Analysis of PITCHf/x Pitched Baseball Trajectories

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Abstract

An analysis is presented of 99 baseball trajectories measured by the PITCHf/x tracking system pitched by Boston Red Sox lefthander Jon Lester in the August 3, 2007, game against the Seattle Mariners at Safeco Field. The analysis technique is described in detail. The drag and lift coefficients and the magnitude and direction of the spin vector are extracted and discussed. A pitch classification scheme is proposed based on the velocity and the magnitude and direction of the spin.

I. ANALYSIS TECHNIQUE

The coordinate system is the usual one used in the PITCHf/x system.[1] Namely, the origin is at the point of home plate, $\hat{y}$ points towards the pitcher, $\hat{z}$ points vertically upward, and $\hat{x} = \hat{y} \times \hat{z}$ (i.e., the x axis points to the catcher's right). The only data utilized was the 9-parameter fit to each trajectory, determining the initial position, velocity, and acceleration for each of the three dimensions. For the particular data set analyzed, the initial tracking location was at $y = 50$ ft. Using those parameters, the trajectory (x,y,z vs. t), was calculated, from which "pseudodata" were generated at 1/60-sec. intervals. Treating the pseudodata as real data, the trajectory was fit to a theoretical model[2] in which the ball is subjected to the forces of gravity

$$\vec{F}_G = -mg\hat{z}\,, \tag{1}$$

the drag force $\vec{F}_D$

$$\vec{F}_D = \frac{1}{2}\rho A C_D v\vec{v}\,, \tag{2}$$

and the Magnus force $\vec{F}_M$

$$\vec{F}_M = \frac{1}{2}\rho A C_L v^2(\hat{\omega} \times \hat{v})\,. \tag{3}$$

Here m is the ball mass, g is the acceleration due to gravity (32.2 ft/s^2), ρ is the air density, A is the cross sectional area of the ball, ω is the angular velocity, and C_D and C_L are the drag and lift coefficients, respectively. Defining $K = 0.5\rho A/m$, which has the numerical value of 5.44×10^{-3} ft^{-1}, the equations of motion are

$$\begin{aligned}
\ddot{x} &= -KC_D v v_x \,-\, KC_L v v_y \sin\psi\sin\phi \,+\, KC_L v v_z \cos\psi \\
\ddot{y} &= -KC_D v v_y \,+\, KC_L v\,(v_x \sin\phi - v_z\cos\phi) \\
\ddot{z} &= -KC_D v v_z \,+\, KC_L v v_y \sin\psi\cos\phi \,-\, KC_L v v_x \cos\psi \,-\, g\,.
\end{aligned} \tag{4}$$

In these expressions, the spin axis makes an angle ψ with the y axis; its projection in the x-z plane makes an angle ϕ with the x axis, with a sign such that $\phi = 90°$ corresponds to the spin pointing upward, along the z axis (see Fig. 1). With three components of acceleration (one for each direction), there is not enough information to determine the four unknown parameters (C_D, C_L, ϕ, and ψ). However, we note that generally $|v_y| \gg |v_x|, |v_z|$ so that the terms in Eq. 4 involving $\cos\psi$ are small. These are "gyroball-like" terms, since they are proportional to the component of spin in the y direction. We therefore make the simplifying assumption that ψ=90° so that these terms vanish and the spin vector lies in the x-z plane. Noting that v_y is negative, it is easy to see that the Magnus force makes an angle $\theta = \phi - 90°$ with the x axis. Therefore $\phi = 0°$ (topspin) results in a downward acceleration, and $\phi = 90°$ (sidespin) results in an acceleration to the catcher's right, exactly as expected. The non-linear least-squares fitting utilizes the Levenberg-Marquardt algorithm,[3] with C_D, C_L, and ϕ as the fitted parameters. With each iteration in the fitting, the trajectory has to be computed by solving the equations of motion numerically using 4th order Runge-Kutta. The fitting is very robust, converges quickly–the fitting code processed the 99 pitches in a few seconds–and is not sensitive to the initial estimate of the parameters. Typical root-mean-square deviations of the fitted trajectory from the pseudodata were 0.04 inches (1 mm) in each dimension.

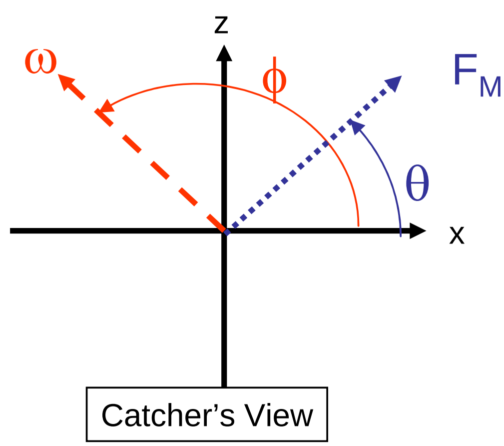

FIG. 1: Coordinate system as viewed by the catcher. The y axis points into the plane, towards the pitcher. The red dashed arrow shows the orientation of the spin axis, which makes an angle ϕ with the x axis, and the blue dotted line shows the direction of the Magnus force, which makes an angle θ with the x axis, where $\theta = \phi - 90°$.

It is worthwhile mentioning that approximate results can be obtained for C_D, C_L, and ϕ without an elaborate fitting procedure. Since the primary motion of the ball is along the $-\hat{y}$ direction (i.e., v_x and v_z are ≈ 0), then Eq. 4 implies that $F_D \approx ma_y$ and $F_M \approx m\sqrt{a_x^2 + (a_z+g)^2}$, from which C_D and C_L can be derived. Moreover, the angle of the Magnus force relative to the x axis is approximately $\arctan{(a_z+g)/a_x}$, and this angle is $\phi - 90°$. These approximate expressions have been verified by direct comparison with the exact ones derived from the fitting procedure. The drag coefficient can alternately be derived approximately by comparing the initial and final velocities since $v_f/v_0 \approx 1. - 0.00538 C_D \Delta y$, where Δy is the distance traversed between the initial and final points. While these approximate results are useful for a cursory look at the relevant parameters, all the results discussed below utilize the full fitting procedure.

II. DRAG, LIFT, AND SPIN

Some of the results of the fitting are plotted in Fig. 2-3, which will now be discussed. We start by looking at the drag and lift coefficients in Fig. 2. The values of C_D fall in the narrow range 0.40-0.50. There seems to be a weak dependence on v_0, starting with 0.45 in the 70-75 mph range, falling to 0.40 near 85-90 mph range, then rising to 0.45 above 90 mph. The value for 95 mph is somewhat high compared to the wind tunnel measurement of Briggs, 0.31, on a non-spinning baseball. Conventional wisdom is that C_D is about 0.50 for $v < 50$ mph and is in the range 0.35-0.40 for $v > 90$ mph. Between 50 and 90 mph, there is some controversy. Adair[4] suggests a gradual decrease in C_D. Sawicki et al.[5] suggests a "drag crisis" in which there is a sharp decrease starting 70-80 mph range, followed by a more gradual increase. Keeping in mind that the present data have no points in the 75-85 mph range, there seems to be no evidence for a drag crisis. Both Adair and Sawicki et al. have $C_D \approx 0.35$ at 95 mph.

The lift coeffient is expected to depend on the so-called spin parameter $S = R\omega/v$, where R is the radius of the ball. Indeed, recent data from Alaways,[2] Nathan,[6] and others[5] show that C_L is approximately equal to S for values of ω and v appropriate for pitched baseballs. Keeping in mind that the spin is not measured in the PITCHf/x system, the values of C_L are quite consistent typical values expected. One way to demonstrate that is to derive an approximate value for ω by assuming that the measured values of C_L are equal to $R\omega/v$, then by seeing if these values of ω seem reasonable. This assumption is reasonably consistent with published data on C_L. Note that this technique for determining ω is not sensitive to a component of spin along the direction of motion (mainly the y direction), since this component does not contribute to the Magnus force. A histogram of these values is plotted in Fig. 3 where it is seen that ω falls into two clusters, centered roughly at 800 and 1900 rpm, values that we consider reasonable. The lift coefficient along with the velocity determines the size of the Magnus force F_M. The distribution of these values, normalized to the weight of the ball, is shown in Fig. 3. These values cover the range 0-0.7. A value larger than 1 would be needed to have a "rising fastball."

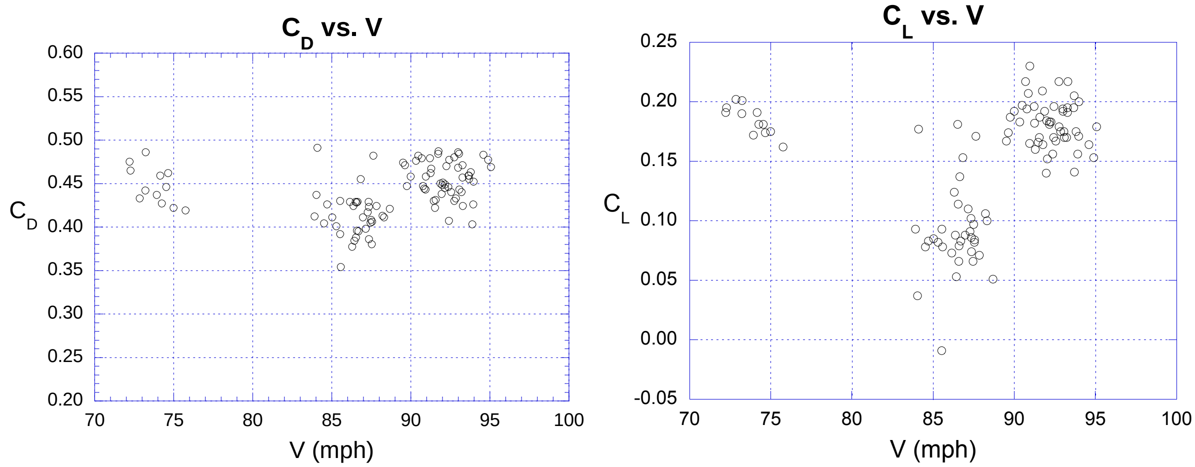

FIG. 2: C_D (left) and C_L (right) vs. initial velocity.

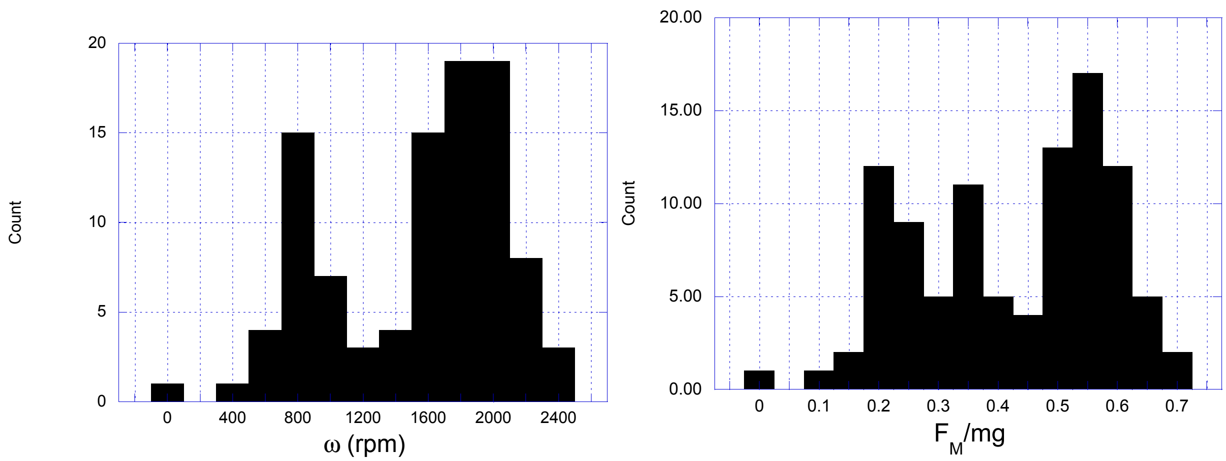

FIG. 3: (left) Distribution of values of the inferred spin ω, which are derived from the lift coefficient assuming $C_L = R\omega/v$. (right) Distribution of F_M normalized to the weight of the ball.

III. PITCH CLASSIFICATION

We next seek the most effective way to classify types of pitches. A promising technique is to classify them according to the velocity v, the spin magnitude ω, and the spin axis ϕ. This technique is investigated in Fig. 4-6. Fig. 4 are scatter plots showing both v and ω

as a function of ϕ. Equivalent information is shown in Fig. 5 plotted as a function of θ, the angle of the Magnus force. Finally, Fig. 6 shows both the break of the pitch, as seen by the catcher, and the correlation between ω and v. The break, dz and dx, is defined to be the deviation of the pitch from a straight-line trajectory due to the spin in the z and x directions, respectively. These quantities are similar to the quantities pfx_z and pfx_x that are part of the PITCHf/x database, except that the latter quantities refer to the break from a starting location 40 ft from the point of home plate whereas the former start from 50 ft. All these plots show four distinct categories of pitches, which we now discuss.[7]

I These pitches are most probably 4-seam fastballs thrown nearly overhand. The velocity v is in the range 90-95 mph, the spin axis is around 170°, corresponding to nearly perfect backspin with a small component of sidespin that makes the ball break in to a left-handed hitter by 3-6 inches. The positive value of vertical break means the ball drops 15-20 inches less than it would under the influence of gravity alone. The spin magnitude is roughly 2000 rpm.

II These pitches are thrown slower (85-90 mph) and with less spin (800 rpm) than the 4-seam fastballs, so that they break less. The spin axis is around 240° resulting in a pitch that breaks away from a left-handed hitter (2-5 inches) and drops about 10 inches more than a 4-seam fastball. It is quite possible that the actual spin is much larger but that it is oriented so that it has a sizeable component in the direction of motion. Such a pitch would be the signature of a slider.

III These pitches are very similar the 4-seam fastball, but thrown slightly slower at 90 mph, with a similar spin of 2000 rpm, but with the spin axis pointing at 135°. Such a pitch drops about 5 inches more than the 4-seam fastball but breaks about 12 inches in to a left-handed hitter. These pitches could be 4-seam fastballs thrown with a 3/4 arm angle, rather than overhand.

IV These pitches are definitely curveballs, thrown at a slower speed (70-75 mph) and less spin (1500 rpm) than the 4-seam fastball and with the spin axis at about 240$\circ$. Such a pitch breaks down (10 inches) and away (5-10 inches) from a left-handed batter.

References

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[1] For an explanation of the PITCHf/x system, see the web site webusers.npl.uiuc.edu/~a-nathan/pob/pitchtracker.html.

[2] L. W. Alaways and M. Hubbard, "Experimental determination of baseball spin and lift," J. Sports Sci. **19**, 349-358 (2001).

[3] William H. Press et al., *Numerical Recipes in Fortran* (Cambridge University Press, New York, 1986), pp xxx-yyy.

[4] R. K. Adair, *The Physics of Baseball* (HarperCollins, New York, 2002) 3rd ed., p.8.

[5] G. S. Sawicki, M. Hubbard, and W. Stronge, "How to hit home runs: Optimum baseball bat swing parametersfor maximum range trajectories," Am. J. Phys. **71**, 1152-1162 (2003).

[6] A. M. Nathan, "The effect of spin on the flight of a baseball," Submitted to Am. J. Phys., July 2007.

[7] Thanks to Dr. Dave Baldwin for his critical look at this section.

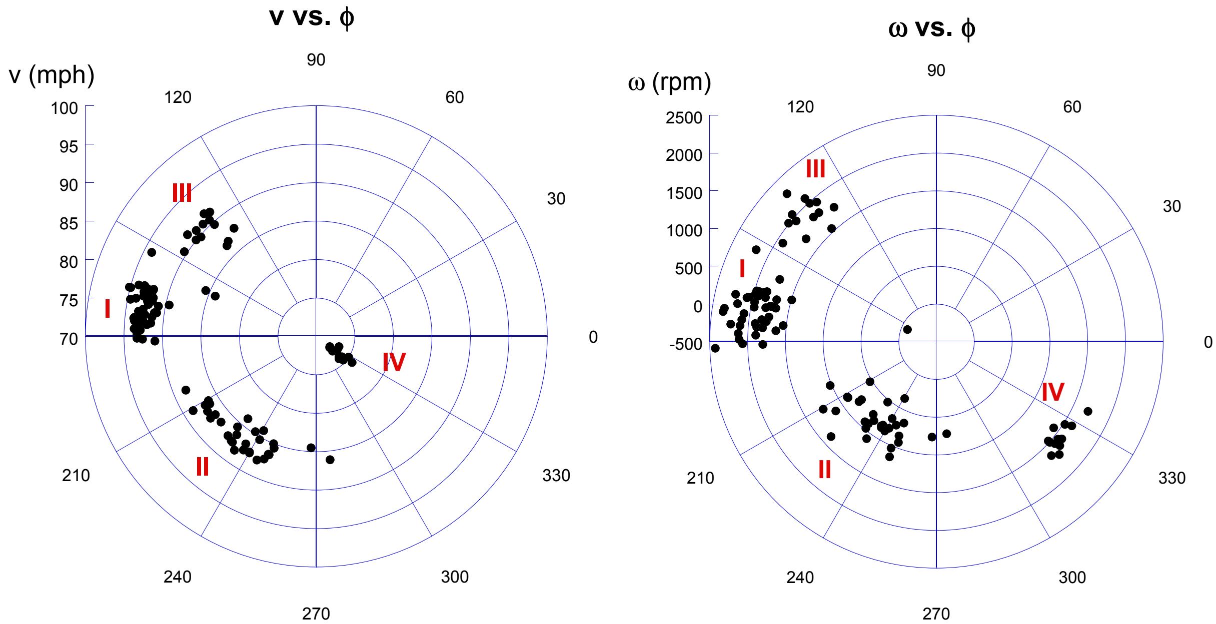

FIG. 4: Polar scatter plot of the pitched ball velocity (left) and the inferred spin (right) vs. ϕ, the angle between the spin axis and the x axis. In both cases quantity plotted (v or ω) increase radially outward from the center of the circle. The Roman numerals denote the pitch category, as discussed in the text.

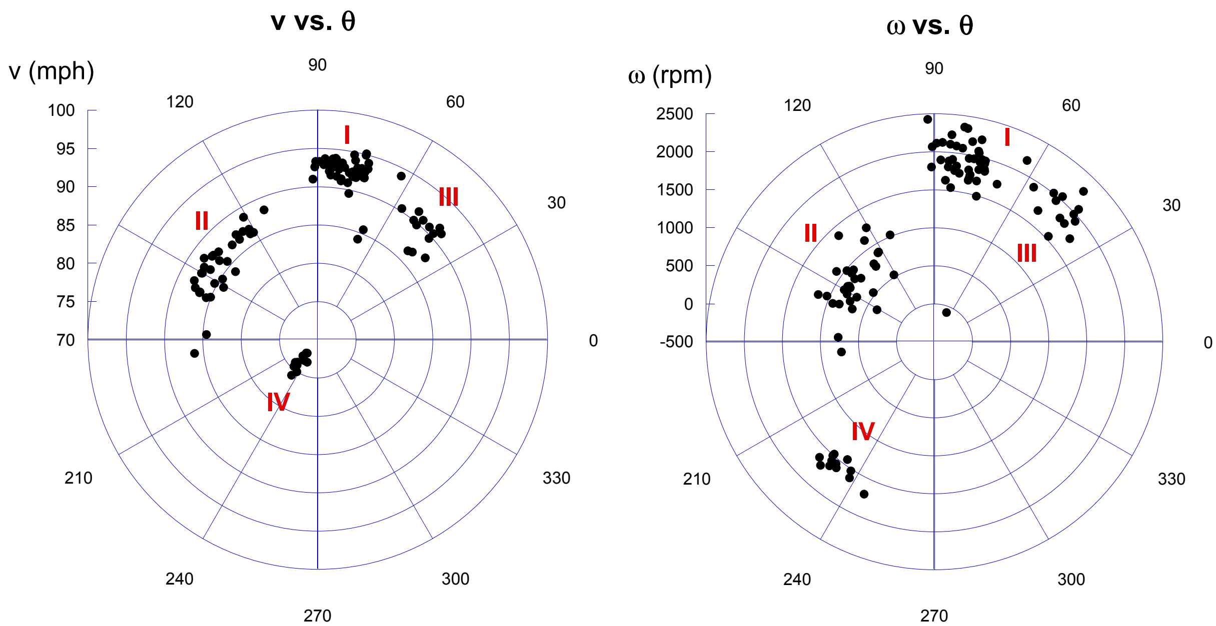

FIG. 5: Polar scatter plot of the pitched ball velocity (left) and the inferred spin (right) vs. θ, the angle between the Magnus force vector and the x axis. In both cases quantity plotted (v or ω) increase radially outward from the center of the circle. Since $\theta = \phi - 90^\circ$, the these plots are obtained by rotating those of Fig. 4 clockwise by 90°. The Roman numerals denote the pitch category, as discussed in the text.

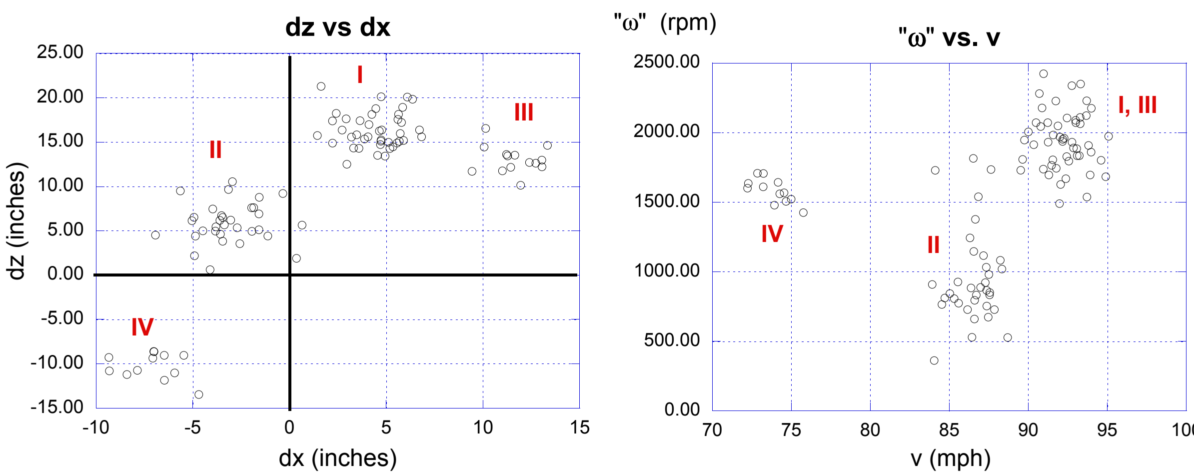

FIG. 6: (left) Scatter plot of the break of the ball as seen by the catcher. For each of the two dimensions, the break is defined as the deviation from a straight line trajectory between the initial tracking point y=50 ft and the front of home plate y=1.417 ft. (right) Scatter plot of pseudo-spin ω vs. initial ball speed. The Roman numerals denote the pitch category, as discussed in the text.