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METHODS FOR MEASURING THE COEFFICIENT OF RESTITUTION AND THE SPIN OF A BALL

By Lyman J. Briggs

ABSTRACT

Four methods for measuring the coefficient of restitution of a ball are discussed and employed experimentally. These methods are:

1. The two-pendulum ballistic method of Thomas, in which the ball is struck by a flat-nosed projectile driven from an airgun.
2. A method based on spark photography, by means of which the ratio of the speed of the ball to that of the projectile is determined.
3. The measurement of the vertical rebound of a ball from a massive horizontal plate, when dropped from a known height, correction being made for air resistance.
4. The measurement of the angle of reflection of a ball rebounding from a smooth inclined plate, the angle of incidence being known.

A correction for spin is necessary in method 4 if the plate is not ideally smooth. Methods are described for measuring the spin velocity, and an approximate method for computing the spin is given, provided the coefficient of restitution is known.

The variation of the coefficient of restitution of golf balls with impact speed and with temperature is experimentally determined, and a method for determining the time interval during which the ball remains in contact with the club is described.

The coefficient of restitution of a golf ball when hit hard is roughly 0.7; the corresponding value for a baseball of prewar construction is about 0.45.

CONTENTS

I. INTRODUCTION

The fundamental laws of impact are given in various books on mechanics, but there is little published information on the measurement of such properties as the coefficient of restitution. The National Bureau of Standards has been asked at various times to make measurements of this kind and if practicable to develop other methods for purposes of comparison. The four methods employed in the course of this work are presented here. The photographic and inclined-plate methods for measuring the coefficient of restitution and the procedure employed in measuring the spin of a ball in flight do not appear to have been used before.

The experimental results for golf balls were obtained in an investigation made at the request of the U. S. Golf Association in 1929, and the results for baseballs were obtained during tests made for the Services of Supply, War Department, and a joint committee of the American and National Baseball Leagues.

II. COEFFICIENT OF RESTITUTION

Let us consider a collision between two spherical bodies, the centers of which before the impact are approaching each other at the relative speed s_1 along a fixed straight line. If the density and the elastic properties of the two balls are symmetrically distributed about their geometrical centers, the rebound and separation occur along the same line as the approach. Let the relative speed of separation be denoted by s_2, and let

$$\frac{s_2}{s_1}=c. \tag{1}$$

Thomson and Tait[1] gave the name *coefficient of restitution* to this ratio.

In practice, it is often easier to measure the speeds of the two moving bodies separately than to measure the speed of one relative to the other, and equation 1 may be put into more convenient form for practical use by introducing the absolute velocities of the two bodies before and after collision.

Let the velocities of one of the bodies before and after the collision be denoted by U and V, respectively, and those of the other body by u and v, the line of motion and the positive direction along it being the same in all four cases, and U being greater than u. Then the relative speed of approach before the collision is

$$s_1=U-u,$$

and the relative speed of separation after it is

$$s_2=v-V,$$

so that the value of c is given by

$$c=\frac{v-V}{U-u}. \tag{2}$$

[1] Natural Philosophy, part I, p. 278 (1896).

The coefficient of restitution, c, is always less than unity, but approaches it as the colliding bodies become more nearly "perfectly elastic." Strictly speaking, c is determined by the elastic properties of both of the colliding bodies. But in systems such as we shall discuss, in which the collision occurs between a golf ball and a massive plate or a dense wooden club, the deformation of the ball is so much greater than that of the plate or club that we may, without sensible error, attribute the coefficient of restitution to the properties of the ball alone.

Sir Isaac Newton concluded from his experiments that, for any one pair of spheres, the ratio c was independent of the striking speed s, so long as the impact was not violent enough to produce a permanent deformation of either body. It will be evident from figure 8 that the coefficient of restitution of a golf ball is by no means independent of the extent to which the ball is momentarily deformed through collision with some other object, so that in reality $c=s_2/s_1=F(s)=f(d)$, where d is the deformation. This is not surprising when we consider that the ball is not homogeneous in structure. Consequently, in comparing balls or in comparing methods, it is essential that the impact speed of the balls be nearly the same; and this speed should preferably be so chosen as to give a deformation comparable with that which occurs in making a long drive.

The deformation is not the only factor to be considered in determining the coefficient of restitution. The time involved in the compression of the ball and its return to its normal shape must be essentially the same as the time interval during which a ball, when struck sharply, remains in contact with the club. In other words, the ball during the restitution test must always be free in the sense that it is not called upon to overcome the inertia of any mass other than its own. For this reason the measurement of the rebound of a heavy weight dropped upon a ball on a massive anvil is not a suitable way to determine the coefficient of restitution.

It is also desirable to bring a ball to some standard temperature before making comparative measurements, because the coefficient of restitution of a ball may change markedly with the temperature, as shown in figure 12.

Unless otherwise specified, the following data apply to the measurements given later for golf balls: Diameter of ball 1.62 inches; weight, 1.62 ounces; temperature, 20° C; impact speed, 140 ft/sec.

III. BALLISTIC METHOD

C. V. Boys [2] was among the first to measure the elastic characteristics of golf balls. He employed a ballistic pendulum of unique design, with five supports. A bag inside the pendulum box kept the ball from rebounding. The excursion of the pendulum was recorded on a smoked glass plate by means of a light scriber.

A novel apparatus for measuring the coefficient of restitution, involving the use of two ballistic pendulums, was developed by H. A. Thomas [3] for the U. S. Golf Association. The apparatus is illustrated in figure 1. A chamber, containing air at the desired pressure, is suddenly connected by means of a quick-acting valve with the breech

[2] Personal communciation. Work not published.
[3] The writer is indebted to Prof. Thomas for permission to include a description of his apparatus in this article.

of a horizontal air gun, which contains a flat-nosed wooden projectile. As the projectile leaves the gun, it strikes the ball, mounted on a rubber tee in front of the muzzle, and drives it through an opening in the first pendulum into the second, where it is caught. The projectile, following the ball, is caught in the first pendulum. The deflection of each pendulum is recorded by a rider sliding on a circular arc below the pendulum.

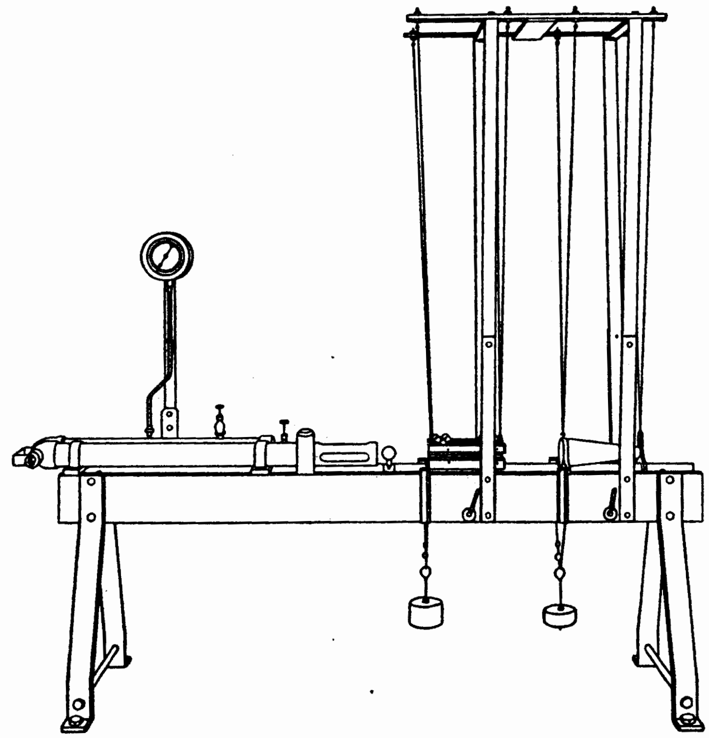

FIGURE 1.—*Two-pendulum apparatus developed by H. A. Thomas for measuring the coefficient of restitution of golf balls.*

From the observed deflection of the first pendulum, the height H through which the bob is raised by the impact of the projectile may be calculated, if the true length of the pendulum is known.

Let

M=mass of projectile

M_1=mass of first pendulum

V=velocity of projectile after ball has left it

V_1=maximum velocity of pendulum M_1.

Then $V_1=\sqrt{2gH}$.

From the equation of momentum, we have

$$MV=(M+M_1)V_1$$

or

$$V=\frac{M+M_1}{M}\sqrt{2gH}. \tag{3}$$

Similarly, the second pendulum of mass M_2 is raised through a height h by the ball of mass m moving with the velocity v; hence

$$v=\frac{m+M_2}{m}\sqrt{2gh}. \tag{4}$$

The total momentum of the ball and the projectile is not changed by the impact. Hence,

$$MU=MV+mv, \tag{5}$$

where U is the velocity of the projectile before impact.

Equations 3, 4, and 5 determine V, v, and U. Since the ball is at rest before being struck by the projectile, $u=0$. The coefficient of restitution may now be found from equation 2, which becomes, when $u=0$,

$$c=\frac{v-V}{U}. \tag{6}$$

In equation 6, U, V, and v are all positive, and $v>V$.

The standard test conditions proposed by Thomas for the ballistic method are as follows: $U=175$ ft/sec; $M/m=4$. Applying the principle of the conservation of momentum, it follows that for the above conditions the ball and projectile must have a common velocity of 140 ft/sec for an instant during the impact, which is defined as the impact speed. The speed v of the ball after rebound from the projectile is $140(1+c)$ ft/sec, or about 230 ft/sec for a ball with a coefficient of restitution $c=0.64$.

The Thomas apparatus is the most practical device known to the writer for the routine measurement of the coefficient of restitution of golf balls. It has also been applied to baseballs, as will appear later on.

IV. SPARK PHOTOGRAPHY AS A MEANS OF DETERMINING THE COEFFICIENT OF RESTITUTION

The absolute measurement of the speed of the projectile and of the ball is unnecessary if the *ratio* of their speeds can be measured in some way. This will be evident if we eliminate U from equations 5 and 6, from which

$$c=\frac{M\,(v-V)}{MV+mv}. \tag{7}$$

If for V in equation 7 we substitute

$$V=kv, \tag{8}$$

we have

$$c=\frac{(1-k)M}{m+kM}, \tag{9}$$

in which the coefficient of restitution c is expressed simply in terms of the known masses M and m of the projectile and ball, respectively, and the speed ratio k.

Spark photography provides one way of determining V/v. Let us assume two instantaneous records of the moving ball and projectile on the same photographic plate, the time interval t between exposures being unknown. Let S be the travel of the projectile (scaled from the plate) during the time t, and s the corresponding travel of the ball. Then $S/s = V/v$.

The apparatus developed by P. P. Quayle [4] was used in making the spark photographs. The spark gap was located in a dark room about 3 feet from the Thomas apparatus and so placed that a line from the spark gap to the ball on its tee was horizontal and normal to the trajectory. The photographic plate was placed in a vertical plane parallel to the trajectory, in such a position that when the ball was illuminated by the spark, its shadow was projected on the plate. By means of an adjustable trigger actuated by the projectile, the spark could be so timed as to illuminate the projectile and ball in various stages of their flight.

A series of records obtained with Quayle's equipment is shown in figure 2. In (a) the projectile is emerging from the barrel of the air gun and is about to strike the ball. In (c) the ball is greatly distorted by the impact of the projectile, but has barely started to move from its tee. In (d) the rebound of the ball from the face of the projectile is nearly complete, whereas in (e) the ball is free and is moving at a speed approximately twice that of the projectile.

Quayle's apparatus was not designed to give two exposures separated by a very small time interval, and consequently photographic records were made of two different flights, including in each the same stationary object, such as the muzzle of the gun, as a point of reference in measuring the records. Although the pressure was adjusted closely to give very nearly the same speed of the ball in the two flights, it is not necessary that the speed be identical in both cases. It may be found from figure 8 that for an impact speed in the neighborhood of 140 ft/sec, an increase of 1 percent in the impact speed decreases the coefficient of restitution, c, by only 0.003.

The coefficients of restitution of the same balls were measured independently by the method just described and by that of Thomas, with the results shown in table 1.

TABLE 1.—*Comparison of results obtained by ballistic and photographic methods.*

Method	Coefficient of restitution	
	Ball S	Ball G
Ballistic (Thomas)	0.629	0.662
Photographic (Briggs)	.626	.659

The determinations are in agreement within 1 part in 200. Each coefficient in the upper line is the mean of three determinations. The coefficients in the lower line are each based on a single determination, involving the measurement of two photographic plates. The two methods may be used simultaneously with the same ball.

The equipment developed by Edgerton and his associates for obtaining very short exposures under controlled conditions would

[4] *Spark photography and its application to some problems in ballistics,* BS Sci. Pap. **20**, 237 (1924) S508.

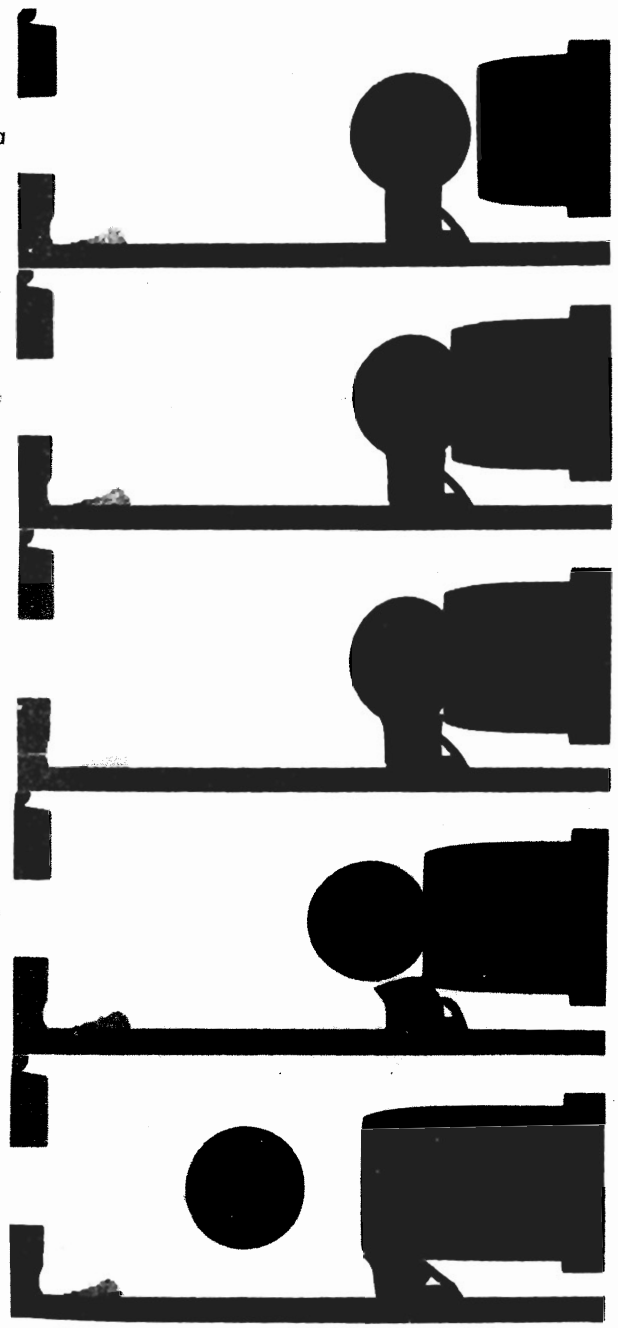

FIGURE 2.—*Deformation and recovery of a ball when struck by a projectile, as recorded by spark photography.*

appear to be very convenient in measuring the coefficient of restitution by the photographic method. (See Flash, published by Hale, Cushman, and Fleet, 1939.)

The velocities v and V are the velocities of the ball and the air-gun projectile when the elastic force between them has fallen to zero and they are moving independently. This begins to be true at the instant of separation; and if it were practicable to make two instantaneous photographs of the same flight, it would evidently be permissible to make the first at the instant of separation. Consequently, if the position of the face of the projectile at that instant can be determined by some other means, only one exposure is needed to determine k and thence c.

In figure 3 let the shaded parts represent the shadows of the ball and projectile as they appear on the photographic plate, and let the dotted part at the right represent the positions as they would have appeared if the plate had been exposed at the instant of separation.

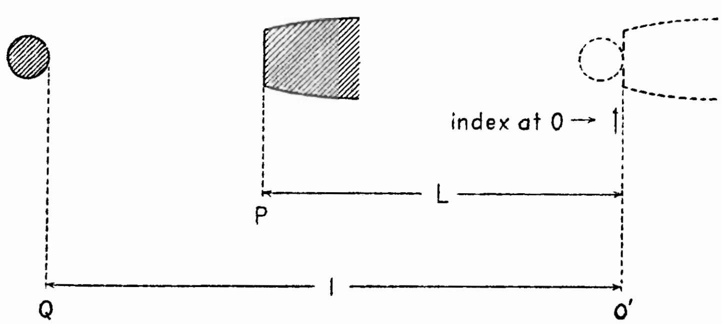

FIGURE 3.—*Determination of the coefficient of restitution by the photographic method.*

During the time elapsed since separation, the projectile has moved the distance $O'P=L$ and the ball has moved the distance $O'Q=l$. Hence, if we can determine the position O' along the trajectory, we have from equation 8

$$\frac{O'P}{O'Q}=\frac{L}{l}=k.$$

Figure 11 shows a double exposure on the same plate, the first recording the undisturbed ball on its tee, and the second so timed that the ball is just leaving the face of the projectile, which is therefore at the desired position O'. For an impact speed of 140 ft/sec, the ball rides in contact with the projectile for a distance less than half its diameter, as shown in figure 11. It may also be seen from the photograph that O' lies 0.1 inch nearer the muzzle of the gun than O, which is the position of the center of the ball before impact; and O' may thus be found without sensible error simply by applying this correction to measurements made with O as origin, the position of which may be shown by a suitable index in the photograph.

V. PERPENDICULAR REBOUND FROM A FLAT PLATE

Measurements of the coefficient of restitution were also made by dropping a ball in a vertical shaft from varying heights (up to 64 ft), allowing it to strike upon a massive horizontal steel plate, and measuring the rebound. If the experiment had been made in a vacuum, we would have

$$c=\frac{s_2}{s_1}=\sqrt{\frac{h}{H}},$$

where s_1 and s_2 represent the striking speed and speed of rebound, respectively, and H and h the length of the drop and rebound.[5] But owing to the resistance of the air, the striking velocity s_1 of the ball is less than $\sqrt{2gH}$, and in turn the height h to which the ball rises on its rebound is less than that corresponding to the velocity of rebound s_2. The error introduced by ignoring the air resistance is shown in table 2.

Experiment shows that for the velocities encountered in the rebound tests, the air resistance is proportional to the square of the velocity. The equation of motion of the falling ball is accordingly

$$m\frac{dv}{dt}=mg-kv^2,$$

where

m = mass of the ball, and
k = coefficient of air resistance.

Let $k=bm$. Then

$$\frac{dv}{bv^2-g}=-dt$$

$$\frac{2vdv}{v^2-\frac{g}{b}}=-2bvdt=-2bds.$$

Integrating,

$$v^2-\frac{g}{b}=Ce^{-2bs}.$$

Let s be measured from the point of release of the ball, positive downward. When $s=0$, $v=0$. Therefore, $C=-g/b$.

Let $v=v_1$ when $s=H$, where H is the height of the point of release. Then

$$v_1=\sqrt{\frac{g}{b}(1-e^{-2bH})}. \qquad (10a)$$

For rebound, the equation of motion is

$$m=\frac{dv}{dt}=-mg-kv^2$$

$$v^2+\frac{g}{b}=Ce^{-2bs}. \qquad (10b)$$

Let s be measured from top of rebound. Then $v=0$ when $s=0$. Therefore, $C=g/b$. Let $v=v_2$ when $s=h$. Then

$$v_2=\sqrt{\frac{g}{b}(e^{-2bh}-1)}.$$

[5] The term "coefficient of resilience" was applied by Lewis Gordon to the ratio $(s_2/s_1)^2$, which is equal to h/H when measured in a vacuum.

Therefore,

$$c \equiv \frac{v_2}{v_1} = \sqrt{\frac{e^{\frac{-2kh}{m}} - 1}{1 - e^{\frac{-2kH}{m}}}}\,. \tag{10c}$$

The coefficient of air resistance, k, was found by measuring the force exerted on a golf ball by the air-stream of a wind tunnel. For speeds up to 65 ft/sec, which exceeded the maximum speed in the dropping experiments, the resistance plotted against the square of the wind speed showed a linear relationship, as assumed in the derivation of equation 10c. The slope of this line is $k = mg/v^2 =$ 0.00030 pound/foot for a golf ball 1.62 inches in diameter, weighing 1.62 ounces.

Using this coefficient, it follows that the maximum speed attainable by the ball in falling in a standard atmosphere is 104 ft/sec; at this speed the air resistance equals the weight of the ball. The terminal speed would actually be somewhat greater, for k begins to decrease for speeds above 75 ft/sec. The deformation on impact at the terminal speed would nevertheless be much below that to which a hard-hit ball is subjected in play.

Table 2 contains a summary of rebound tests made with 16 brands of golf balls, the results being expressed as a ratio of rebound to drop (h/H). The first column gives the drop in feet; the second, the ratio of rebound to drop for the brand showing the highest rebound (mean of 12 observations); and the third, the ratio for the brand showing the lowest rebound. The mean value for the 16 brands is given in column 4.

TABLE 2.—*Determining the coefficient of restitution by measuring the vertical rebound*

Drop	Ratio of rebound to drop			Coefficient of restitution (from mean values)	
	Maximum	Minimum	Mean	Uncorrected for air resistance	Corrected for air resistance
ft					
13.4	0.654	0.574	0.620	0.787	0.814
43.9	.559	.493	.533	.730	.808
65.6	.500	.430	.467	.683	.787

The coefficient of restitution, based on the mean ratio and uncorrected for air resistance, is given in column 5, where each coefficient is the square root of the corresponding number in column 4; for if the air resistance is zero, $c = \sqrt{h/H}$. The last column gives the coefficient of restitution as determined by equation 10c. It will be noted that there is a marked difference, the uncorrected coefficient being 13 percent too small in the case of a ball dropped from a height of 65 feet.

If the correction for air resistance is made, the rebound method is reliable for speeds up to say 70 ft/sec. Above this limit, k can no longer be assumed to have a constant value. This places a practical limitation on the method, for the deformation produced by a fall from a height of 70 feet is only about one-half that to which a hard-driven ball is subjected.

VI. REBOUND FROM AN INCLINED PLATE

The coefficient of restitution may also be determined by measuring the angle of reflection of a ball after it strikes a smooth massive plate set at a known angle to the flight path.

Let *ao* (fig. 4) represent the path of the centroid of the ball as it approaches plate *M* with velocity *u* along *ao*, and let *P* represent the position of the centroid for maximum compression. After impact, the ball rebounds from the plate, the centroid moving along the path *ob* with the velocity *v*. Let the angle of incidence *aon* be represented by *i* and the angle of reflection *nob* by *r*.

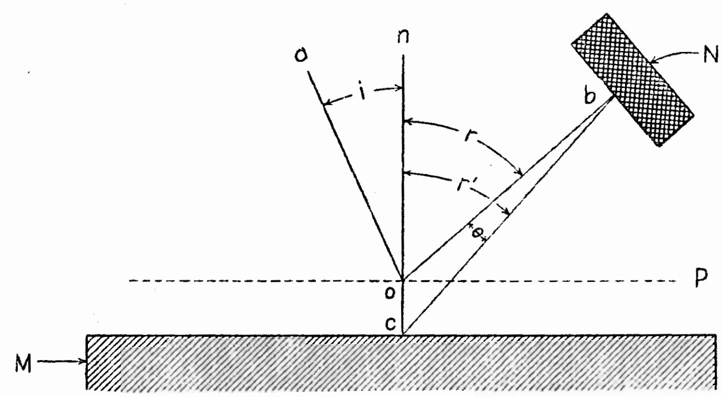

FIGURE 4.—*Rebound of a ball from an inclined plate.*

We shall assume that the plate is ideally smooth. Under such conditions no spin will be imparted to the ball by the impact, and the component of velocity of the ball parallel to the surface of the plate will be the same after impact as before. Hence

$$u \sin i = v \sin r. \tag{11}$$

The component of velocity normal to the plate after impact depends upon the coefficient of restitution, *c*, the numerical value of the normal component after impact being *c* times that of the normal component before impact.

Therefore,

$$cu \cos i = v \cos r. \tag{12}$$

Dividing equation 11 by equation 12,

$$c = \frac{\tan i}{\tan r}, \tag{13}$$

which determines *c* if *i* and *r* are known.

In making the measurements, a heavy block of steel was clamped to the frame of the Thomas machine in place of the second (ball) pendulum. The surface of this block was plane and vertical and was adjustable in azimuth about a vertical axis, the normal to the surface forming the angle *i* with the path of the approaching ball.

The ball after bounding horizontally from the inclined plate was caught in a bed of plastic clay (*N*, fig. 4). The clay was packed in a shallow tray and was stiff enough to retain its position when the tray was placed on end. The tray carrying the clay was adjustable in azimuth about the vertical axis of the steel block. The angle r' between the normal to the plate and a line drawn from the center of the imbedded ball to its imprint on the dusted surface of the plate plus the small computed angle θ, figure 4, gave the angle of reflection r.

Measurements were made (1) with the surface of the plate smooth and coated with vaseline, (2) smooth and dry, and (3) corrugated with vertical V-grooves 1 mm deep and 3 mm between centers. The speed of approach, u, was kept nearly uniform (175 ft/sec) by controlling the air pressure and by observing the deflection of the projectile pendulum. The results of the measurements are given in table 3.

TABLE 3.—*Determination of the coefficient of restitution from the angle of reflection*

Surface of plate	Angle of incidence	Angle of reflection	$\frac{\tan i}{\tan r}=c$	Ballistic method
	Degrees	*Degrees*		
Smooth and lubricated	19.5	30.6	0.60	0.60
	26.8	40.1	.60	.62
	43.5	55.9	.64	.67
Smooth and dry	26.7	31.6	.82	.62
	43.5	51.0	.77	.67
Corrugated	26.6	24.7	1.09	.62
	44.1	41.0	1.11	.67

The angles of reflection represent in each instance the mean of at least six determinations. The coefficient of restitution $\tan i/\tan r$ as determined by this method is given in the fourth column of the table. For comparison, the last column gives the coefficient as determined by the ballistic method for an impact speed equal to the normal component ($u \cos i$) of the speed of approach. These values were obtained from figure 8 by reading from the graph the coefficient corresponding to an impact speed of 175 cos i ft/sec.

It will be noted that the ratio $\tan i/\tan r$ for balls rebounding from a smooth lubricated surface is in fair agreement with the coefficient of restitution as determined by the ballistic method. With this surface, the theoretical assumptions (1) that the tangential component of the velocity is unchanged by impact, and (2) that the ball acquires no spin velocity through impact, appear to have been realized. The method does not, however, give reliable results if the surface of the plate is smooth and dry or if it is rough (see table 3). Under such conditions, spin momentum is imparted to the ball during rebound and equation 13 no longer applies. It will be noted from table 3 that with a corrugated surface the angle of reflection may actually be less than the angle of incidence, leading (according to equation 13) to the conclusion that the coefficient of restitution is greater than unity, which is impossible.

VII. MEASUREMENT OF SPIN

P. G. Tait [6] appears to have made the first measurements of the spin of a golf ball. He fastened one end of a long light tape firmly to the ball, and after removing all twist, secured the other end to the

[6] Trans. Roy. Soc. Edinburgh 39, pt. II, 494 (1896–99).

ground at a point such that the tape was perpendicular to the direction of driving. The ball was then driven into a vertical clay bed 4 feet from the tee. The tape was found to be twisted from one to two full turns after a drive, "indicating something like 60 to 120 turns per second."

1. MEASUREMENT OF SPIN BY SPARK PHOTOGRAPHY

Spark photography provides an accurate means for measuring spin, if the translational speed of the ball is known. The angular rotation is determined from the change in position of a light marker inserted in the ball (see fig. 5), and the corresponding linear travel is scaled from the photographic record. In figure 5 (e) the ball was 4.87 inches from the tee when the record was made; while traveling this distance it rotated 90° about a horizontal spin-axis normal to the flight path, corresponding to an angular motion of 0.62 turn per foot. The linear speed could not be measured by the ballistic method in this case, because the reaction of the ball with the inclined face of the projectile threw both ball and projectile out of line with the ballistic pendulums. The linear speed was estimated to be about 175 ft/sec on the basis of similar experiments with square-nosed projectiles, which gave a spin velocity of 108 rps.

2. MEASUREMENT OF SPIN FROM RESIDUAL TURNS

The following method, used in measuring the spin imparted to a ball in rebounding from an inclined plate, avoids the necessity of attaching anything to the ball.

Identifying marks are made on the surface of a ball at opposite ends of a diameter, and the ball is so located on its tee that these marks lie in the projection of the horizontal flight path. After the ball strikes the vertical plate (inclined to the flight path by an angle i), the marks rotate about the vertical spin-axis of the ball until it is brought to rest in a bed of plastic clay. By comparing the azimuthal position of one of the marks on the ball in the clay bed with its azimuthal position on the tee, the fractional part of a revolution made by the ball during its flight from the plate to the clay bed may be determined. If these fractions are measured for two or three suitably chosen distances of the clay bed from the plate, the whole number of turns may be established with certainty by the procedure illustrated in figure 6.

Let the observed fractional turn of the ball, after traveling 1.5 feet, be 0.6; and after traveling 2.5 feet, 0.3. The fraction is added to integral numbers in sequence and the result plotted against the distance. The only straight line that can be drawn through a pair of these points and the origin is shown in the graph. Its slope gives the number of turns per foot of travel; and this, multiplied by v, gives the spin velocity.

Measurements of spin by this method were made at two angles of incidence (table 4). The spin observed after rebound from a corrugated plate (i=26.6°) was 139 rps; and for i=44.1°, 240 rps. From a smooth plate (i=26.7°) the observed spin was 158 rps. The latter measurement was also checked by the tape method, which gave approximately the same result.

It will be shown later (fig. 11) that when a ball is struck by a moving 's

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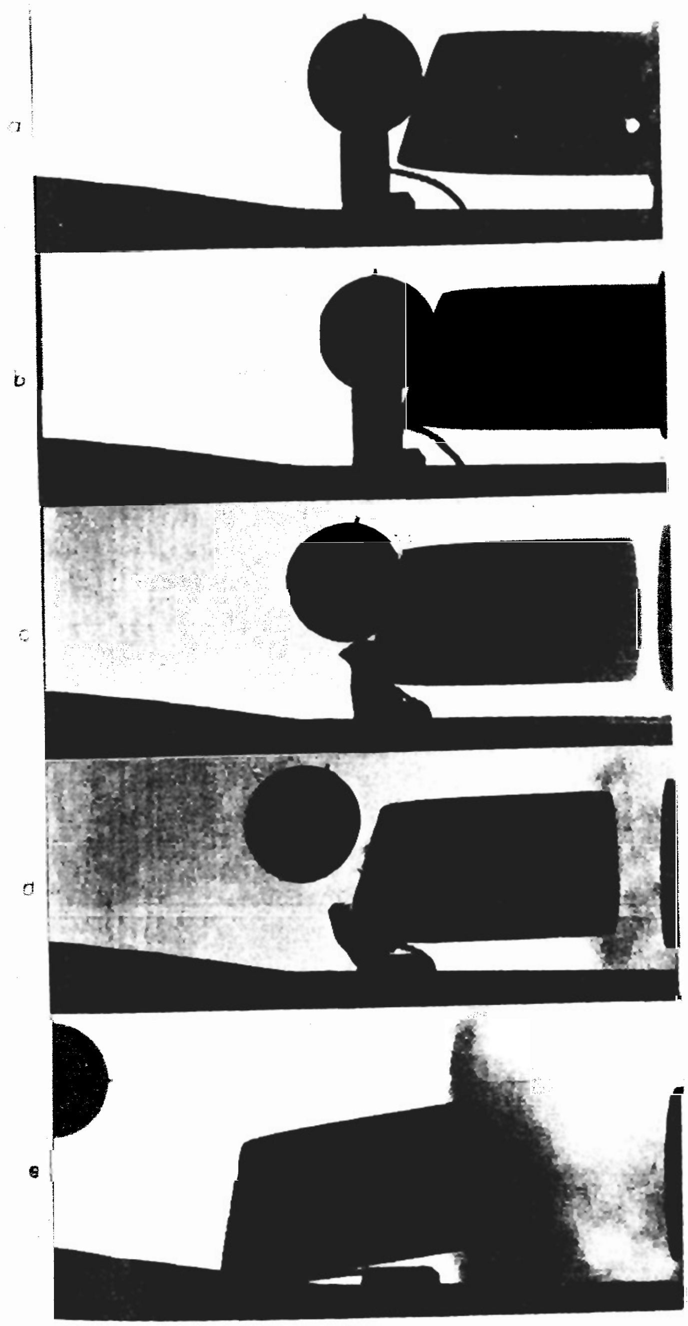

FIGURE 5.—*Measurement of spin by spark photography.*

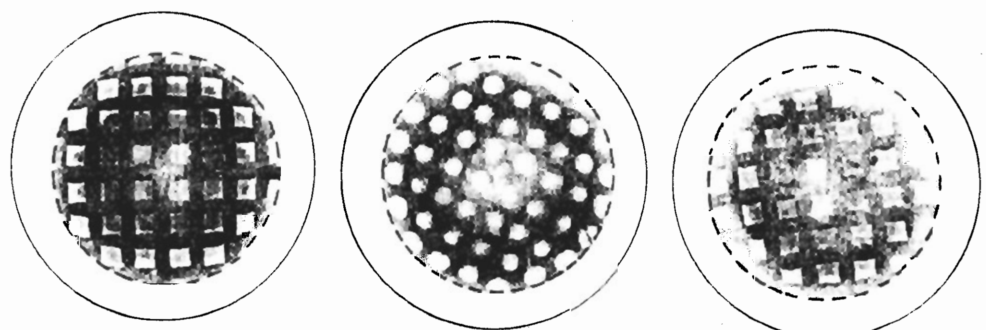

FIGURE 9.—*Imprints made by golf balls striking a flat plate at a speed of 140 feet per second.*

Outer circle represents maximum cross section of the undeformed ball.

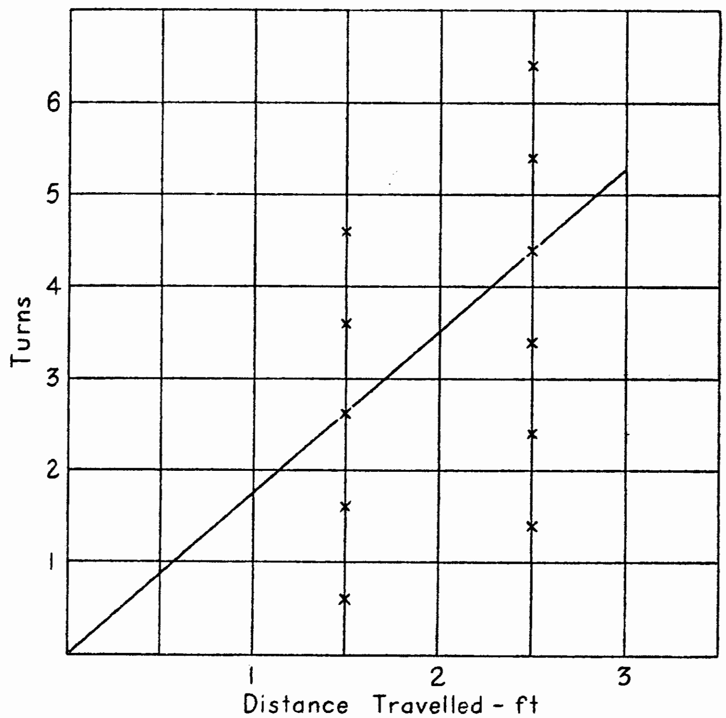

FIGURE 6.—*Graph illustrating the use of residual turns in computing the spin of a ball.*

projectile the time interval of contact is about 0.0004 second. From this we may compute the approximate rotation that a ball undergoes while in contact with an inclined plate from which it is rebounding. For a spin of 158 rps (i=26.7°), the rotation is $2\pi \times 158 \times 0.0004 = 0.4$ radian, or 23°. The rotation of the ball shown in figure 5 (d) was measured directly and found to be 23°. The normal to the face of the projectile used in this case was inclined at an angle of 20° to the longitudinal axis of the projectile.

3. COMPUTATION OF SPIN

The spin velocity may be determined approximately by computation if the speed of approach, u, the angle of incidence, i, the angle of reflection, r, and the coefficient of restitution, c, are known.

Let v be the velocity after rebound and v_n and v_p the components of v normal to and parallel to the plate, respectively. Then the component of the impact speed normal to the plate is

$$u_n = u \cos i$$
$$v_n = cu_n = cu \cos i$$

$$v = \frac{v_n}{\cos r} = cu \frac{\cos i}{\cos r}. \tag{14}$$

Therefore,

$$v_p = v \sin r = cu \cos i \tan r. \tag{15}$$

The velocity v_p represents the translational speed of the ball as it rolls along the surface of the plate during the contact. The rough surface of the plate is assumed to prevent slipping. Consequently, the spin velocity, v_s, is the ratio of v_p to the effective circumference $(2\pi a)$ of the ball as it rolls on the plate.

Let d represent the distance of the centroid of the ball from the plate at the instant of maximum deformation. It is evident that the numerical value of the effective radius a must lie somewhere between d and the radius, r, of the undeformed ball. The closer a approaches d, the greater the spin; but the deformation represented by d is attained only momentarily, and the spin inertia of the ball prevents it from responding fully. In the absence of more definite information, we assume

$$a = r - (2/3)(r-d),$$

and the computed values of spin given in table 4 are based on this assumption.

To find $(r-d)$, the deformation of a ball was measured for different static compression loads and the corresponding area of contact recorded by means of carbon paper. The relation is shown in figure 7. Similar carbon prints (see fig. 9) were obtained from a ball striking a flat plate at a velocity u. Interpolation of the latter results in figure 7 gave the deformation $(r-d)$ corresponding to u.

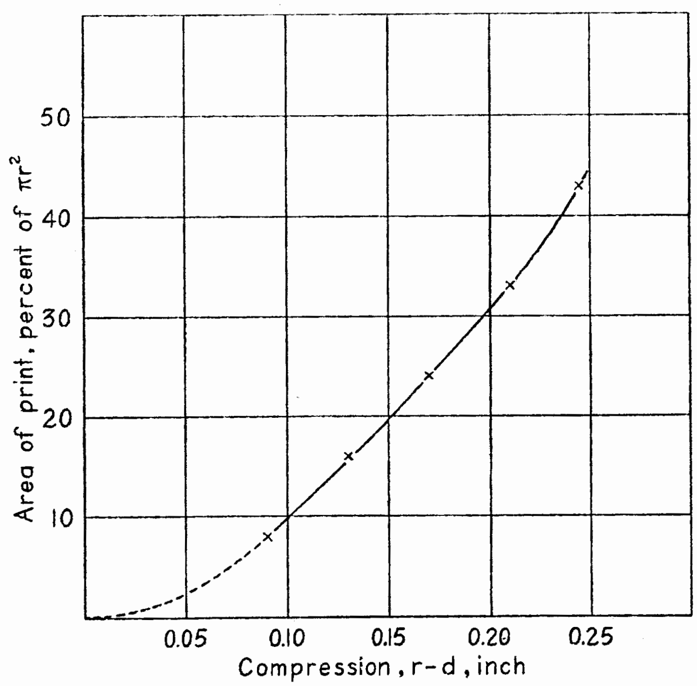

FIGURE 7.—*Relation, during impact, between area of contact on a flat surface and the compression measured along a radius normal to the surface.*

It will be seen from table 4 that the computed values of spin for the rough plate accord with the observed values of spin within 10 percent.

For the smooth plate, the computed value is too large, as might be predicted, because the ball skids to some extent.

Table 4.—*Observed and computed spin velocity of a ball rebounding from an inclined plate*

[Speed of approach, 175 ft/sec]

Surface of plate		r	c	v_p	$2\pi a$	Spin velocity	
						Computed	Observed
	Degrees	*Degrees*		*ft/sec*	*ft*	*rps*	*rps*
Corrugated	26.6	24.7	0.62	44.5	0.324	138	139
Do	44.1	11.0	.67	73.1	.330	222	240
Smooth	26.7	31.6	.62	59.5	.324	184	158

VIII. CORRECTION FOR SPIN

The spin energy imparted to a ball when it rebounds from an inclined plate is derived at the expense of the energy of translation parallel to the plate. If the loss in energy of translation could be considered as confined to that which reappears as spin energy, it would be possible to determine the coefficient of restitution of a ball rebounding with spin at an angle r, provided the spin velocity is known. The impact of a ball on a plate is, however, accompanied by frictional energy losses (skidding) brought about by the spin inertia of the ball, and these frictional losses cannot be measured directly.

The spin energy of a ball of mass m, radius a, and angular speed ω is $m\omega^2a^2/5$. If we add this to the parallel component of the energy of translation after rebound, we obtain for the parallel component of energy before impact (ignoring the frictional loss)

$$\frac{1}{2}mu_p^2=\frac{1}{2}mv_p^2+\frac{1}{5}ma^2\omega^2, \qquad (16)$$

where u_p and v_p are the components of velocity parallel to the plate before and after impact.

Solving for v_p,

$$v_p=\sqrt{u_p^2-\frac{2}{5}a^2\omega^2}. \qquad (17)$$

But

$$u_p=u \sin i$$

and

$$v_p=v \sin r$$

Therefore,

$$v=\frac{\sqrt{u^2 \sin^2 i-\frac{2}{5}a^2\omega^2}}{\sin r}. \qquad (18)$$

From equation 14

$$c=\frac{v \cos r}{u \cos i}.$$

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Therefore,

$$c=\frac{\tan i}{\tan r}\sqrt{1-\frac{2a^2\omega^2}{5u^2\sin^2 i}}. \tag{19}$$

When the spin velocity is zero, the right-hand term under the radical vanishes and equation 19 reduces to equation 13.

Substituting in equation 19 the necessary data from table 3, we obtain for a ball rebounding with spin from a smooth, dry, inclined plate ($i=26.7°$) the value $c=0.68$. The corresponding value by the ballistic method is $c=0.62$ (table 3). The former discrepancy between the inclined plate and ballistic methods (table 3) is greatly reduced by the correction for spin, but the influence of the neglected friction term is still in evidence. The discrepancy is greater in the experiments with a rough plate, as one would expect if external friction reduces the velocity parallel to the plate.

The experiments with an inclined plate lend a singular reality to the analytical conception of a velocity resolved into two mutually perpendicular components. The normal and parallel components of the velocity of a ball are both modified by the impact of the ball on an inclined plate, but for quite different reasons. The normal component after impact is reduced to c times its former value, because the ball is not perfectly elastic (internal friction); and the parallel component is smaller after impact than before, because during impact spin energy is imparted to the ball, and external frictional forces come into play.

IX. EFFECT OF IMPACT SPEED ON THE COEFFICIENT OF RESTITUTION

Newton concluded from his experiments that the coefficient of restitution of a ball is independent of the deformation, provided the ball is not permanently deformed by the impact. While this may be true for a ball that is uniform in structure throughout, such as a steel ball, it does not hold for a golf ball, with liquid core, rubber winding, and vulcanized cover.

The coefficient of restitution of a golf ball is largest for small deformations of the ball and decreases as the deformation increases, that is, as the impact speed increases. The relationship as experimentally determined is shown in figure 8. The graph is based on average values, and consequently the coefficient of restitution of an individual brand and its rate of change with impact speed may depart from the relationship shown. The data for speeds up to 60 ft/sec were obtained by observing the height of rebound (table 2); for higher speeds, the ballistic method was used.

When the rebound method is used, the impact speed may be computed from equation 10a. In the ballistic method, the impact speed is the speed of the ball at the instant of its greatest deformation, which is the instant at which the ball and projectile have the same velocity. Hence, from the equation of momentum, the impact speed is

$$v_0=\frac{MU}{M+m}.$$

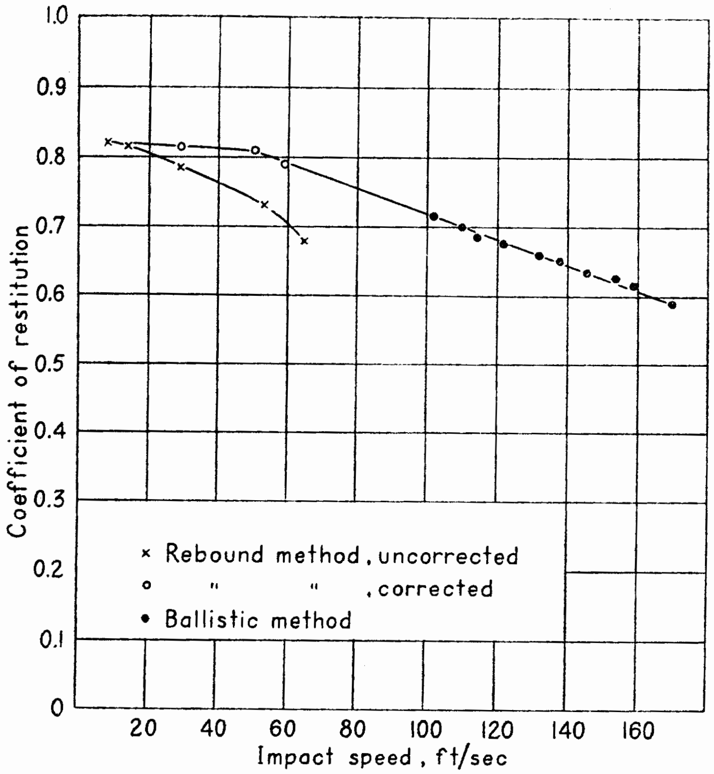

FIGURE 8.—*Reduction in the coefficient of restitution with increase in the impact speed (increase in deformation).*

In the ballistic measurements, the mass M of the projectile was 4 times the mass of the ball; hence $v_0=0.8U$.

The shorter graph in figure 8 represents the results obtained by the rebound method (table 2, mean values), with no correction for air resistance. The corrected values, which in figure 8 lie above and slightly to the left of the uncorrected points, conform fairly to a smooth curve passing through the ballistic determinations.

It is of interest to note that the decrease in the coefficient of restitution with increase in impact speed automatically handicaps a strong player to some extent; for the harder a ball is hit, the less is the percentage gain in speed resulting from the rebound of the deformed ball from the face of the club.

X. CONTACT AREA OF A DEFORMED BALL

In discussing figure 2, attention was called to the marked deformation of a golf ball during impact. This may also be shown by carbon-paper prints of the area of contact. Figure 9 represents the actual size of the imprint made by a ball 1.62 inches in diameter in striking a flat plate at a speed of 140 ft/sec. The circle around each imprint encloses the projected area of the undeformed ball. The area of contact at this impact speed is more than one-half the maximum cross section of the undeformed ball.

The relation between contact area and impact speed is shown graphically in figure 10, each point representing the mean of observations made with three brands of balls. For speeds up to 100 ft/sec, the contact area, within experimental limits, is proportional to the speed. At higher speeds this linear relationship begins to break down, as the ball cannot flatten out indefinitely.

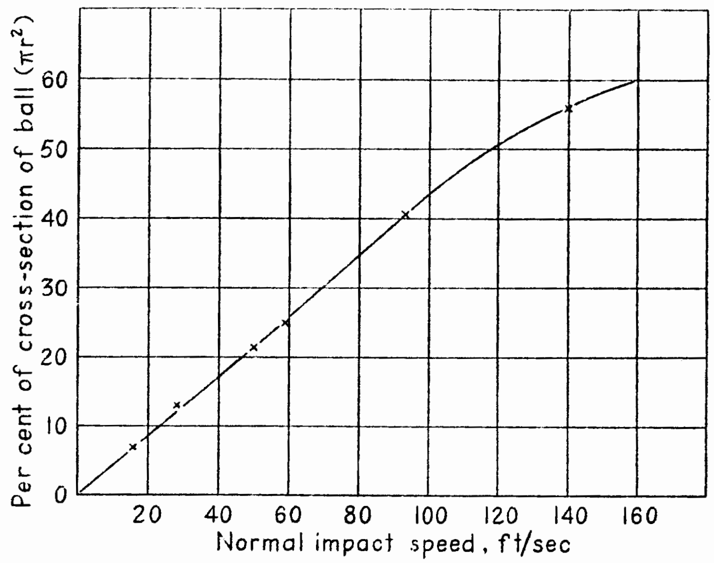

FIGURE 10.—*Contact area on impact is approximately proportional to impact speed up to 100 feet per second.*

XI. DURATION OF CONTACT BETWEEN BALL AND PROJECTILE

Figure 11 represents a spark photograph in which two exposures were made on the same plate. The first exposure was made with the ball on its tee, and this position of the ball is indicated by a circle defined for the most part by a black area with its center at O. The second exposure was so timed as to record the position of the ball very nearly at the instant of its separation from the projectile after the impact. The position of the ball at this instant is shown by a second circle, defined partly in gray and partly in black, the latter part representing the overlapping of the two positions. The outline of the projectile will be seen in gray at the right, only part way out of the black muzzle of the gun. O' lies in the vertical line of contact between the ball and the projectile.

Figure 11 provides a basis for computing the time interval Δt during which the ball and projectile remain in contact. The initial speed of the projectile was 175 ft/sec; its final speed, 120 ft/sec. We assume an average speed of 148 ft/sec during the time interval Δt. The distance traveled by the projectile during Δt may be scaled from the figure. It is the distance from the face of the projectile to that point

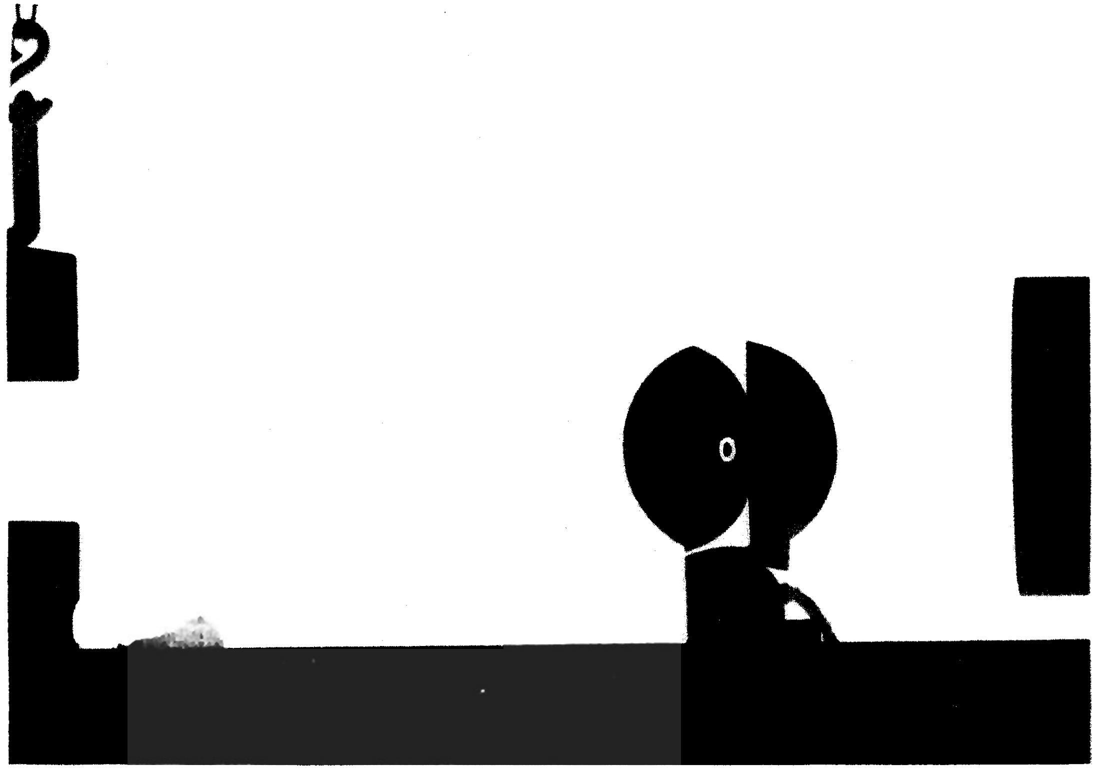

FIGURE 11.—*Spark photograph with double exposure.*

The first exposure shows the ball on its tee (center at *O*). The second exposure shows the ball just about to leave the face of the projectile. Total time of contact between ball and projectile is only 0.0004 second.

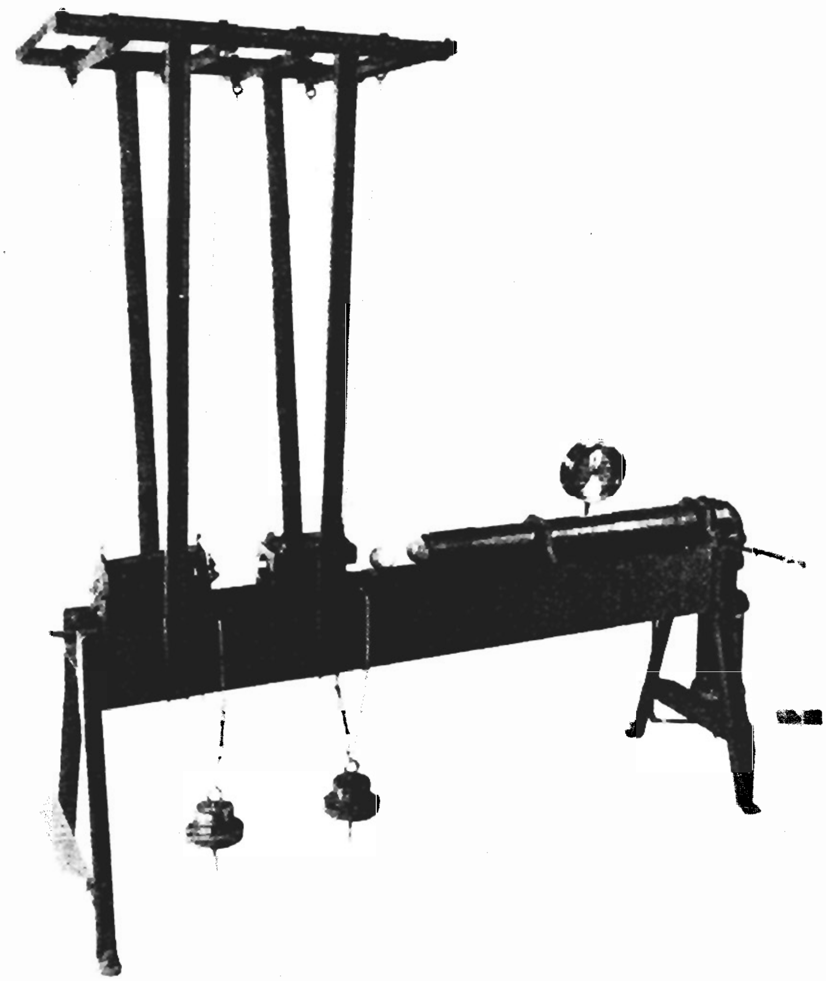

FIGURE 13.—*Thomas apparatus constructed at the National Bureau of Standards for measuring the coefficient of restitution of baseballs.*

on the undisturbed ball first touched by the projectile (correction being made for the slight magnification of the photographic record). This distance was found from figure 11 to be 0.70 inch. Hence $\Delta t = 0.70/(12 \times 148) = 0.0004$ second, approximately.

The use of a harder ball would decrease somewhat the time interval during which the projectile is in contact with the ball, because the deformation of the ball would be less, and consequently the distance traveled by the projectile in deforming the ball would be diminished.

XII. INFLUENCE OF TEMPERATURE ON THE COEFFICIENT OF RESTITUTION

Temperature has a marked effect on the coefficient of restitution, as will be seen from figure 12, which gives the results obtained with three brands of balls, each point representing the mean of observations on six balls. The line is drawn through the points for one brand. The balls were brought to the desired temperature in a liquid bath, removed one at a time, dried and tested as quickly as possible. For the lower temperatures, the bath consisted of gasoline in a vacuum-jacketed flask, to which solid carbon dioxide was added at a rate sufficient to maintain the desired temperature. A projectile speed of 175 ft/sec was used in all the measurements.

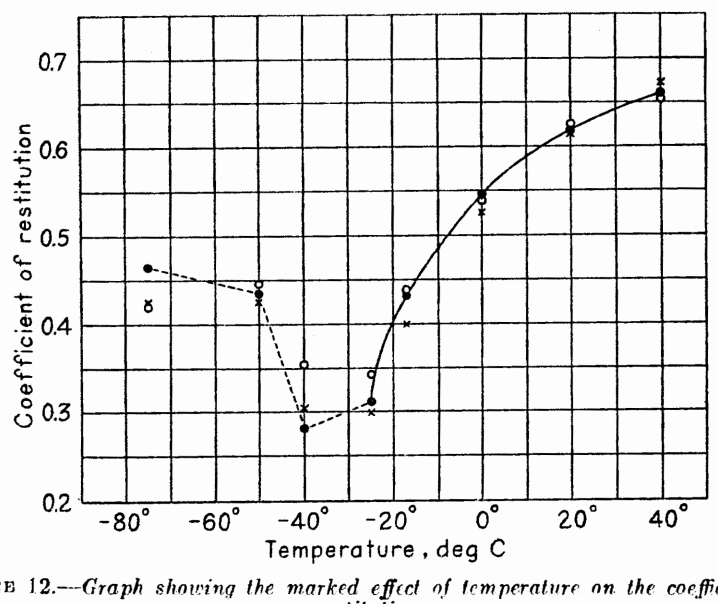

FIGURE 12.—*Graph showing the marked effect of temperature on the coefficient of restitution.*

The reversal in the graph in the neighborhood of $-30°$ C is probably associated with the solidification ("freezing") of the rubber, the increase in c at lower temperatures being attributable to an increase in rigidity. At $-50°$ C the value of c was about the same as at $-15°$ C, but a ball at $-50°$ C sounded like a stone when dropped on the floor. At $-75°$ C the cover of a ball often flew into pieces when struck by the projectile.

The effect of temperature changes on c is so great that it is advisable to make all routine measurements at a standard temperature (20° C). A correction for small departures may be made by means of the equation

$$c_{20}=c_t-0.003\ (t-20),$$

in which c_t is the coefficient of restitution determined at t° C.

It will be seen from figure 12 that at 0° F (−18° C) the coefficient of restitution of a golf ball is only about two-thirds of what it would be on a hot summer day. A lively ball in summer thus becomes a dead ball in winter, unless it is kept warm.

XIII. VARIATION IN THE COEFFICIENT OF RESTITUTION OF GOLF BALLS

We have finally to consider the effect of size, weight, and construction on the coefficient of restitution of golf balls, and the probable effect of such differences on the distance carried in play. The variation observed in c at low speeds has already been shown in table 2. Later tests of 18 brands (3 balls each; diameter 1.62 in.; weight 1.62 oz; impact speed, 140 ft/sec) by the ballistic method gave for the mean of the series c=0.62, the range being from 0.54 to 0.64.

By driving balls first into a ballistic pendulum, and then over a measured course by means of a driving machine, Thomas found that the carry of a drive was roughly proportional to the speed of the ball as it left the tee. On this basis, increasing c from 0.54 to 0.64 would represent a gain of about 7 percent in distance (carry), assuming the balls to be alike in other respects.

Observed values of c for balls of various diameters and weights are given in table 5, the coefficient in each instance being the mean value found from single measurements of 12 balls. It will be noted in the case of each brand that the increase in size is accompanied by a slight increase in the value of c, that is, the larger ball is the livelier. The greater air resistance and decreased weight of the 1.68:1.55 ball, which preceded the 1943 official ball, was thus offset in part by an increase in the coefficient of restitution.

TABLE 5.—*Comparison of balls of various diameters and weights*

Brand and year	Diameter	Weight	c	Average deviation	Maximum deviation	
	in.	*oz*			+	−
A—1929	1.62	1.62	0.620	0.008	0.015	0.020
	1.68	1.55	.657	.010	.015	.023
B—1929	1.62	1.62	.620	.004	.012	.010
	1.68	1.55	.689	.003	.008	.008
C—1929	1.62	1.62	.630	.005	.014	.013
	1.68	1.55	.649	.007	.014	.011
A—1931	1.68	1.62	.726	.011	.022	.030
	1.68	1.55	.666	.005	.009	.015

A comparison was also made (for a single brand only) of the 1.68:1.55 ball and the 1.68:1.62 ball authorized in 1932. It will be seen from table 5 that in this case the coefficient of restitution of the new ball is again slightly greater than that of the 1.68:1.55 ball, which it superseded. Attention is also called to the uniformity of manufacture of the 1.68:1.55 balls, the earlier determination agreeing (within the average deviation) with that made over 2 years later. These measurements were made in each instance soon after the balls were manufactured.

The last three columns of table 5 show the deviation of observations by the ballistic method. Departures from the mean similar to those shown for B may reasonably be classed as experimental errors associated with the method. The larger deviations are attributable to actual differences in the balls. When we consider the complex structure of a golf ball (liquid core, rubber winding, vulcanized cover), the degree of uniformity attained is noteworthy.

XIV. COEFFICIENT OF RESTITUTION OF BASEBALLS

The ballistic method of Thomas has also been used in the measurement of the coefficient of restitution of baseballs with different types of centers.[7]

A new machine was built for this purpose with heavier pendulums and with a barrel 3.5 inches in internal diameter (see fig. 13). The weight of the wooden projectile was 1 pound.[8] A muzzle velocity of 175 ft/sec was used in these measurements, equivalent to an impact speed of 130 ft/sec. As before, the projectile was propelled by compressed air stored in a chamber that could be connected with the gun through a quick-acting valve.

In the first series of measurements the coefficients of restitution of American and National League balls were compared. Six balls of each brand were used, each ball being measured three times. The results were practically identical for the two brands, the average coefficient of restitution being 0.46 in each case, with a mean deviation of less than 0.01 in each set.

This result was confirmed by additional driving tests made at a ball park. The gun was mounted near the home plate, the pendulums removed, and the muzzle elevated 30° above the horizon. At this angle the projectile moving at a speed of 175 ft/sec knocked the ball into the center-field bleachers. The speed was accordingly reduced so that the carry could be measured. Each of the six balls in each lot was driven three times. The average carry was 366 ± 8 feet for one lot and 367 ± 5 feet for the other. The figure following the $\pm$ sign represents the average deviation.

Some of the balls in each group were livelier (had a higher coefficient of restitution) than others , but the individual differences were not large. The results again showed that the American League and the National League balls were on the whole practically identical in performance.

Laboratory and driving tests of new baseballs constructed in accordance with the specifications used in 1924 were also included in the

[7] These measurements were made by members of the staff of the Division of Mechanics and Sound, National Bureau of Standards, to whom the writer is indebted for permission to incorporate their results in this paper. (See also Technical News Bulletin NBS, April 1938.) The work was first undertaken at the request of a joint committee of the American and National Baseball Leagues and later for the Services of Supply, War Department. The latter measurements were made to determine the effect of proposed changes in the construction of baseballs for Army camps, necessitated by the scarcity of rubber.

[8] The official limits for the circumference of a baseball are 9 to 9.25 inches; for the weight, 5 to 5.25 ounces. The mass of the projectile was thus about three times that of the ball.

tests of American and National League baseballs. The average coefficient of restitution of balls of this type of construction was 0.47, showing that, despite popular belief, no marked change in the "liveliness" of baseballs occurred between 1924 and 1942. On the driving tests, the average carry of the baseballs made to 1924 specifications was 372 ± 10 feet, which is in substantial agreement with the corresponding tests on baseballs manufactured under 1938 specifications.

It may be remarked at this point that the coefficient of restitution of a baseball is not as stable as that of a golf ball. All the balls of a given lot sometimes drift slowly in one direction or the other. The cause of this has not been determined, but it seems not unlikely that it is associated with the moisture content of the ball. In contrast with golf balls, the leather cover of a baseball is readily permeable to moisture. Baseballs conditioned first at 20-percent relative humidity and later at 90-percent relative humidity gained about a half ounce in weight in the process. Very humid or very dry weather may thus result in changes in the tightness of the winding of the wool yarn which constitutes five-sixths or more of the volume of the ball.

The measurements carried out for the War Department, using the laboratory procedure outlined above, gave the results reported in table 6.

TABLE 6.—*Coefficient of restitution of baseballs*

Type of center	Coefficient of restitution
Reclaimed rubber, 1943	0.42
Balata-cork; official, 1943	.40
Cushioned cork; official, 1938	.46

It will be seen that the official 1943 baseball has a coefficient of restitution of 0.40 compared with 0.46 for the prewar ball. These values of course do not reflect directly the performance of the balls in play. In the course of the deformation experienced by a ball when struck by a club, there is one instant at which the ball and club have the same speed. Let us call this speed unity. The ball immediately starts to restore itself to its spherical form, and in doing so rebounds from the club and thereby gains an additional fractional speed equal to its coefficient of restitution. The ratio of the relative speeds of the 1943 and 1938 balls under the given experimental conditions is then $(1+0.40)/(1+0.46)=0.959$.

It remains to determine the probable effect of the reduction in the coefficient of restitution on the performance of the 1943 ball in play. Balls of prewar construction have not been available for direct comparison by means of driving experiments with the official ball of 1943. Recourse must be had to earlier driving experiments in which balls with similar coefficients of restitution were compared.[9] In these experiments, the horizontal distances traversed by balls with coefficients of restitution of 0.39 and 0.44 were 328 and 353 feet, respectively, under the same driving conditions. The ratio of the speeds

[9] The coefficient of restitution of these balls was measured at the National Bureau of Standards and the driving experiments were made by A. G. Spaulding & Bros. at Chicopee, Mass., in 1938.

was (1+0.39)/(1+0.44)=0.965. On this basis, a hard-hit fly ball with a 1943 center might be expected to fall about 30 feet short of the prewar ball hit under the same conditions.

I record my indebtedness to Richard L. Lloyd, who carried out many of the foregoing measurements; to Hugh L. Dryden for his aid in determining the corrections to be applied for the air resistance of a falling ball; and to the late Edgar Buckingham for his constructive suggestions.

WASHINGTON, February 14, 1944.