(7)	43.9(5) [98.3(11)]	42.4(2) [94.9(5)]	0.221(11)	0.531(9)	1848
M5	44.4(4)	43.0(3) [96.1(7)]	42.6(2) [95.4(5)]	0.197(4)	0.505(5)	2233

bats. This is an important conclusion deserving further discussion.

First an approximate expression for the bat recoil factor is written, valid for impacts within 10-20 cm (4-8 in.) from the barrel tip [11]:

$$r \approx \frac{m_{ball} z_{15}^2}{I_{15}}\,, \tag{6}$$

where z_{15} is the distance from the impact location to a point on the bat 15 cm (6 in.) from the knob and I_{15} is the moment of inertia about that point. The latter can be computed from I_{knob} using the parallel axis theorem. The combination of Eqs.1, 2, and 6 shows that the properties of a bat that determine its performance are the BBCOR, I_{15}, and the swing speed. On the other hand,

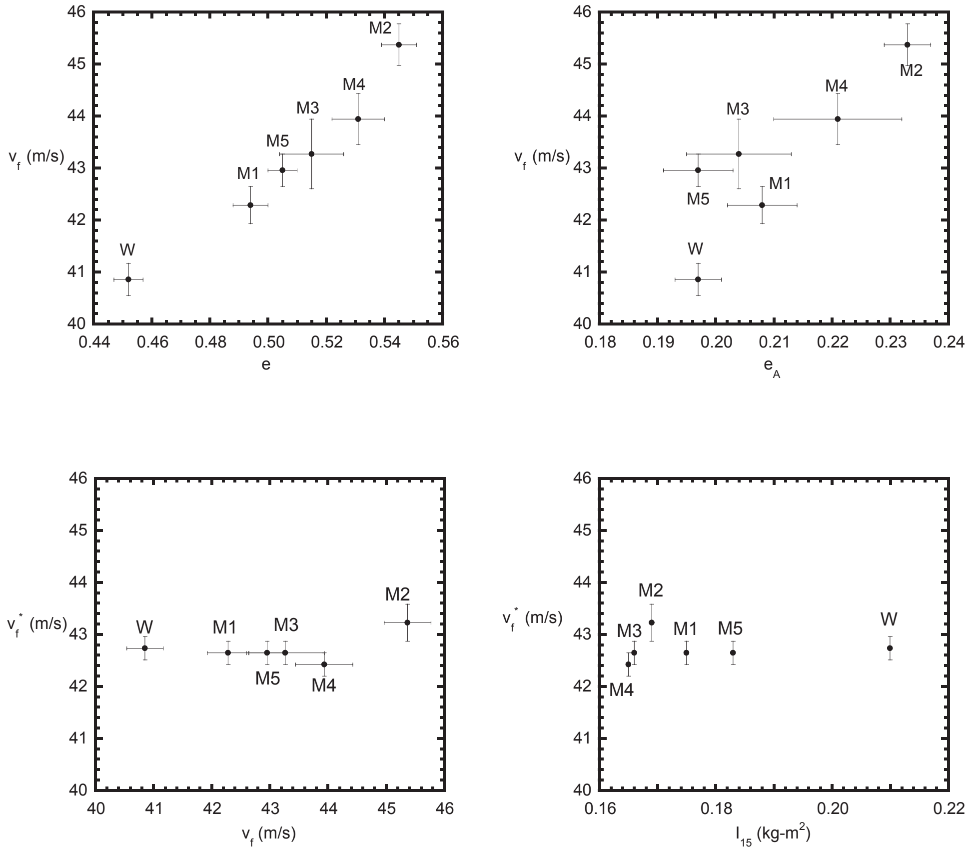

Fig. 3 Correlations between v_f and e (upper left), v_f and e_A (upper right), v_f^* and v_f (bottom left), and v_f^* and the swing weight I_{15} (bottom right).

as will be discussed in Section 3.4, the swing speed of a bat depends on I_{knob}, which itself is a nearly fixed fraction of I_{15}. For the bats in Table 1, the fraction ranges from 1.6 to 1.7. Indeed, I_{15} is often referred to as the "swing weight" of a bat [11]. Therefore the *only two* distinguishing characteristics of a bat that determine its performance are the BBCOR and the swing weight. However, the effect of swing weight on batted ball speed enters in two opposing ways. As an example, consider a bat of a given BBCOR performing at a certain level of batted ball speed. Now consider increasing the swing weight of the bat, leaving everything else the same. This could be achieved, for example, by inserting some additional weight at the endcap of the bat. Increasing the swing weight will both *increase* the collision efficiency and *decrease* the swing speed, resulting in partially canceling effects and reducing the dependence of batted ball speed on the swing weight. Therefore, it makes sense physically that the single parameter of a bat that determines its performance is the BBCOR, a conclusion strongly supported by the present data. Although this result had been anticipated previously on theoretical grounds [11], to our knowledge the present analysis of the batting cage data provides the first experimental confirmation. The strong correlation between batted ball speed and e has led the NCAA to adopt the BBCOR as their primary metric of performance [12].

3.3 Comparing batting cage and laboratory performance

Fig. 4 is a profile plot of e_A vs. z, along with the values from the laboratory study. For the batting cage data, the plot was created for each bat by dividing the batting cage data into five 2.5 cm bins of impact location z. The points within each bin are then averaged and plotted as a single point along with a vertical bar representing the standard error of that point. Superimposed on each plot are the $e_A(z)$ values from the laboratory measurements, which follow the general trend of the batting cage values both in overall magnitude and in the z dependence. The agreement between the two sets of measurements tends to be expecially good for z

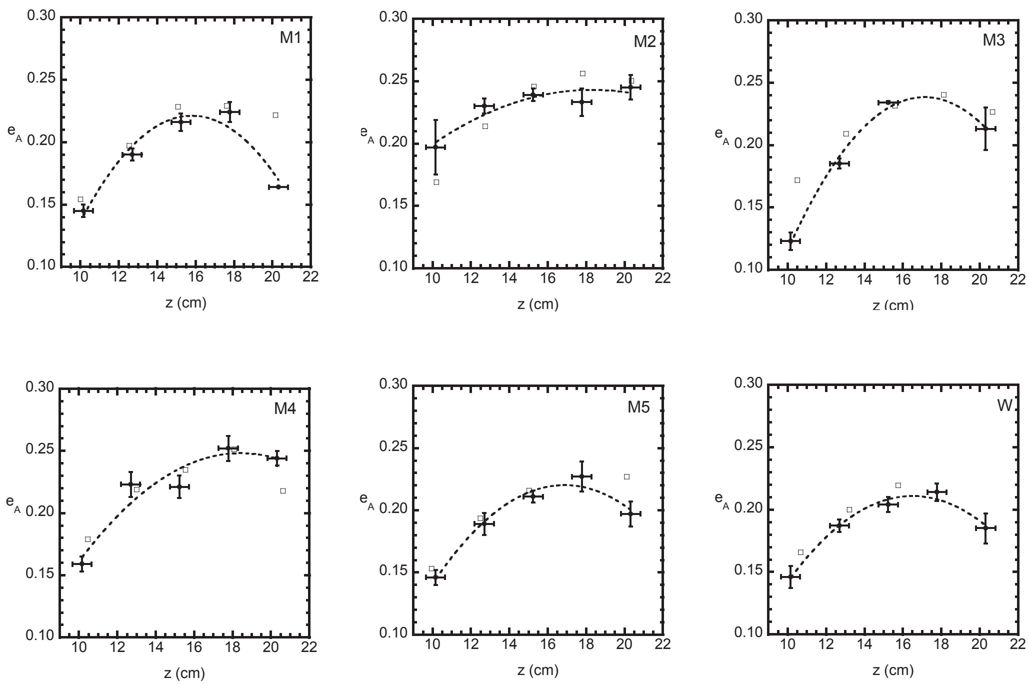

Fig. 4 A plot comparing e_A vs. impact location z for the batting cage and laboratory data. The circles are batting cage values and are displayed as a profile plot by averaging over 1-inch intervals of z. The vertical bar represents the standard error of the mean value and the horizontal bar represents the averaging interval. The open squares are the laboratory average values, with a standard error less than the size of the square. The dashed line is a parabolic fit to the batting cage profile data.

in the range 12-18 cm from the tip. This agreement is in accord with both theoretical expectations [4,7] and previous experimental findings [13].

3.4 Swing Speed Analysis

For this part of the analysis, all 503 impacts are used. The goal is to use the batting cage data to develop a universal formula for bat speed that can be used in Eq. 1 along with laboratory measurements of e_A to predict v_f in the field. Guided by the results of a similar study for slow-pitch softball [14], it is anticipated that bat speed will depend on I_{knob}, the impact location, and the rotation axis. For each impact, the batting cage data set includes the bat speed v_{bat} at the impact point, the angular velocity ω about the rotation axis, and the instantaneous rotation axis. The latter quantity is shown in Fig. 5, which is a composite plot for all bats and all impacts. The data show that the mean rotation axis is at the location z_P=1.65 cm (0.65 in.) and x_P=7.5 cm (3.0 in), where z_P is the distance along the long axis of the bat measured from the knob end and x_P is the perpendicular distance. This result shows that the axis lies close to the wrist of the lower (left) hand of the right-handed batter. Therefore the hands are barely moving at the time of contact, suggesting that the swing is very efficient with the maximum amount of energy transferred to the bat.

Having verified that the rotation axis is close to the knob, the next step is to determine $\omega_{\rm knob}$ for each bat, averaged over all impacts and over batters of different skill level [2], as shown in the histograms in Fig. 6. A Gaussian fit to each histogram results in the values of $\omega_{\rm knob}$ presented in Table 2 and plotted against $I_{\rm knob}$ in Fig. 7. Following the procedure of Smith [14], the values of $\omega_{\rm knob}$ are fitted to the function

$$\omega_{\rm knob} = \omega_0 \left(\frac{I_0}{I_{\rm knob}}\right)^n , \tag{7}$$

obtaining n=0.29±0.04 and ω_0=45.2±0.2 rad/s, with the reference MOI I_0 fixed at 0.293 kg/m^2 (1.6×10^4 oz-in^2). The value for the exponent does not depend on the exact prescription used to obtain the $\omega_{\rm knob}$ (e.g., Gaussian fit or mean values). The value of ω_0 implies a mean bat speed of 32.0 m/s (71.7 mph) at a location 0.71 m (28 in.) from the knob. The value of n is consistent with values determined from other studies [14–17]. It falls nearly midway between the extreme values $n = 0$, which implies a bat swing speed independent of $I_{\rm knob}$, and $n = 0.5$, which implies a bat kinetic energy independent of $I_{\rm knob}$ [18].

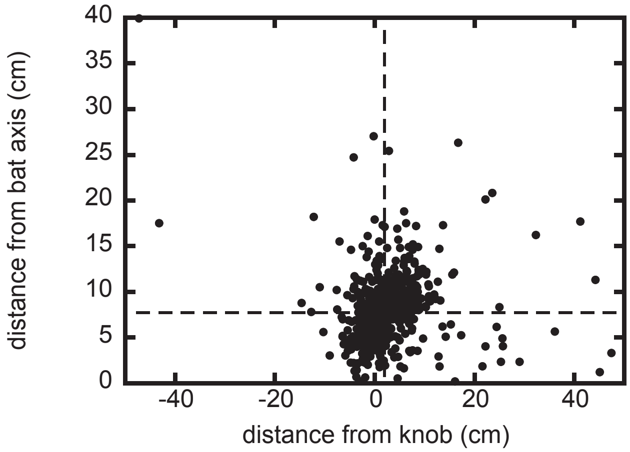

Fig. 5 Composite scatter plots for all bats and all impacts showing the instantaneous rotation axis of the bat just prior to the collision, plotted as the off-axis (x) versus the on-axis distance (z). The latter is measured from the knob of the bat and the former is positive when pointed towards the catcher. The dashed lines show the mean distances, with an intersection at z=1.65 cm (0.65 in.) and x=7.5 cm (3.0 in.), corresponding approximately to the location of the wrist of the bottom (left) hand of the right-handed batters.

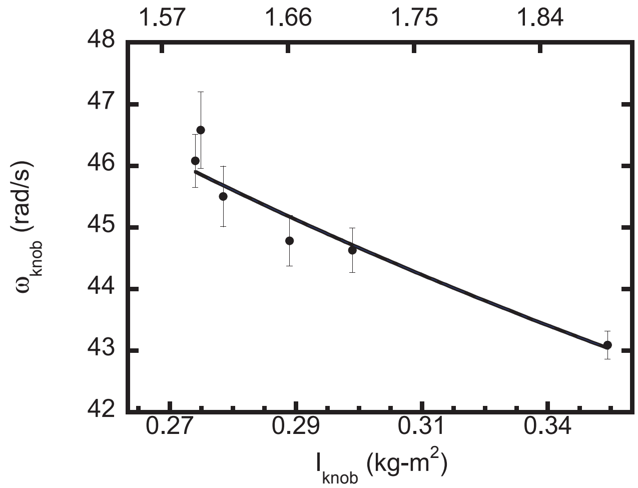

Fig. 7 Plot of the angular velocity ω_{knob} of the bat about the knob just prior to impact versus the I_{knob}, the MOI of the bat about the knob. Each point represents the angular velocity of a given bat, averaged over all impacts, as determined from Gaussian fits, with values given in Table 2. The curve is a power-law fit of the form $\omega_{\mathrm{knob}} \sim 1/I^n_{\mathrm{knob}}$, with $n = 0.29 \pm 0.04$. The upper horizontal scale is I_{knob} in units of 10^4 oz-in^2.

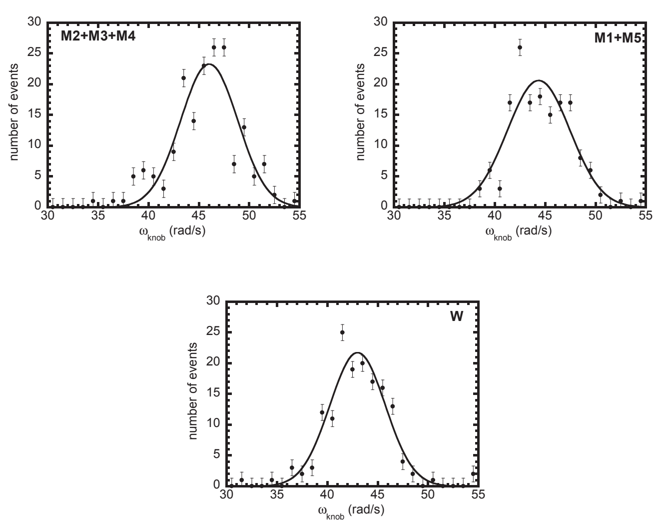

Fig. 6 Histograms of the angular velocity of the bat about the knob just prior to impact, where bats with similar values of MOI have been grouped together. The curves are Gaussian fits. The result of Gaussian fits for each of the six bats is given in Table 2 and plotted in Fig. 7.

As a first attempt at a universal swing speed formula, Eq. 7 can be combined with the mean location of the rotation axis to arrive at the result

$$v_{bat} = v_0 \left[\frac{\sqrt{(L - z - z_P)^2 + x_P^2}}{0.71 \ \mathrm{m}} \right] \left(\frac{I_0}{I_{\mathrm{knob}}} \right)^{0.29} , \quad (8)$$

where L is the bat length, z is the impact location relative to the tip, and z_P and x_P are 1.65 cm (0.65 in.) and 7.5 cm (3.0 in.), respectively. For the particular data analyzed, v_0=32.0 m/s (71.7 mph). As a numerical example, bat M1 is predicted to be swung with a bat speed of 30.5 m/s (68.2 mph) at z=15 cm (6 in.). In the approximation that the rotation axis is exactly at the knob (i.e., x_P and z_P=0), then the formula simplifies to

$$v_{bat} = v_0 \left[\frac{L - z}{0.71 \ \mathrm{m}} \right] \left(\frac{I_0}{I_{\mathrm{knob}}} \right)^{0.29} , \quad (9)$$

While Eq. 9 adequately describes bats over the range of I_{knob} given in Table 1, it clearly cannot work for arbitrarily small I_{knob} since it diverges. An improved model more solidly grounded in both physics and biomechanics has been suggested by Adair [18]. The basis for the model is that the batter must accelerate not only the bat but his arms. A reasonable assumption is that the batter converts a fixed amount of energy generated from the muscles into kinetic energy of the bat-plus-arms system. Assuming the system is rotating at the angular velocity ω_{knob} at the time of impact, Eq. 7 is replaced by the formula

$$\omega_{\mathrm{knob}} = \omega_0 \left(\frac{I_0 + I_P}{I_{\mathrm{knob}} + I_P} \right)^{1/2} , \quad (10)$$

where I_P is the equivalent MOI of the arms. In effect, a fixed energy supplied by the batter is shared between the bat and the arms, with the fraction going to the bat equal to $I_{\mathrm{knob}}/(I_{\mathrm{knob}}+I_P)$. The factor I_P in the numerator is inserted to assure that $\omega=\omega_0$ when I_{knob} equals the reference value of I_0. A formula similar to Eq. 10 has been proposed by Cross [19], who demonstrates that it works very well fitting the speed of balls of different mass thrown overhand. The essential physics in the throwing case is essentially the same as for the present case; namely, a fixed energy is converted to kinetic energy shared by the ball and the arm.

While Eq. 7 cannot be valid over a broad range of MOI, it can easily be shown to be equivalent to Eq. 10 over some limited range. In the present context, equivalent means that, given the experimental value of n, there is some choice of I_P such that $(I_{\mathrm{knob}}/\omega)d\omega/dI_{\mathrm{knob}}$ is numerically the same for the two expressions. It is straightforward to derive the necessary expression:

$$I_P = I_{\mathrm{knob}}\left(\frac{1}{2n}-1\right) . \qquad (11)$$

For $n = 0.29$ and $I_{\mathrm{knob}} \approx I_0$, $I_P \approx 0.238$ kg-m^2 (1.3×10^4 oz-in^2). If Eq. 10 is fit to the present batting cage data, an identical value is found for I_P, with the same value of ω_0 and the same fixed value of I_0. The fitted curve is indistinguishable from that shown in Fig. 6 over the range of I_{knob} shown, although the two curves diverge from each other at much lower I_{knob}. Note that since I_P is nearly as large as I_0, only about half of the total kinetic energy is in the bat, with the remainder in the arms. Again assuming a rotation axis at the knob of the bat, Eq. 10 leads to a swing speed formula

$$v_{bat} = v_0\left[\frac{L-z}{0.71\ \mathrm{m}}\right]\left(\frac{I_0+I_P}{I_{\mathrm{knob}}+I_P}\right)^{1/2} , \qquad (12)$$

which is taken to be the final result of this study.

In applying this formula to any given situation, it is important to keep in mind that both v_0 and I_P are likely to be batter-dependent quantities. For the college-level batters tested here, $v_0 \approx 31$ m/s (70 mph) and I_P=0.238 kg-m^2 (1.3×10^4 oz-in^2). For younger players, one might expect smaller values for both quantities, to be determined by further testing.

3.5 BBCOR and the Hoop Mode

According to our understanding of the trampoline effect, a more flexible barrel leads to less overall energy loss and therefore a higher BBCOR [5]. One measure of the barrel flexibility is the frequency of the lowest "hoop mode". All other things the same, the more flexible the wall of the bat, the lower the frequency of the hoop mode. Therefore, a correlation is expected between the BBCOR of a hollow bat and the frequency of the lowest hoop mode [8]. Such a correlation is shown in Fig. 8, which shows the BBCOR increasing as the frequency decreases, in agreement with the qualitative explanation.

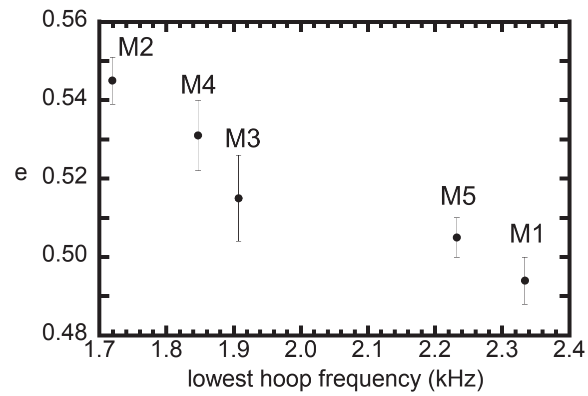

Fig. 8 Correlation between e and the frequency of the lowest hoop mode. For reference, the wood bats has e=0.452.

4 Summary and Conclusions

This paper has described an analysis of batting cage data that seeks to compare the performances among six different bat models. The analysis was supplemented with additional laboratory measurements of the bats, including both impact measurements of the collision efficiency and modal analysis of the vibrations. Our findings are summarized as follows:

1. The batted ball speed is strongly dependent on the BBCOR of the bat. The BBCOR was shown to be the primary distinguishing feature among the bats that determines field performance.
2. The batting cage measurements of the collision efficiency using a hand-held bat are in generally good agreement with the laboratory measurements using a stationary bat pivoted at the handle, both in mean value and in the dependence on impact location. This result confirms theoretical expectations [7,4] and previous experimental findings [13].
3. The bats are shown to be rotated about a point near the knob just prior to collision with the ball. The rotational speed of each bat is shown to vary as $1/I_{knob}^{0.29}$, in agreement with similar experiments on the swinging of sporting instruments [14–17]. An

explicit formula for the dependence of bat speed on the properties of the bat is derived and the limits of its validity are discussed.

4. The ball-bat coefficient of restitution is strongly correlated with the frequency of the lowest hoop mode for the hollow metal bats, as predicted by a simple model for the trampoline effect [8].

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