The effect of spin on the flight of a baseball

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Measurements are presented of the Magnus force on a spinning baseball. The experiment utilizes a pitching machine to project the baseball horizontally, a high-speed motion analysis system to determine the initial velocity and angular velocity and to track the trajectory over 5 m of flight, and a ruler to measure the total distance traversed. Speeds in the range $v=50-110$ mph and spin rates ω (topspin or backspin) in the range 1500–4500 rpm were utilized, corresponding to Reynolds numbers of $\mathrm{Re}=(1.1-2.4)\times 10^5$ and spin factors $S\equiv R\omega/v$ in the range 0.090–0.595. Least-squares fits were used to extract the initial parameters of the trajectory and to determine the lift coefficients. Comparison is made with previous measurements and parametrizations, and implications for the effect of spin on the flight of a baseball are discussed. The lift coefficient C_L is found not to depend strongly on v at fixed values of S.
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I. INTRODUCTION

It is well known to players, fans, and even physicists that a spinning baseball curves. In the language of physics, a spinning baseball in flight experiences the Magnus force, which contributes along with gravity to the deviation of a baseball from a straight-line trajectory. The earliest experimental investigations of the Magnus force were conducted by Isaac Newton,[1] who studied the motion of spinning tennis balls. A recent review of the Magnus force on a variety of different sports balls (baseball, tennis, golf, cricket, volleyball, and soccer) has been given by Mehta and Pallis.[2] Were Newton alive today, he would surely know that the Magnus force is responsible for much of the subtlety in the battle between pitcher and batter, such as the sideways break of a slider or cutter, the drop of a curveball with topspin, and the "hop" of a fastball with backspin. He would also understand that home run hitters typically undercut the baseball to put backspin on the struck ball because they instinctively know that the Magnus force on a ball with backspin is primarily vertically upward, so that balls hit on a home run trajectory stay in the air longer and travel farther.

It might surprise Newton that more than 300 years after his initial investigations, we still do not have a completely quantitative description of the Magnus force and its effect on the flight of a baseball, despite the many experimental and computational studies that have appeared in the literature. Computational studies of the effect of spin on the flight of hit baseballs have been reported by Rex,[3] Watts and Baroni,[4] and Sawicki *et al.*[5] These studies utilized models of the Magnus force based on the available experimental information, which will be reviewed in Sec. II. Additional computations are reported in the books of Watts and Bahill[6] and Adair.[7] Experimental data on the effect of spin on the flight of pitched baseballs have been reported in Refs. 8–11. To our knowledge, there are no experimental data on the effect of spin on the flight of hit baseballs.

The most recent and extensive computational study was reported in Refs. 5 and 12 in which the optimum bat-swing parameters for producing the maximum range of a batted baseball were determined. They used models for the ball-bat collision and the aerodynamic forces on a baseball and tracked the ball from collision to landing. For given initial parameters of the pitch (speed, angle, and spin), the bat swing angle and undercut distance were varied to maximize the range. The study found the surprising result that an optimally hit curveball travels farther than an optimally hit fastball, despite the higher batted-ball speed of the fastball. The physics underlying this result is that, in general, a baseball will travel farther if projected with backspin. It will also travel farther if it is projected with higher speed. A fastball will be hit with a higher speed, and a curveball will be hit with greater backspin.[13] It then becomes a question as to which effect wins. The calculations of Ref. 5 showed that the latter effect wins and the curveball travels farther. This conclusion depends critically on the magnitude of the Magnus force on a spinning baseball. A particular model was used for the Magnus force based largely on experimental data (see Sec. II). This model and conclusions have been criticized by Adair,[14] who claims that the effect of spin on the flight of a baseball was overestimated in Ref. 5.

The question of whether a curveball can be hit farther than a fastball is not an issue of primary importance to a physicist. The issue is the quantitative effect of spin on the flight of a baseball, regardless of whether the trajectory is that of a long fly ball, a popup to the infield, or a pitched baseball. The goals of the present paper are to report new experimental data for the Magnus force and to use these data to update and extend the earlier investigations of the effect of spin on the flight of pitched or batted baseballs. The present experiment, including the data reduction and analysis, is described in Sec. III. The results are compared to previous determinations of the Magnus force in Sec. IV and the implications for the flight of a baseball are discussed. A summary of our conclusions is given in Sec. V.

II. PREVIOUS DETERMINATIONS OF THE MAGNUS FORCE

When a spinning baseball travels through the atmosphere, it experiences the force of gravity in addition to the drag and Magnus forces, F_D and F_M, as shown in Fig. 1. Conventionally the magnitudes of these forces are parametrized as

$$F_D=\frac{1}{2}C_D\rho A v^2, \tag{1}$$

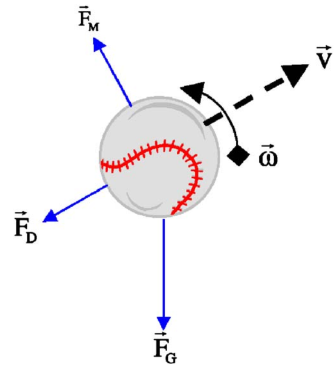

Fig. 1. Forces on a spinning baseball in flight. The drag force F_D acts in the $-\mathbf{v}$ direction, the Magnus force F_M acts in the $\hat{\omega}\times\hat{v}$ direction, and the force of gravity F_G acts downward.

$$F_M = \frac{1}{2}C_L\rho A v^2, \tag{2}$$

where A is the cross sectional area of the ball, v is its speed, ρ is the air density $(1.23\ \mathrm{kg/m^3})$, and C_D and C_L are the drag and lift coefficients, respectively.[15] We will focus only on C_L. Data on other spherical sports balls suggest that C_L is mainly a function of the spin factor $S=R\omega/v$, although it may also be a function of the Reynolds number $\mathrm{Re}=2\rho Rv/\mu$.[2,16] Here R is the radius of the ball, ω is the angular velocity, and μ is the dynamic viscosity of the air. For a standard ball and air at normal temperature and pressure, $\mu=1.85\times10^{-5}$ N-s/m^2, so that $\mathrm{Re}=2180v$ and $S=8.53\times10^{-3}\omega/v$ with v in mph and ω in rpm.

Previous determinations of C_L for a baseball are shown in Fig. 2, along with the present data that we will discuss. The earliest data are those of Briggs,[17] in which the deflection of a spinning baseball falling under the influence of gravity in a horizontal wind tunnel was measured, from which F_M and C_L were determined. The data cover the range $S=0.1-0.3$, with velocities in the range 45–90 mph, and were the data used in the study by Rex.[3] The experiment found the surprising result that F_M is proportional to ωv^2, implying that C_L is proportional to vS rather than to S. More recently, it was argued that an important correction needs to be applied to the Briggs data,[17] increasing the C_L values by approximately 50% and bringing them more in line with the expectation that C_L is proportional to S.[12] The corrected values are shown in Fig. 2. A more extensive set of data was taken by Watts and Ferrer[18] by using strain gauges to measure the force on a spinning baseball in a wind tunnel. These data cover the range $S=0.4-1.0$, but only for speeds up to about 37 mph. These data are roughly consistent with $C_L=S$, independent of Re, which is the parametrization used by Watts and Baroni.[4,6] The data of Alaways[8,9] were obtained using a high-speed motion analysis technique to measure the spin vector and track the trajectory of a pitched baseball. From these data C_L was extracted for S in the range $S=0.1-0.5$ and for speeds up to approximately 75 mph. The data of Jinji[11] were taken using standard video cameras to track the trajectory of a pitched baseball and a high-speed camera to measure the spin and spin axis just after release. Data were taken using both live pitchers and a pitching machine. Only the pitching machine data are shown in Fig. 2 because the live data display too much scatter to be useful. The plotted data cover the narrow range $S=0.2-0.3$ and $v=65-78$ mph.

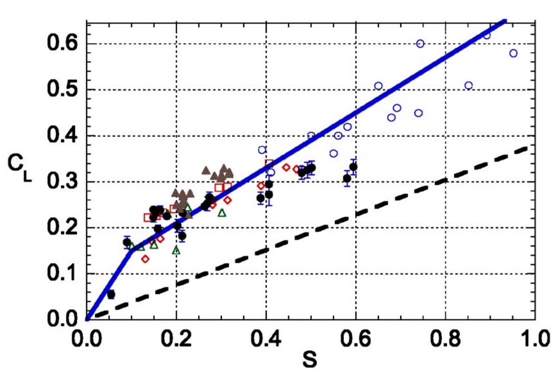

Fig. 2. Experimental results for C_L. The closed circles are from the present experiment. Open circles are from Watts and Ferrer (Ref. 18), open triangles are from Briggs (Refs. 12 and 17), open diamonds and squares are from Alaways two- and four-seam (Ref. 23), respectively (Refs. 8 and 9), and closed triangles are from the pitching machine data of Jinji (Ref. 11). Also shown are the parametrizations of Ref. 5 (solid) and Eq. (3) (dashed), the latter calculated for a speed of 100 mph.

Not shown in Fig. 2 are the wind tunnel data of Refs. 19 and 20 in which C_L was measured over a broad range of $S=0.2-0.9$ and $v=32-73$ mph. For $S<0.4$, these data follow the general trend of the Alaways data,[8,9] but fall well below the Watts and Ferrer data[18] at higher S values. Interestingly, the data of Refs. 19 and 20 show little or no dependence on Re for fixed S in the range investigated. More interesting is that a reverse Magnus effect (that is, $C_L<0$) was observed for a smooth ball with the dimensions of a baseball. For example at $v=55$ mph, $C_L<0$ occurs for $S=0.15-0.55$. A reverse Magnus effect was also observed on a smooth ball by Briggs.[17] No such effect has been reported for a real baseball.

The parametrizations used in Refs. 5 and 7 are also shown in Fig. 2. The parametrization of Ref. 5 is a bilinear function of S, which is an approximate fit to the Alaways[8,9] and Watts and Ferrer[18] data, assuming that C_L is independent of Re for a fixed S, as suggested by the available data. The parametrization of Ref. 7 comes from the heuristic argument that the Magnus force on a spinning ball is related to the difference in drag between two sides of the ball that pass through the air at different surface speeds due to the rotation, an argument first posed by Newton,[1] but more recently criticized in Ref. 12. Such an argument leads to the expression[7]

$$C_L = 2C_D S\left[1 + \frac{v}{2C_D}dC_D/dv\right]. \tag{3}$$

Adair argued[14] that his prescription for C_L provides a natural explanation for the reverse Magnus effect observed by Briggs,[17] which occurs whenever the term in brackets is negative. For a smooth ball, it is well known[21] that at a critical value of Re a "drag crisis" occurs in which C_D decreases sharply $(dC_D/dv<0)$. Equation (3) predicts that a drag crisis might lead to a reverse Magnus effect. Whether such an effect occurs on a real baseball is not known. Equation (3) leads to a Magnus force in good agreement with the Watts and Ferrer data[18] at low speed, where C_D is relatively constant and equal to ≈0.5. For these conditions $C_L\approx S$, in good agreement with the Watts and Ferrer data[18] and Ref. 5. However, at the high speeds that are typical of pitched and hit baseballs, the differences between the two parametrizations are very pronounced, as shown in Fig. 3, where the

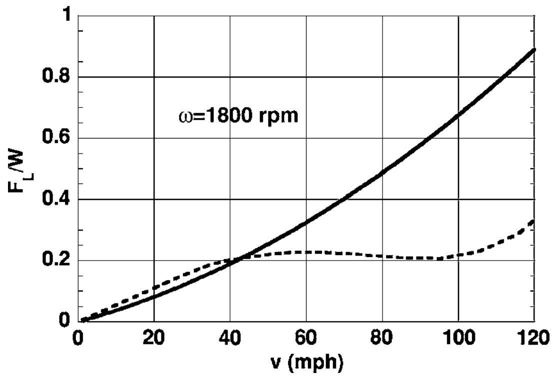

Fig. 3. Calculated ratio of the Magnus force to weight for $\omega=1800$ rpm. The solid and dashed curves utilize the parametrizations of Refs. 5 and 7, respectively, the latter essentially reproducing Fig. 2.2 of Adair (Ref. 7).

calculated ratio of the Magnus force to weight is plotted as a function of speed for fixed $\omega=1800$ rpm. Although the two curves agree quite well for $v<40$ mph, they diverge for larger speeds, and in the vicinity of $v=100$ mph ($S\approx 0.15$), the Magnus force found in Ref. 5 is about three times larger than that given by Eq. (3).

It is the region of v and S that is most relevant to the game of baseball, $v=75-100$ mph and $S=0.15-0.25$, where the disagreement is largest between the two parametrizations. The only data in this regime are the older wind tunnel data of Briggs,[17] which are subject to rather large corrections. Extending the data to over 100 mph is one of the primary motivations for the present experiment. A second motivation is to investigate the dependence of C_L on Re (or, equivalently, v) for fixed S. A third motivation is to address the question of whether Eq. (3) is a reasonable parametrization of the lift coefficient.

III. EXPERIMENT AND DATA REDUCTION

The experimental technique is to project an official Major League baseball (mass=0.145 kg, radius=36.4 mm) approximately horizontally with backspin or topspin and to use a motion analysis system to measure the initial velocity and angular velocity and to track the trajectory over approximately 5 m of flight. For these conditions the vertical motion is particularly sensitive to the Magnus force, which leads to a downward acceleration smaller or larger than g, the acceleration due to gravity, when the ball is projected with backspin or topspin, respectively. Additional information is obtained by measuring the total distance traversed by the baseball before hitting the floor, which is approximately 1.5 m below the initial height. The projection device was an ATEC two-wheel pitching machine,[22] in which the speed of the ball and the spin can be independently adjusted by varying the rotational speed of the two wheels. The geometry of the machine is such that the spin axis of the ball is constrained to be perpendicular to the velocity vector. A total of 22 pitches were analyzed, all in the "two-seam" orientation,[23] with speeds in the range $v=50-110$ mph and angular speeds in the range $\omega=1500-4500$ rpm. The corresponding ranges of the Reynolds number and spin factor were $\text{Re}=(1.1-2.4)\times 10^5$ and $S=0.090-0.595$, respectively.

To measure the initial velocity and angular velocity of the ball, a motion analysis system was used. The system consisted of ten Eagle-4 cameras operating at 700 frames per second and 1/2000 s shutter speed and the EVaRT4.0 reconstruction software.[24] Each camera shines infrared LED light onto the ball, attached to which is a circular dot of retro-reflective tape. The tape reflects the light back to the cameras, which records the coordinates of the dot in the CCD array. The reconstruction software determines the spatial coordinates in a global coordinate system by using triangulation among the ten cameras. The cameras were positioned at staggered heights and overlapping fields of view along a line approximately parallel to and 6 m from the line of flight of the ball. To accomplish the triangulation, the precise position and lens distortions of each camera were determined using the following calibration scheme. A global coordinate system is first defined by positioning an L-shaped rod in the viewing volume of the ten cameras simultaneously. The rod has four reflective dots located at precise relative locations to each other. With these distances, the software determines a first approximation to the location of each camera. These locations are further refined and the lens distortions determined by waving a wand throughout the tracking volume of the cameras. The wand has three reflective dots at known relative distances. Although the particular calibration software is proprietary, it almost surely uses some variation of the direct linear transformation technique.[25,26] A typical root-mean-square (rms) precision for tracking a single dot is 1−2 mm. Additional calibrations and consistency checks were performed. A plumb line was used to establish that the y-axis of the global coordinate system made an angle of 0.16° with the vertical. The clock of the motion capture system was checked against a precisely calibrated strobe light and found to be consistent within 0.5%. A nonspinning baseball was tossed lightly in the tracking volume and the vertical acceleration was measured to be g to within 1.5%.

The setup for this experiment is similar to that used in the pioneering experiment of Alaways,[8] but different in some key respects. One difference concerns the deployment of cameras. Alaways used two sets of motion capture cameras. One set tracked the ball over the first 1.2 m of flight and was used to establish the initial conditions; a second set tracked the center of mass trajectory over the last 4 m of flight, starting approximately 13 m downstream from the initial position. This arrangement allowed a very long "lever arm" over which to determine the acceleration but a short lever arm for determining the initial spin. In the present setup, only one set of cameras was used and distributed spatially so as to track over the largest distance possible, approximately 5 m. The same cameras determined both the initial conditions and the acceleration. Tracking over a larger distance is useful for measuring the spin for small S, because the angle through which the ball rotates over the distance D is proportional to SD. For all of the pitches analyzed in the current experiment, the ball completed at least one complete revolution over the tracking region. Some redundant information was obtained by measuring the total distance R traversed by the baseball while falling through a height of about 1.5 m. The second difference concerns the deployment of reflective markers. Alaways utilized four dots on the ball with accurately known relative positions, allowing the direction of the spin axis to be determined. In the present experiment, tracking more than one dot proved difficult and unreliable because the automatic tracking software would often confuse the dots when operating at high frame rates. Therefore only a single dot was used, offset from the spin axis by approximately 15 mm. Consequently, it was not possible to measure the spin axis, which

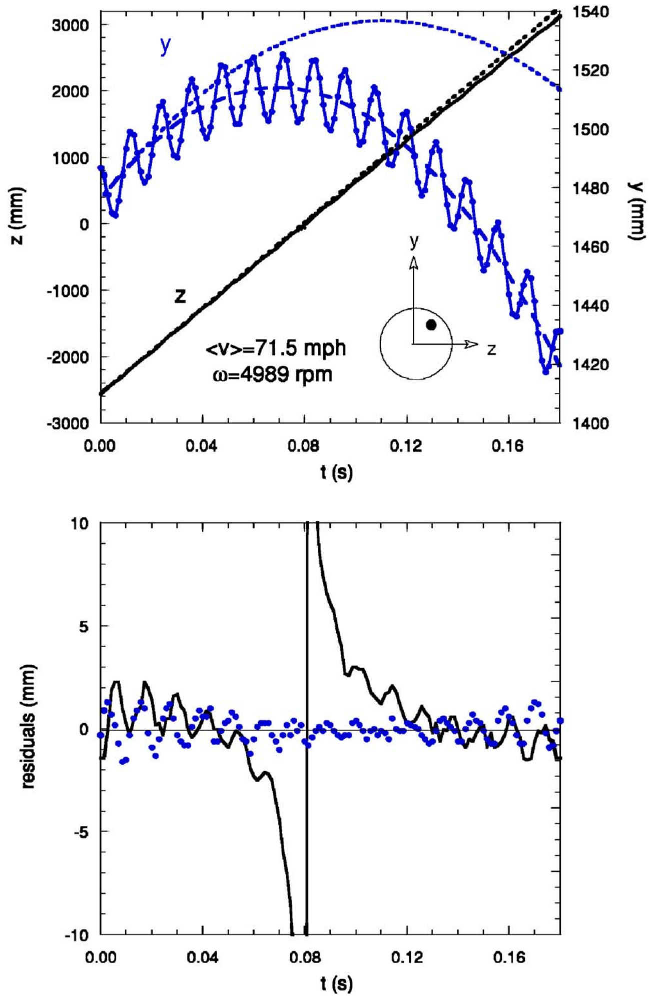

Fig. 4. Trajectory data (top) for one of the pitches, where y and z are the coordinates of the dot on the ball in the coordinate system shown in the inset. The ball is projected at a slight upward angle to the $+z$ direction and is spinning clockwise (topspin) about an axis perpendicular to the y-z plane. Solid curves are least-square fits to the data using Eq. (4b), resulting in $C_D=0.44$ and $C_L=0.33$. The long dashed curve is the center-of-mass trajectory for the y coordinate, which is consistent with a downward acceleration of $1.58g$ due to the combined effects of gravity and the Magnus force. The short dashed curves are the center-of-mass coordinates for both y and z with both C_D and C_L set to zero, indicating that the data are very sensitive to C_L but not to C_D. The fit residuals for the y (points) and z (curve) coordinates are shown in the bottom plot.

was assumed to be constrained by the pitching machine to lie in the horizontal plane, perpendicular to the direction of motion. Subsequent analysis of the data showed no horizontal deflection of the baseball, implying that the vertical component of the spin is consistent with zero. Finally, the present experiment operated at nearly three times the frame rate as that of Alaways,[8,9] allowing measurements over a wider range of Re and S.

A plot of $y(t)$ and $z(t)$ for one of the pitches is shown in Fig. 4; y and z are the vertical and horizontal coordinates, respectively, of the reflective marker. The motion in the y coordinate is interesting, because the oscillatory motion of the dot is clearly superimposed on a parabolic trajectory. The downward acceleration associated with the parabolic trajectory is due to the combined Magnus and gravitational forces. The motion in the z coordinate also has oscillatory motion superimposed on the parabolic motion, but it is very difficult to discern from the plot because of the large z component of the velocity. The quantities C_L and C_D are largely decoupled from each other and are determined mainly by the acceleration in the y and z directions, respectively.

The Levenberg-Marquardt nonlinear least-squares fitting algorithm[27] was used to fit these data to functions of the form

$$y(t) = y_{\text{cm}}(t) + A\sin(\omega t + \phi), \tag{4a}$$

$$z(t) = z_{\text{cm}}(t) \pm A\cos(\omega t + \phi), \tag{4b}$$

where the first term on the right-hand side is the center-of-mass coordinate of the ball and the second term is the location of the rotating dot with respect to the center of mass. The center-of-mass coordinates are calculated by numerically integrating the equations of motion using the fourth-order Runge-Kutta method, given the initial components of the center of mass position and velocity (y_{cm}, z_{cm}, $v_{y,\text{cm}}$, and $v_{z,\text{cm}}$) and the lift and drag coefficients C_L and C_D. The rotation of the dot is characterized as an oscillation with amplitude A, angular frequency ω, and initial phase ϕ, with the $\pm$ signs for backspin or topspin, respectively. The fitting procedure was used to determine nine parameters: the four initial center-of-mass values, the three rotation parameters, and C_L and C_D, the latter assumed to be constant over the 5 m flight path. Depending on the initial speed, each fit had 100–300 data points, including the range R. This procedure is similar to that used by Alaways.[8]

The curves in Fig. 4 are the results of the fit to the data. Residuals for the fit are also shown in Fig. 4. The oscillations in the residuals are out of phase with the primary motion by 90°, suggesting that the spin axis might be tilted slightly relative to the normal to the y-z plane. The rms deviation of the fit from the data in Fig. 4 is 0.4 mm for $y(t)$ and 12.7 mm for $z(t)$. These values are typical of the fits for the other pitches.

The method used to obtain C_L and C_D was checked using an alternate simplified fitting procedure that avoids a numerical solution of the equations of motion by assuming constant acceleration a_y and a_z for the two center-of-mass coordinates. The fitting procedure determined the initial position, velocity, and acceleration for each of the two center-of-mass coordinates, in addition to A, ω, and ϕ. The values of the lift and drag coefficients were derived by assuming that a_y is due to the combined effects of the Magnus and gravitational forces and a_z is due to the drag. These assumptions should be reasonably well satisfied for the nearly horizontal trajectories in this study. The simplified procedure results in values of C_D and C_L that are within 10% of those obtained by the more elaborate calculation, giving us confidence in the accuracy of the latter procedure.

The inferred values of C_L, which represent the main results of our experiment, are presented in Figs. 5 and 6. The error bars on the values are estimates based on the rms deviation of the fits from the data, the accuracy of the calibration procedure, and the difference between the two fitting techniques. Results for C_D are not presented. There is considerably more scatter in our results for C_D than for C_L, reflecting the generally larger rms deviation of the fit for z than for y. It is easy to see from Fig. 4 that our technique is much more sensitive to C_L, which is determined by the cur-

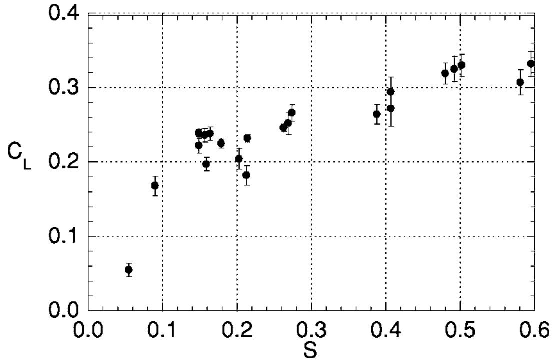

Fig. 5. Results for C_L from the present motion capture experiment.

vature of $y(t)$, than to C_D, which is determined by the curvature of $z(t)$; the latter is nearly completely masked by the large linear term.

IV. RESULTS AND DISCUSSION

A. Results for C_L

Our results for C_L are plotted in Fig. 5 as a function of the spin factor S. The C_L values rise sharply for small S, then lie on a smooth curve for S larger than about 0.10, increasing approximately linearly with S. Figure 6 shows whether there is a dependence of C_L on Re for S approximately constant by showing the dependence of C_L on v for S in the range 0.15–0.25. It is clear from the plot that C_L has no strong dependence on v over the range 50–110 mph, corresponding to Re in the range $(1.1-2.4)\times 10^5$. This result is one of the primary conclusions of this work.

The present results are compared with previous data in Fig. 2, along with the parametrization in Ref. 5. For $S \lesssim 0.3$, which is the region most relevant for the game of baseball, the present results are in excellent agreement with the Alaways data[8,9] and the parametrization of Ref. 5, although the values of Jinji[11] are about 20% larger. The trend of both the present and the Alaways data,[8,9] extrapolated to larger S, fall below the Watts and Ferrer[18] points. Also shown is the parametrization of Eq. (3), calculated at 100 mph using C_D values from Adair.[7] In the vicinity of $S=0.17$, corresponding to a backspin of 2000 rpm, this curve falls well below the

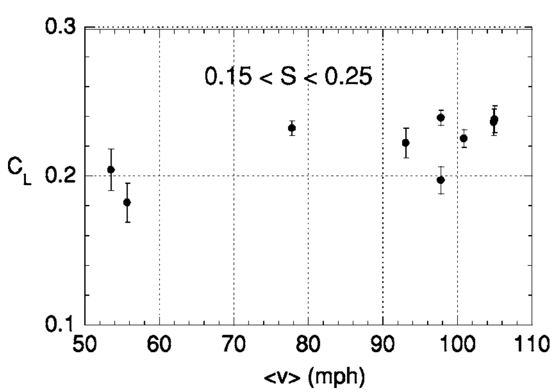

Fig. 6. Results for C_L from the present experiment for S in the range 0.15–0.25, demonstrating that C_L does not depend strongly on v (or Re) for fixed values of S.

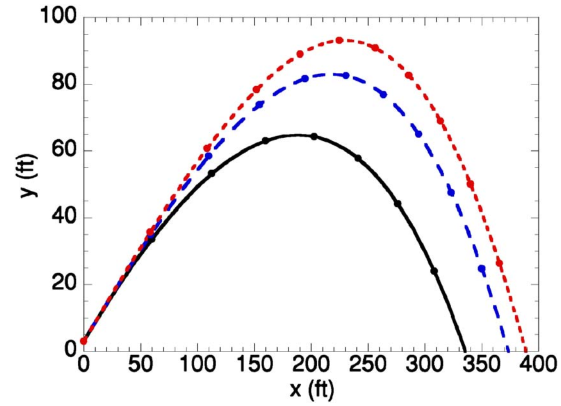

Fig. 7. Calculated trajectories of a hit baseball with an initial speed of 100 mph, angle of 30°, height of 3 ft, and backspin of 0 rpm (solid), 1000 rpm (long-dashed), and 2000 rpm (short-dashed). The points indicate the location of the ball in 0.5 s intervals. These calculations utilize the C_D values of Adair (Ref. 7) and the C_L values from the parametrization of Sawicki *et al.* (Ref. 5).

data. We find that the present data do not support the parametrization of Eq. (3), another conclusion of this study.

B. Implications for the flight of a baseball

In Fig. 7 a calculation is shown of the trajectory of a typical hit baseball as a function of the initial backspin. For the particular choice of initial conditions, increasing the backspin from 1000 to 2000 rpm has the expected effect of keeping the ball in the air longer, increasing the maximum height, and increasing the total distance. In Fig. 8 a calculation is shown of the range of a fly ball as a function of the initial angle for differing amounts of backspin. Again the expected results are observed, the larger backspins resulting in a smaller optimum initial angle for the maximum range. Plots with similar quantitative results were shown by Watts and Bahill.[6] The deflection of a pitched baseball due to the Magnus force is presented in Table I for speeds and spins typical of fastballs (90 mph) and curveballs (75 mph). The total deflections are in accord with experimental measurements.[11,28]

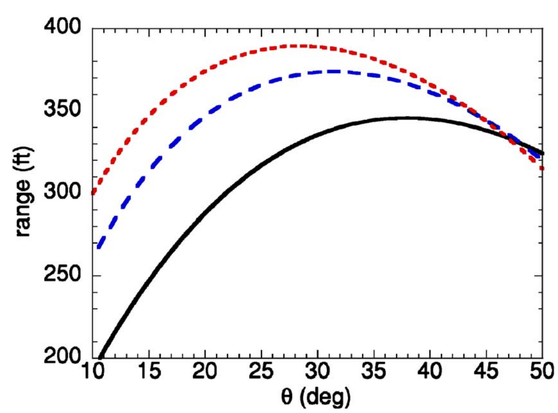

Fig. 8. Calculated range of a hit baseball with an initial speed of 100 mph, height 3 ft, and backspin of 0 rpm (solid), 1000 rpm (short-dashed), and 2000 rpm (long-dashed), as a function of the initial angle θ. These calculations utilize the C_D values of Adair (Ref. 7) and the C_L values from the parametrization of Sawicki *et al.* (Ref. 5).

Table I. Calculated deflection d of a pitched baseball thrown with an initial horizontal velocity v, spin ω, and spin factor S after traversing 55 ft. These calculations utilize the C_D values of Adair (Ref. 7) and the C_L values from the parametrization of Sawicki *et al.* (Ref. 5). These deflections are in accord with experimental observations (Refs. 11 and 28).

v (mph)	ω (rpm)	S	d (in.)
75	1000	0.11	16
75	1800	0.20	21
90	1000	0.09	14
90	1800	0.17	19

V. SUMMARY AND CONCLUSIONS

An experiment has been performed utilizing high-speed motion analysis to determine the effect of spin on the trajectory of a baseball. From these data, values for C_L over the range $50 < v < 110$ mph and $0.1 < S < 0.6$ are determined. These values are in excellent agreement with those of Alaways[8,9] in regions where they overlap. They are the only precise values for C_L in the region most relevant to the game of baseball, namely $0.1 < S < 0.3$ and $80 < v < 110$ mph. The parametrization of Ref. 5 is found to give an excellent description of the data in this regime. Moreover, C_L is found not to depend strongly on v between 50 and 100 mph, for fixed values of S in the range 0.15–0.25, contradicting expectations based on the model of Adair.[7]

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